PROCEEDINGS OF SCIENCE

Improved method of searching for flares of neutral particles from point sources

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Certain classes of astrophysical objects produce flares that might also generate ultra-high-energy particles. If some of these particles are neutral, they could be detected in groups that would be closely correlated in both arrival direction and time. Finding such clusters in cosmic ray data would be an evidence for the existence of neutral ultra-high-energy particles, and could help determine their origins. We present the so-called stacking method to search for these space-time clustering of ultra-high-energy air showers. This method combines a time-clustering analysis with an unbinned likelihood approach. In the case of photon-induced events, background events (initiated by hadrons) can be more efficiently eliminated by using a photon tag that employs probability distribution functions to categorize each event as more likely to be either a photon or a hadron. Monte Carlo tests show that this method effectively separates photon-initiated events from hadron-initiated ones and can accurately reproduce both the number of photon events and the duration of the flare(s). With the photon tag, the stacking method can detect photon flares with only a few signal events. This approach can help identify cosmic ray sources and improve limits on the fluxes of ultra-high-energy photons.

7th International Symposium on Ultra High Energy Cosmic Rays (UHECR2024)
17-21 November 2024
Malargüe, Mendoza, Argentina

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1. Introduction

Certain cosmic phenomena, particularly active galactic nuclei (AGN) flares or gamma-ray bursts, are potential sources of ultra-high-energy cosmic rays (UHECRs). These particles may form clusters in the data obtained by cosmic ray observatories, exhibiting correlations in their arrival directions and times. Such space-time clustering should be especially noticeable for neutral particles like photons. Unlike charged particles, photons are not deflected by cosmic magnetic fields, maintaining straight-line trajectories from their sources to Earth. This property makes them ideal for source identification: we expect that their directions would point to their sources with the accuracy of the angular resolution of the detector systems. If such clusters could be identified in the data – sets of particles arriving closely together in both time and direction – it would indicate the presence of ultra-high-energy (UHE) photons, defined as photons with energies exceeding 10^{17} eV.

In this paper, we propose an improved algorithm for finding the space-time clusters in UHECR data. The method builds on the standard unbinned likelihood method [1] and is designed to expand it to search for multiple weak flares from point sources of UHE photons. This so-called stacking method has already been used in studying correlations between arrival directions of neutrinos detected at the IceCube Observatory and the directions of their potential sources [2]. Monte Carlo tests demonstrate that the improved method effectively identifies time-space-correlated UHECR clusters. This approach can advance both the identification of cosmic ray sources and the determination of flux limits for ultra-high-energy neutral particles.

2. The stacking space-time clustering method

Building upon the standard unbinned likelihood approach [1], we propose an improved method for space-time clustering analysis [2, 4]. By employing stacking analysis of doublets (pairs of consecutive events) rather than multiplets (multiple consecutive events) as in the standard approach, the proposed method achieves significantly faster computational performance. Additionally, the improved method can detect faint multiple flares of arbitrary shapes from point sources. In this framework, identifying a point source of photons (a flaring event) means detecting a statistically significant cluster of space-time correlated events from a specific direction in the sky. This so-called stacking method consists of 3 main steps. Before describing these steps in details, we define the signal and background probability distribution functions (PDFs), which are essential to our analysis.

For an event i, the signal PDF (s_i) and background PDF (b_i) are each defined as products of their spatial and temporal components, i.e. $s_i = s_i^{\text{space}} s_i^{\text{time}}$ and $b_i = b_i^{\text{space}} b_i^{\text{time}}$. The spatial probability s_i^{space} is a Gaussian function

$$s_i^{\text{space}} = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{|\vec{r}_i - \vec{r}_s|^2}{2\sigma_i^2}\right), \tag{1}$$

where σ_i is the angular reconstruction uncertainty of considered event and $\vec{r}_i$ its direction, respectively. The source direction is given by $\vec{r}_s$. For each tested doublet time window $\Delta t_j = t_j^{\text{max}} - t_j^{\text{min}}$, a temporal signal PDF is given by

$$s_i^{time} = \frac{H\left(t_j^{max} - t_i\right) H\left(t_i - t_j^{min}\right)}{\Delta t_j}, \tag{2}$$

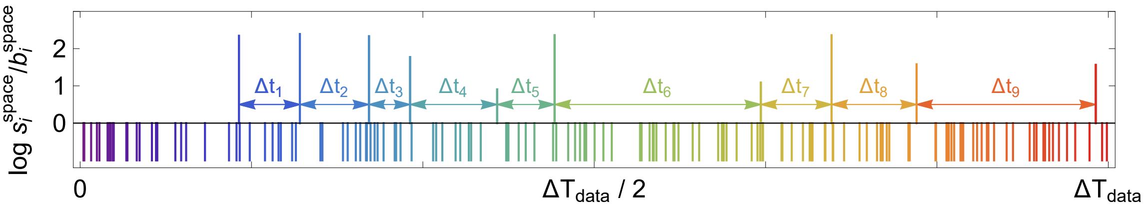

Figure 1: Visualization of the timeline of the observation period ΔT_{data}, where each event is represented by a vertical line. Time of events is color coded: purple means earliest time and red latest time. The horizontal black line at $\log(s_i^{\mathrm{space}}/b_i^{\mathrm{space}}) = 0$ (S/B threshold equal to 1) separates signal from background events. Consecutive signal-like events form doublets, and their temporal separations are labeled as Δt_j.

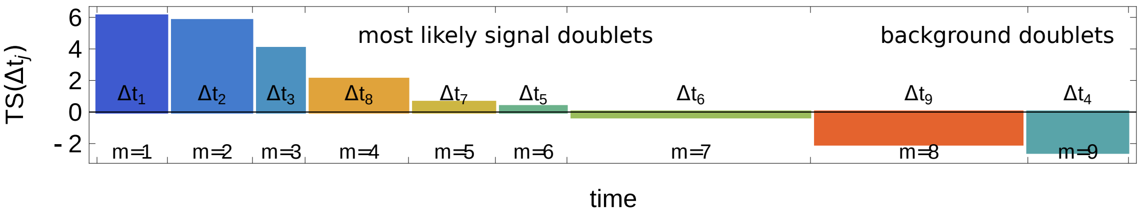

Figure 2: Doublets re-arranged in the order of decreasing test statistics $\mathrm{TS}_{\Delta t_j}$. Each doublet is shown as a colored box, where its width and color represent the length and position of the doublet in the timeline, respectively. The doublets are given new numbers (multiplicity index m) based on their significance, rather than the time of observation.

where H is the Heaviside step function and t_i is the arrival time of event. Under this definition, only events within the considered time window Δt_j are counted, with $s_i = 0$ for all events outside this interval.

Background PDF can be calculated purely from data. In general, due to analysis cuts, Earth absorption effects and the detector geometry, it may depend on zenith and azimuth angles. Here, because we assume uniform background distribution (both in space and time), we define the spatial background PDF using solely the total solid angle $\Delta\Omega$ subtended by the considered part of the sky. Similarly, to calculate the temporal background PDF we use only the length of the entire data taking period, or more precisely, the uptime ΔT_{data}. The resulting spatial and temporal background PDFs are $b_i^{\mathrm{space}} = 1/\Delta\Omega$ and $b_i^{\mathrm{time}} = 1/\Delta T_{\mathrm{data}}$. Note that additional observable quantities, such as energy, can also be incorporated into the PDFs for enhancing the ability of the method to differentiate the clustering of astrophysical UHECR signals from the background noise. For simplicity, we neglect energy information. This may be reinstated in the final analysis.

The initial step of the stacking method involves selecting potential flare candidates (signal-like events) from all of the events numbered by index i. The selection criterion is a ratio between their spatial signal PDF and the spatial background PDF, which must be larger than an adjustable threshold S/B, i.e. $s_i^{\mathrm{space}}/b_i^{\mathrm{space}} > \mathrm{S/B}$. Next we extract all doublets of consecutive signal-like events to identify all possible time windows Δt_j that compose the flares contribution. This procedure is illustrated in Figure 1. Events collected over the time period ΔT_{data} are represented by vertical lines, with colors indicating their timing: purple for earlier times and red for later times. Signal-like events are shown as lines extending above the black horizontal line, which represents $\log(s_i^{\mathrm{space}}/b_i^{\mathrm{space}}) = 0$ (where we set $\mathrm{S/B} = 1$). Lines below this threshold represent background events. The time intervals between consecutive signal-like events (consecutive doublet time windows) are denoted as Δt_1, Δt_2, Δt_3, and so on.

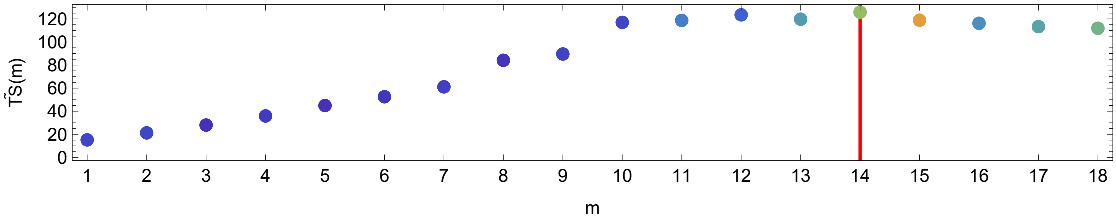

Figure 3: Graph showing the maximum values of the test statistic $\widetilde{\mathrm{TS}}$, which are obtained assuming that the flare consists of m most significant doublets. As in the previous plots, the color indicates the time of observation. In this single flare example, the test statistic peaks when 14 most significant doublets are considered, thus optimal value of multiplicity is $M_{\mathrm{opt}} = 14$. The graph shows a distinctive single-step profile: the test statistic rises with the number of doublets included, and then it levels off at a plateau.

In the second step, for each signal-like doublet, we calculate its significance by maximizing the corresponding test statistic with respect to the number of signal events n. We use the test statistic with addition of a marginalization term $\Delta T_{\mathrm{data}}/\Delta t_j$ to ensure more uniform sensitivity for finding doublets of different time widths [2], i.e.

$$\mathrm{TS}_{\Delta t_j}(n) = -2\log\left[\frac{\Delta T_{\mathrm{data}}}{\Delta t_j}\frac{\mathcal{L}(0,\Delta t_j,\vec{r}_s)}{\mathcal{L}(n,\Delta t_j,\vec{r}_s)}\right], \tag{3}$$

where $\mathcal{L}$ denotes the likelihood. To evaluate the likelihood, we combine the signal and background PDFs for all observed events (numbered by index i), i.e.

$$\mathcal{L}(n,\Delta t_j,\vec{r}_s) = \prod_{i=1}^{N}\left(\frac{n}{N}s_i + (1-\frac{n}{N})b_i\right), \tag{4}$$

where N is the total number of events in the data sample and n is the assumed number of signal events. A background-only likelihood $\mathcal{L}(0,\Delta t_j,\vec{r_s})$, representing the absence of signal events, can also be calculated using this equation. By construction of the signal PDF s_i (see equation 2), only events within the time window of the considered doublet Δt_j contribute to the likelihood, thus in this step complete space-time information is used.

All doublets are then sorted and indexed (re-numbered introducing multiplicity index m) by their significance, which is determined by the maximum value of $\mathrm{TS}_{\Delta t_j}(n)$, denoted as $\mathrm{TS}_{\Delta t_j}$. This procedure is illustrated in Figure 2, where doublets are represented by colored boxes sorted by their maximum test statistic values. The most significant doublet has multiplicity index $m = 1$. The width of each box represents the time window of the corresponding doublet, while the color indicates its temporal position. Doublets on the left side are the most likely signal candidates, while those on the right are more likely to be background events. The key challenge is determining the optimal multiplicity index M_{opt} that effectively separates signal from background doublets. Once M_{opt} is determined, we can infer that flare or flares likely comprise M_{opt} doublets, ranging from $m = 1$ to $m = M_{\mathrm{opt}}$. The total flare duration ΔT is then calculated as the sum of time windows corresponding to these most probable signal doublets, i.e. $\Delta T_{\mathrm{flare}} = \sum_{m=1}^{M_{\mathrm{opt}}}\Delta T_m$. The value M_{opt} is searched for in the last step of the method we are presenting.

The third and final step of the proposed method is application of the stacking analysis to find M_{opt}. One-event signal probability s_i is replaced by the weighted sum of signal sub-terms over m

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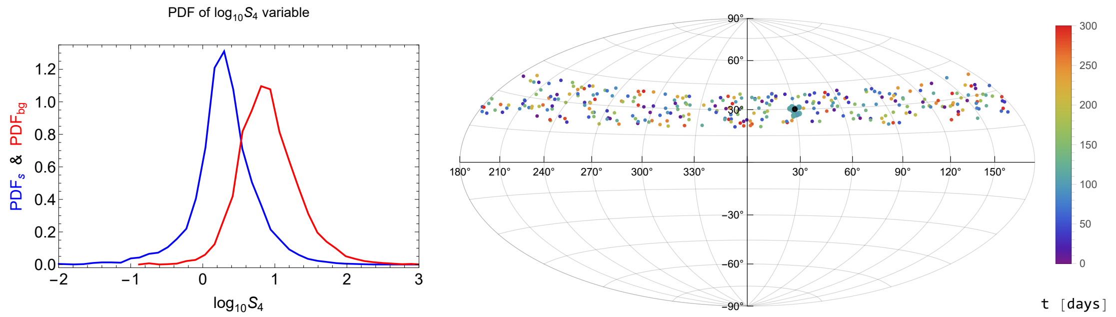

Figure 4: (**Left**) Probability distribution functions of $\log_{10} S_4$, obtained from simulations of air showers initiated by two types of primaries: photons (considered as the signal, blue) and protons (background, red) [5]. (**Right**) An example of a scrambled sky map. To speed up calculations, we consider only a narrow band of declination, just wide enough to include all the signal events around the source, instead of covering the entire sky. The position of the source is marked by a black dot, while the events are presented as colored dots. The larger dots close to the source represent signal events produced by a flare. The colors of the dots indicate observation time of each event.

doublets, with weights $w_j = \mathrm{TS}_{\Delta \mathrm{t_j}}$. Specifically, we make the replacement:

$$s_i \longrightarrow s_i^{\mathrm{tot}}(m) = \sum_{j=1}^{m} w_j s_i^j \Big/ \sum_{j=1}^{m} w_j, \tag{5}$$

where s_i^j is the signal probability of the i-th event calculated for the j-th doublet ($s_i^j = 0$ if the event is outside the time window of the considered doublet). We then employ the standard likelihood and test statistic with the stacking term $s_i^{\mathrm{tot}}(m)$, thus making the following replacements: $\mathcal{L}(n) \to \mathcal{L}(n, m)$ and $\mathrm{TS(n)} \to \widetilde{\mathrm{TS}}\,(\mathrm{n, m}) = -2\log\left[\mathcal{L}(0)/\mathcal{L}(\mathrm{n, m})\right]$. Finally, we maximize $\widetilde{\mathrm{TS}}\,(n, m)$ to find the optimal value of multiplicity of flare(s) $\mathrm{M_{opt}}$, for which our test statistic reaches a maximum. This procedure is shown in Figure 3 for a single flare scenario. $\mathrm{M_{opt}}$ is the number of the most significant doublets, which are not necessarily consecutive. This feature allows the method to detect multiple flares. As mentioned before, $\mathrm{M_{opt}}$ defines the total flare(s) duration through $\Delta T_{\mathrm{flare}} = \sum_{m=1}^{M_{\mathrm{opt}}} \Delta T_m$. We estimate the number of signal events n_s by maximizing the final test statistic $\widetilde{\mathrm{TS}}\,(n, \mathrm{M_{opt}})$ with respect to n. The statistical significance of the obtained results is assessed by comparing value of the maximum test statistic with a distribution of maximum values of the test statistic derived from many randomly generated scrambled maps mimicking the original dataset. Note that by construction, n_s is a real number since the test statistic is maximized over the full range of real numbers, not over a discrete set of integers. However, we might expect integers to occur more frequently, thus forming multi-peak distributions of n_s.

3. S_b photon tag

The S_b variable, commonly used to distinguish between photon-initiated and hadron-initiated showers, is defined as $S_b = \sum_k S_k\,(R_k/1000\ \mathrm{m})^b$ [3, 5]. Here, the summation runs over all detectors with non-zero signals S_k, and R_k represents the distance of the k-th detector from the shower axis. While the parameter b is somewhat flexible, we use $b = 4$, which is a typical choice for Auger photon searches. To improve capability of the stacking method for photon search, we apply a

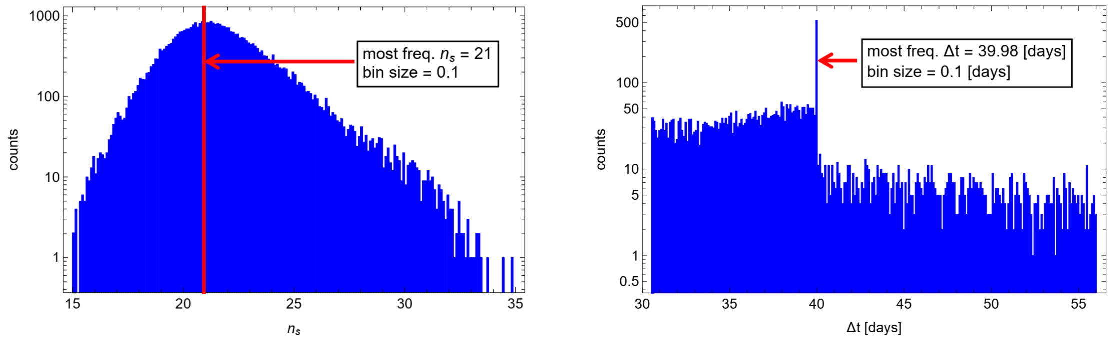

Figure 5: Distributions of n_s (**Left**) and Δt (**Right**) obtained from Monte Carlo simulations with a triple flare (consisting of 3 separated flares: 10, 10, and 20 days long) with $N_s = 20$ signal events in total. Both plots include a box indicating the most common value – the highest peak – in their distributions. The number of injected signal events $N_s = 20$ and total duration of all parts of the triple flare $\Delta T_{\rm flare} = 40$ days are well recovered. The distribution of the reconstructed values of the Δt shows higher counts at bins with $\Delta t < 40$ days, which shows that the stacking method remains more sensitive to shorter flares, even with a marginalization term $-2\log\left(\Delta T_{\rm data}/\Delta t_j\right)$ included in the test statistic (see equation 3).

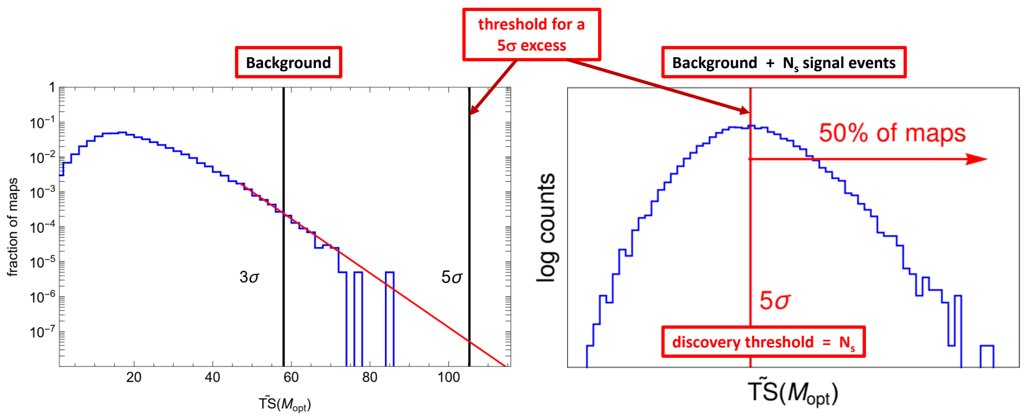

Figure 6: (**Left**) Probability distribution of the global test statistic $\tilde{\rm TS}\,(\rm M_{opt})$ for background-only simulations. The red line is an exponential fit to the distribution used to obtain the threshold for 5σ excess over the background, shown as a vertical black line. (**Right**) Distributions of the test statistic $\tilde{\rm TS}\,(\rm M_{opt})$ for background with N_s signals injected. Discovery threshold is equal to N_s when the median of the distribution equals the 5σ threshold.

photon tag using probability density functions (PDFs) that assess whether events are more likely to be photon-initiated or background. Specifically, we use PDFs of $\log_{10} S_4$ derived from simulations of photon-initiated (signal; $\rm PDF_s$) and proton-initiated (background; $\rm PDF_{bg}$) showers [5], see the blue and red curves in the left panel of Figure 4, respectively. To implement the S_4 photon tag, we apply the transformations: $s_i^{\rm space} \to s_i^{\rm space}\rm PDF_s(S_4^i)$ and $b_i^{\rm space} \to b_i^{\rm space}\rm PDF_{bg}(S_4^i)$, where S_4^i represents the S_4 value for the i-th event.

4. Monte Carlo test

To validate the stacking method, we generated a large number of sky maps containing both signal and background events. Background events were distributed uniformly across both the sky

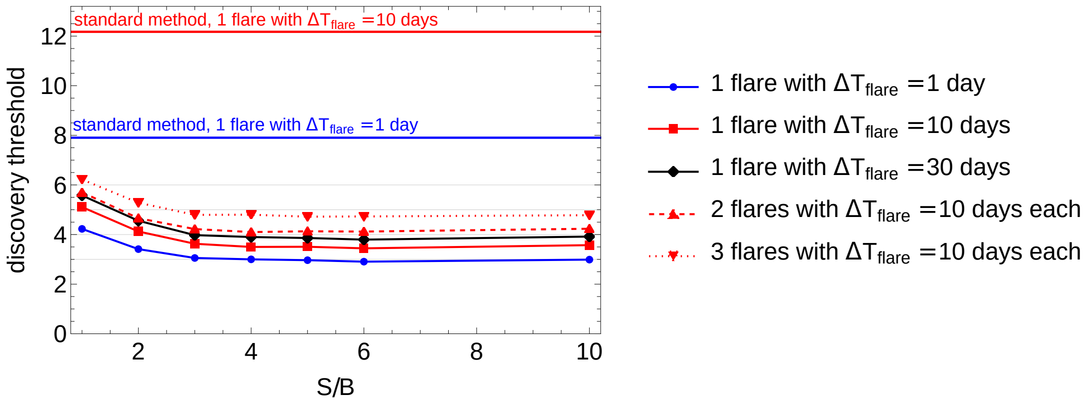

Figure 7: Discovery thresholds for the stacking method using the S_4 photon tag are presented as a function of the S/B threshold. Results are shown for both single and multiple flares of varying lengths. As can be seen, fewer events are needed to detect flares as the S/B threshold increases. These results are compared with the standard unbinned likelihood method without photon tag [1]. In this case, no pre-selection S/B threshold is applied. Shown are two scenarios: a one-day flare (blue horizontal line) and a ten-day flare (red horizontal line). All the results have been obtained assuming $\Delta T_{\rm data}$ = 3150 days and 595 background events.

and the observation period $\Delta T_{\rm data}$. For signal events, we positioned N_s events using a Gaussian distribution centered on the source, and their corresponding times were distributed uniformly over the specified flare duration $\Delta T_{\rm flare}$. The start time of each flare was also selected randomly. An example of such a map is presented in right panel of Figure 4. For each map, we obtain three key parameters: an estimate of the number of signal events (n_s), estimated time of the flare duration (Δt), and the maximum value of the test statistic ($\widetilde{\rm TS}({\rm M_{opt}})$) that measures the statistical significance of the results.

To evaluate how well the stacking method works, we ran various Monte Carlo simulations testing the ability of the method to detect flares and accurately measure both the number of events and flare durations. Figure 5 presents results from one such test, where we simulated a triple flare consisting of one 20-day flare and two 10-day flares, with a total of 20 signal events. The plots show distributions of values obtained for different random sky maps, with boxes indicating the most frequent value (the highest peak) – these peak values serve as the best estimates of the actual parameters. The results show that the method successfully recovered both the number of signal events and the total 40-day duration of the flares.

5. Discovery potential

The discovery potential quantifies the number of signal events required to claim discovery of a cluster of events in data. It serves as a useful metric for comparing different analysis methods. By definition, the discovery threshold is the number of signal events needed to achieve a p-value below 2.87×10^{-7} (one-sided 5σ) in 50% of the analyzed sky maps. To determine this threshold, we first generate many scrambled sky maps to obtain the probability distribution of the global test statistic $\widetilde{\rm TS}\,({\rm M_{opt}})$ for background-only simulations. An exponential fit is then applied to the tail of this distribution to identify the test statistic value corresponding to the 5σ excess threshold, see left panel of Figure 6. Next, we examine the distributions of $\widetilde{\rm TS}\,({\rm M_{opt}})$ for scenarios where different numbers of signal events are injected into the background. By comparing the median of the test statistic distribution with the 5σ threshold for the background, we determine whether the number of injected signal events is below or above the discovery threshold, see right panel of Figure 6. Discovery threshold must be determined separately for single and multiple flares of varying durations.

Figure 7 presents the obtained discovery thresholds for the stacking method with the S_4 photon tag as a function of the S/B threshold for signal-like events. As this threshold increases, fewer events are needed to detect flare(s). For comparison, discovery thresholds for standard unbinned likelihood method without a pre-selection S/B threshold and without a photon tag [1] are also shown. In this case, more signal events are required to claim discovery, especially for longer flares.

6. Summary

We proposed an improved method, called the stacking method, to investigate space-time clustering in UHECR (air-shower) data. This method could potentially provide evidence of ultra-high-energy (UHE) photons that may be produced by flares in astrophysical sources. Additionally, it can help refine the limits on UHE photon flux and support the search for sources of neutral UHECRs. The stacking method improves the existing techniques in several respects. It is computationally faster, more efficient at detecting weak flares regardless of their temporal shape, and capable of identifying multiple flares from one source. It can accurately reproduce both the total number of signal events and the duration of the flare(s). Most notably, when combined with the S_4 photon tag, the method needs remarkably few events to detect photon flare(s).

7. Acknowledgments

The authors would like to thank the colleagues from the Pierre Auger Collaboration for all the fruitful discussions. We want to acknowledge the support in Poland from the National Science Centre, grants No. 2020/39/B/ST9 /01398 and 2022/45/B/ST9/02163 as well as from the Ministry of Science and Higher Education, grant No. 2022/WK/12.

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