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Discrete Leptonic Flavor Symmetries in the SMEFT

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Standard Model effective field theory (SMEFT) serves as a powerful and rigorous framework for systematically characterizing deviations from the Standard Model. However, due to its model-independent nature, there is an inevitable trade-off resulting in a significant increase in the number of independent parameters. We discuss the charting of the SMEFT parameter space by incorporating a range of flavor assumptions based on the global continuous symmetry groups. The interplay between the flavor symmetries and the UV mediators, yielding valuable phenomenological insights, is outlined accordingly. Expanding upon this analysis, we extend the list of flavor symmetries in the leptonic sector by adding three well-motivated discrete groups, namely A_4, A_5, and S_4. The analysis of the relevant UV mediators under these flavor assumptions is performed, which includes the extraction of flavor tensors as well as the derivation of SMEFT matching relations. Particular emphasis is placed on the leptonic directions, for which a comprehensive phenomenological analysis is conducted.

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1. Introduction

In the absence of direct evidence for new physics (NP) states, which suggests a clear separation between the electroweak (EW) scale and the scale associated with NP mediators, it becomes crucial to establish a robust theoretical framework to systematically characterize deviations from the Standard Model (SM). This framework is provided by the Standard Model Effective Field Theory (SMEFT) [1–3]. The SMEFT Lagrangian is expressed as an infinite series of local higher-dimensional operators with canonical mass dimensions greater than four. These operators are constructed using SM fields, ensuring gauge and Poincaré symmetry. Wilson coefficient (WC) is associated to each operator, which encapsulates short-distance effects and remains independent of specific NP models, offering a versatile, model-independent framework for diverse phenomenological applications. A defining feature of the SMEFT Lagrangian is that, at each order in the expansion in inverse powers of the cutoff scale, only a finite number of independent operators with a given canonical mass dimension can be constructed. However, while each order is characterized by a limited set of higher-dimensional operators, the number of independent operators grows rapidly with increasing mass dimension. This growth arises not only from the greater variety of operator structures permitted by gauge and Poincaré symmetries but also, more significantly, from the presence of three generations (flavors) of fermionic fields. For instance, considering baryon-number-conserving operators at dimension six, the number of independent parameters is 59 with a single active flavor, whereas when all three flavors are active, this number escalates to 2499. Here, our goal is to investigate how this number is affected in presence of flavor symmetries and, furthermore, study the interplay of flavor symmetries and the UV mediators.

2. Flavor structure of the SMEFT

At the level of the Standard Model (SM), and in the absence of Yukawa interactions, the kinetic Lagrangian for the five distinct fermionic gauge representations exhibits invariance under a $U(3)^5$ flavor symmetry. However, the introduction of Yukawa interactions breaks this large global symmetry, reducing it to the global baryon number and three individual lepton numbers. Despite this symmetry breaking, empirical evidence from observed quark masses and mixing angles continues to suggest the presence of an approximate flavor symmetry, which can serve as a framework for interpreting the observational data. Furthermore, from a Beyond the Standard Model (BSM) perspective, precise flavor experiments impose stringent constraints on new physics (NP) [4]. To ensure that NP remains within the reach of current or future experiments (not far from the TeV scale), approximate flavor symmetries must not be significantly violated.

In what follows, we provide the list of some viable flavor symmetries and outline the procedure how the flavor parameter counting is performed [5]. Starting from the $U(3)^5$ global flavor symmetry group

$$\mathcal{G}_F = U(3)^5 \equiv U(3)_q \times U(3)_u \times U(3)_d \times U(3)_\ell \times U(3)_e \,, \tag{1}$$

we construct the list of the viable flavor subgroups, both for the quark (Q) and lepton (L) sector separately, which we take into account in the context of our subsequent parameter-counting exercise:

$$\begin{aligned}
\mathcal{G}_F^{(Q)} &\to \left\{U(3)^3,\, U(2)^2 \times U(3)_d,\, U(2)^3 \times U(1)_{d_3},\, U(2)^3\right\} \,, \\
\mathcal{G}_F^{(L)} &\to \left\{U(3)^2,\, U(3)_V,\, U(2)^2 \times U(1)^2,\, U(2)^2,\, U(2)_V,\, U(1)^6,\, U(1)^3\right\} \,.
\end{aligned} \tag{2}$$

We note that the list of the flavor subgroups given by Eq. (2) is not exhaustive and that there are more viable options to be considered (see Sec. 4 for the discussion involving discrete flavor symmetries).

Once the flavor symmetry subgroups have been specified, the next step involves associating the flavor irreps to the SM fermions, determining the set of spurions for each of the subgroups such that the Yukawa matrices and CKM mixing patters are reproduced as well as constructing the SMEFT invariants at different orders in spurion expansion.[1]

To illustrate the steps of the procedure outlined above, we analyze a specific example involving $\mathcal{G}_F^{(Q)} = U(3)^3 \equiv U(3)_q \times U(3)_u \times U(3)_d$ flavor symmetry in the quark sector. The SM quark fields are then taken to transform as

$$q^i \sim (\mathbf{3},\mathbf{1},\mathbf{1}) \equiv \mathbf{3}_q\,, \qquad u^i \sim (\mathbf{1},\mathbf{3},\mathbf{1}) \equiv \mathbf{3}_u\,, \qquad d^i \sim (\mathbf{1},\mathbf{1},\mathbf{3}) \equiv \mathbf{3}_d\,. \tag{3}$$

Furthermore, since the Yukawa couplings in the Lagrangian

$$-\mathcal{L}_Y \supset H\bar{q}^i Y_d^{ij} d^j + \tilde{H}\bar{q}^i Y_u^{ij} u^j + \text{h.c.} \tag{4}$$

break the flavor symmetry, they are promoted into spurions in order to make the Lagrangian in Eq. (4) formally invariant under the flavor symmetry:

$$Y_u^{ij} \sim (\mathbf{3},\bar{\mathbf{3}},\mathbf{1}) \equiv (\mathbf{3}_q,\bar{\mathbf{3}}_u)\,, \qquad Y_d^{ij} \sim (\mathbf{3},\mathbf{1},\bar{\mathbf{3}}) \equiv (\mathbf{3}_q,\bar{\mathbf{3}}_d)\,. \tag{5}$$

Rotating q and u fields in flavor space in order to bring the Y_u matrix into the diagonal form $\hat{Y}_u$ and using the rotation of the d field in order to partially diagonalize Y_d, the flavor symmetry breaking pattern can be denoted in two steps as

$$\begin{aligned}
&\text{Step I:} \quad && Y_u \to \hat{Y}_u \quad && \Longrightarrow \quad && U(3)_q \times U(3)_u \to U(1)^3_{q+u}\,,\\
&\text{Step II:} \quad && Y_d \to V\hat{Y}_d \quad && \Longrightarrow \quad && U(1)^3_{q+u} \times U(3)_d \to U(1)_B\,,
\end{aligned} \tag{6}$$

where V denotes the CKM matrix. Using Eqs. (3) and (5), we can construct the flavor invariants at different orders in spurion expansion entering the SMEFT operators. This point is exemplified by examining the allowed flavor invariants in case of SMEFT operators of the schematic form $O_1 \sim \bar{q}q$ and $O_2 \sim (\bar{q}q)(\bar{q}q)$:

$$\begin{aligned}
O_1 \to\ & \alpha_1(\bar{q}q) + \alpha_2(\bar{q}Y_uY_u^\dagger q) + \alpha_3(\bar{q}Y_dY_d^\dagger q)\,,\\
O_2 \to\ & \beta_1(\bar{q}q)(\bar{q}q) + \beta_2(\bar{q}_iq^j)(\bar{q}_jq^i) + \beta_3(\bar{q}q)(\bar{q}Y_uY_u^\dagger q) + \beta_4(\bar{q}q)(\bar{q}Y_dY_d^\dagger q)\\
& + \beta_5(\bar{q}_iq^j)(Y_uY_u^\dagger)^i{}_k(\bar{q}_jq^k) + \beta_6(\bar{q}_iq^j)(Y_dY_d^\dagger)^i{}_k(\bar{q}_jq^k)\,.
\end{aligned} \tag{7}$$

These decompositions can be used to extract the number of independent invariants for all SMEFT operator classes and for all flavor subgroups listed in Eq. (2). See Ref. [5] for more details. In Tab. 1 we provide the numbers of $O(1)$ flavor invariants, i.e. leading order structures allowed in absence of flavor spurions, for different combinations of flavor subgroups.

[1] Spurions account for the explicit flavor symmetry breaking and also enable the establishment of the flavor power counting. See Refs. [5, 6] for more details.

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SMEFT $O(1)$ terms (dim-6, $\Delta B = 0$)		Lepton sector															
		$U(3)^2$		$U(3)_V$		$U(2)^2{\times}U(1)^2$		$U(2)^2$		$U(2)_V$		$U(1)^6$		$U(1)^3$		No symm.	
Quark sector	$U(3)^3$	41	6	45	9	59	6	62	9	67	13	81	6	93	18	207	132
	$U(2)^2 \times U(3)_d$	72	10	78	15	95	10	100	15	107	21	122	10	140	28	281	169
	$U(2)^3 \times U(1)_{d_3}$	86	10	92	15	111	10	116	12	123	21	140	10	158	28	305	175
	$U(2)^3$	93	17	100	23	118	17	124	23	132	30	147	17	168	38	321	191
	No symmetry	703	570	734	600	756	591	786	621	818	652	813	612	906	705	1350	1149

Table 1: Overview of the number of independent $O(1)$ terms for the dimension-6 SMEFT operators ($\Delta B = 0$) for different assignments of quark and lepton symmetries considered in the paper. The left (right) entry in each column gives the number of CP even (odd) coefficients for each symmetry combination.

3. Flavor symmetries and UV mediators

The analysis of the flavor symmetries can be extended beyond the SMEFT operators by considering a plethora of linear new physics (NP) extensions to the SM. As demonstrated in Ref. [7], these NP mediators can be classified by their spin and by their gauge quantum numbers, which determine the set of interaction Lagrangians with the SM fields. Under the assumption of scale separation between the NP mediators and electroweak scale, these NP mediators can be integrated out at tree level to obtain the corresponding SMEFT matching relations.

Using the counting of the $O(1)$ flavor invariants given by Tab. 1 along with the decompositions, the flavor-symmetric SMEFT operator basis can be formed for each symmetry configuration. For the remainder of the analysis we focus on the $U(3)^5$-symmetric SMEFT basis, which can be found in Appendix B of Ref. [5]. Moreover, a significant fraction of these flavor-symmetric SMEFT operators can be generated at tree level, assuming a renormalizable UV completion. The goal of present analysis is to examine the impact of imposing $U(3)^5$ flavor symmetries at the level of the interaction Lagrangians of the UV mediators with the SM fields and, consequently, study the resulting tree-level SMEFT matching relations.

As an illustrative case, we examine the $\omega_4 \sim (\mathbf{3}, \mathbf{1})_{-4/3}$ scalar mediator, whose gauge configuration allows for the interaction terms of the form [7]

$$-\mathcal{L}_{\text{UV}} \supset [y^{ed}_{\omega_4}]_{rij} \omega^{\dagger}_{4r} \bar{e}^c_i d_j + [y^{uu}_{\omega_4}]_{rij} \omega^{\alpha\dagger}_{4r} \varepsilon_{\alpha\beta\gamma} \bar{u}^{\beta}_i u^{c\gamma}_j + \text{h.c.}\,, \tag{8}$$

where i, j and r denote the flavor indices of the SM and UV fields, respectively. Greek indices denote color, whereas c labels the charge-conjugated fields. Imposing $U(3)^5$ flavor symmetry on the interaction terms in Eq. (8), in order to preserve the invariance under the flavor symmetry, ω_4 mediator has to transform as

$$\begin{aligned} -\mathcal{L}_{\text{UV}} &\supset [y^{ed}_{\omega_4}]_{rij} \omega^{\dagger}_{4r} \bar{e}^c_i d_j + \text{h.c.} &&\Longrightarrow && \omega^{ij}_4 \sim (\mathbf{3}_e, \mathbf{3}_d)\,, \\ -\mathcal{L}_{\text{UV}} &\supset [y^{uu}_{\omega_4}]_{rij} \omega^{\alpha\dagger}_{4r} \varepsilon_{\alpha\beta\gamma} \bar{u}^{\beta}_i u^{c\gamma}_j + \text{h.c.} &&\Longrightarrow && \omega^{i}_4 \sim \mathbf{3}_u\,. \end{aligned} \tag{9}$$

In the next step, for each of these flavor representations we can extract the flavor structure of the coupling tensor, which can be written as

$$\omega^{ij}_4 \sim (\mathbf{3}_e, \mathbf{3}_d) \implies [y^{ed}_{\omega_4}]^{i_e k_d}_{j_e l_d} = y^{ed}_{\omega_4} \delta^{i_e}_{j_e} \delta^{k_d}_{l_d}\,, \qquad \omega^i_4 \sim \mathbf{3}_u \implies [y^{uu}_{\omega_4}]^{i_u j_u k_u} = y^{uu}_{\omega_4} \varepsilon^{i_u j_u k_u}\,, \tag{10}$$

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where we introduce the additional label for the flavor indices in order to make the flavor contractions more apparent. Lastly, using the tree-level SMEFT matching results [7] and the flavor coupling tensors given by Eq. (10), we obtain

$$\omega_4^{ij} \sim (\mathbf{3}_e, \mathbf{3}_d) \implies \mathcal{L}_{\text{SMEFT}} \supset \frac{|y_{\omega_4}^{ed}|^2}{2M_{\omega_4}^2} O_{ed}\,, \qquad \omega_4^{i} \sim \mathbf{3}_u \implies \frac{|y_{\omega_4}^{uu}|^2}{M_{\omega_4}^2}(O_{uu}^D - O_{uu}^E)\,, \tag{11}$$

where the SMEFT operators are $U(3)^5$-symmetric and are defined as

$$O_{ed} = (\bar{e}_i \gamma^\mu e^i)(\bar{d}_j \gamma_\mu d^j)\,, \quad O_{uu}^D = (\bar{u}_i \gamma^\mu u^i)(\bar{u}_j \gamma_\mu u^j)\,, \quad O_{uu}^E = (\bar{u}_i \gamma^\mu u^j)(\bar{u}_j \gamma_\mu u^i)\,, \tag{12}$$

where i, j denote the flavor indices. This procedure can be applied to all of the remaining mediators and a comprehensive set of allowed flavor irreps accompanied by the tree-level matching relations can be derived [5]. It should be noted that for the majority of the flavor irreps, the tree-level matching relations reduce to a single linear combination of the flavor-symmetric SMEFT operators multiplied by a single parameter. The matching relations of this type are referred to as *leading SMEFT directions* and they can be used to perform a phenomenological analysis, whose main goal is to determine the lower bound on the scale of the given UV flavor irrep. See Ref. [5] for a comprehensive overview of these bounds as well as Ref. [8] for important RGE effects within this framework.

4. Discrete leptonic directions in the SMEFT

As indicated in Sec. 2, the list of examined flavor symmetries given by Eq. (2) can be expanded by considering some of the well-motivated discrete flavor symmetries, focusing particularly on the leptonic sector. Discrete flavor symmetry groups, which are often used in portraying the lepton sector as well as in various approaches to neutrino mass model building, involve A_4, A_5 and S_4 (see Refs. [9–11] and the references therewithin).

These discrete symmetries, however, can be examined in a manner similar to the analysis outlined in Sec. 3 [12]. As an example, let us focus on the A_4 discrete flavor symmetry group. A_4 group represents a finite, alternating group on four objects, which consists of all even permutations and it contains $4!/2 = 12$ elements. From the point of view of a geometric interpretation, A_4 is isomorphic to the symmetry group of a regular tetrahedron, which implies that this group describes the rotational symmetries of a regular tetrahedron, excluding reflections. In addition to this, from the perspective of representation theory, A_4 is the smallest finite non-Abelian group which allows for a triplet representation ($\mathbf{3}$), along with three distinct singlets, which are denoted as $\mathbf{1}$, $\mathbf{1}'$ and $\mathbf{1}''$. The tensor products involving A_4 singlets can be written as

$$\mathbf{1} \otimes \mathbf{1} = \mathbf{1}\,, \qquad \mathbf{1}' \otimes \mathbf{1}' = \mathbf{1}''\,, \qquad \mathbf{1}'' \otimes \mathbf{1}'' = \mathbf{1}'\,, \qquad \mathbf{1}' \otimes \mathbf{1}'' = \mathbf{1}\,, \tag{13}$$

while the tensor product involving two triplets yields

$$\mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}'' \oplus \mathbf{3}_S \oplus \mathbf{3}_A\,, \tag{14}$$

where S and A indicate the symmetric and antisymmetric combinations of the triplet components, respectively. Taking $\alpha \equiv (\alpha_1, \alpha_2, \alpha_3)^T \sim \mathbf{3}$ and $\beta \equiv (\beta_1, \beta_2, \beta_3)^T \sim \mathbf{3}$, the irreps obtained as a result of taking the tensor product involving two A_4 triplets can be expressed in terms of the components of α and β. The singlets in the decomposition given by Eq. (14) are given as[2]

$$[\alpha\beta]_{\mathbf{1}} = \alpha_1\beta_1 + \alpha_2\beta_3 + \alpha_3\beta_2\,, \quad [\alpha\beta]_{\mathbf{1'}} = \alpha_3\beta_3 + \alpha_1\beta_2 + \alpha_2\beta_1\,, \quad [\alpha\beta]_{\mathbf{1''}} = \alpha_2\beta_2 + \alpha_1\beta_3 + \alpha_3\beta_1\,, \tag{15}$$

while the symmetric and antisymmetric triplet combinations can be written as

$$[\alpha\beta]_{\mathbf{3_S}} = \frac{1}{3}\begin{bmatrix} 2\alpha_1\beta_1 - \alpha_2\beta_3 - \alpha_3\beta_2 \\ 2\alpha_3\beta_3 - \alpha_1\beta_2 - \alpha_2\beta_1 \\ 2\alpha_2\beta_2 - \alpha_1\beta_3 - \alpha_3\beta_1 \end{bmatrix}, \qquad [\alpha\beta]_{\mathbf{3_A}} = \frac{1}{2}\begin{bmatrix} \alpha_2\beta_3 - \alpha_3\beta_2 \\ \alpha_1\beta_2 - \alpha_2\beta_1 \\ \alpha_3\beta_1 - \alpha_1\beta_3 \end{bmatrix}. \tag{16}$$

Using these relations we can analyze the subset of the UV mediators coupling to the SM leptons in presence of the discrete symmetries (see Ref. [12] for the analogous discussion on A_5 and S_4 symmetries as well as a comprehensive analysis of the relevant UV mediators). As a representative example we take the $\Xi_1 \sim (\mathbf{1}, \mathbf{3})_1$ scalar with the interaction Lagrangian given as

$$-\mathcal{L}_{\rm UV} \supset [y_{\Xi_1}]_{rij} \Xi^{a\dagger}_{1r} \bar{\ell}_i \sigma^a i\sigma_2 \ell^c_j + \text{h.c.}\,. \tag{17}$$

Taking ℓ to transform as A_4 triplet, the set of allowed A_4 invariants can be written as

$$\text{inv}_{\Xi_1} = [\bar{\ell}\ell^c]_{\mathbf{1}}[\Xi_1]_{\mathbf{1}} \oplus [\bar{\ell}\ell^c]_{\mathbf{1'}}[\Xi_1]_{\mathbf{1''}} \oplus [\bar{\ell}\ell^c]_{\mathbf{1''}}[\Xi_1]_{\mathbf{1'}} \oplus [\bar{\ell}\ell^c]_{\mathbf{3_S}}[\Xi_1]_{\mathbf{3}}\,, \tag{18}$$

Using the rules of tensor product decomposition, we can extract the flavor tensors and, consequently, determine the tree-level SMEFT matching relations [12]. As in the case of *leading SMEFT directions*, the tree-level matching relations for most mediators under the assumption of the discrete flavor symmetries also reduce to the similar form, leading to comprehensive phenomenological analysis, including an extended set of observables comprised of cLFV $|\Delta L| = 1$ and $|\Delta L| = 2$ transitions [13, 14].

5. Conclusion and outlook

We analyzed the impact of various flavor assumptions on the parameter space of SMEFT operators. Furthermore, the discussion is extended to the plethora of UV mediators and, assuming the scale separation, the tree-level matching relations are studied. The comprehensive classification of these mediators, along with the phenomenological implications is performed assuming the unbroken $U(3)^5$ flavor symmetry. Similar analysis is carried out for a set of well-motivated discrete symmetries in the leptonic sectors.

Promising avenues for future studies involve examining the UV mediators under $U(2)^5$ flavor symmetry, which is particularly well-motivated in the quark sector. Along the similar lines, extending the analysis beyond the tree-level matching relations would be of interest. Lastly, in case of discrete flavor symmetries, promoting them to modular symmetries and going beyond the leading order in Yukawa insertions would lead to a more comprehensive set of phenomenological implications.

[2] We use the notation of the form $[\mathcal{R}_1\mathcal{R}_2]_{\mathcal{R}_3} \equiv (\mathcal{R}_1 \otimes \mathcal{R}_2)_{\mathcal{R}_3}$, where $\mathcal{R}_1$ and $\mathcal{R}_2$ are the irreps entering the tensor product and $\mathcal{R}_3$ is a single irrep we pick from the tensor product expansion.

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