

## $\Lambda$ and $\Sigma$ potentials in neutron stars, hypernuclei, and heavy-ion collisions

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With an appropriate  $YNN$  force, the  $\Lambda$  single-particle potential ( $\Lambda$  potential) can be made strongly repulsive at high density, and one can solve the hyperon puzzle of neutron stars. We investigate the consistency of such a  $\Lambda$  potential, evaluated recently from  $YN$  and  $YNN$  forces based on chiral effective field theory, with hypernuclear data and heavy-ion collision data. It is found that model calculations with such a  $\Lambda$  potential can reproduce the data of the  $\Lambda$  hypernuclear spectroscopy and the  $\Lambda$  directed flow in heavy-ion collisions. Also, we evaluate the  $\Sigma$  potential, which can be calculated by using the same hyperon forces as for the  $\Lambda$  potential. Specifically, we show that the low-energy constants characterizing the strength of the  $YNN$  force can be chosen to suppress the appearance of the  $\Lambda$ 's in neutron stars while at the same time the empirical value of the  $\Sigma$  potential is reproduced.

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## 1. Introduction

The hyperon puzzle of neutron stars refers to the observation that most equations of state (EOSs) with hyperons are too soft to support the observed massive neutron stars [1]. One promising solution is that the three-baryon forces (3BFs) among hyperon ( $Y$ ) and nucleons ( $N$ ) are strongly repulsive so that the hyperons cannot appear in neutron stars. For example, in Ref. [2],  $YNN$  forces have been constructed based on the decuplet dominance approximation [3] and employed together with a  $YN$  potential derived within chiral effective field theory ( $\chi$ EFT) [4]. With such a combination, a  $\Lambda$  single-particle potential ( $\Lambda$  potential) fulfilling that scenario can be obtained. A similar  $\Lambda$  potential can be obtained based on the hypernuclear spectroscopy [5]. An analysis employing the up-to-date tuned chiral  $YN$  potential [6], which is based on the combined analysis of the  $p\Lambda$  scattering and recent  $p\Lambda$  femtoscopy data [7], suggests that additional repulsion by the  $YNN$  force is required to reproduce the empirical value of the  $\Lambda$  potential at the saturation density.

In this contribution, we examine whether the  $\Lambda$  potentials published in Ref. [2] are consistent with the  $\Lambda$  separation energies of hypernuclei and with the directed flow  $v_1$  of  $\Lambda$  in heavy-ion collisions. Furthermore, we investigate the impact of the  $YNN$  interactions proposed in Ref. [2] on the  $\Sigma$  single-particle potential ( $\Sigma$  potential).

Let us emphasize that our work is in line with a recent trend in nuclear matter studies [8–10], conducting a unified approach integrating nuclear experiments and neutron star observations with the modern nuclear force from  $\chi$ EFT to obtain a well-constrained EOS of dense matter. This approach contributes to a more comprehensive understanding of both nuclear experiments and astrophysical observations. For a microscopic description of EOS, the properties of hyperons in nuclear matter should also be constrained. We utilize the experimental data involving hyperons to evaluate the validity of the existing  $\Lambda$  potential, which is important in determining the onset density of strangeness.

## 2. Evaluating the repulsive $\Lambda$ potential from $\Lambda$ hypernuclear data

First, we utilize the  $\Lambda$  hypernuclear spectroscopy to examine the  $\Lambda$  potential. We consider three  $\Lambda$ -potential models (Chi3, Chi2, and LY-IV) as follows (see Ref. [11] for more details): We constructed the Chi3 potential by fitting the result of  $\chi$ EFT with  $YN$  and  $YNN$  forces [2, 12] to the Skyrme-type  $\Lambda$  potential [11]. The  $YN$  force is chosen as NLO13(500) [4], while the  $YNN$  force is constructed by the decuplet dominance approximation [3]. For reference, the Chi2 potential was similarly constructed without the 3BF. The LY-IV potential is a conventional  $\Lambda$  potential [13] attractive at high density, with which  $\Lambda$ 's appear in dense neutron star matter. The density dependence of the  $\Lambda$  potentials is plotted on the left panel of figure 1. The momentum dependencies for Chi2, Chi3, and LY-IV in the lower momentum region  $k \leq 1.0 \text{ fm}^{-1}$  exhibit behaviors similar to those of Kohno2, Kohno3, and LY-IVmomSoft shown on the right panel of figure 1, respectively (see Ref. [11] for the comparison). We employ the Skyrme-Hartree-Fock method using the above-mentioned three different  $\Lambda$  potentials. One parameter that cannot be determined from the uniform-matter results is tuned to reproduce the  $\Lambda$  binding energy data of  $^{13}_{\Lambda}\text{C}$ .

We compare the model calculations with data on the separation energies of  $\Lambda$  hypernuclei [11] on the left panel of figure 2. Chi3 reproduces the data as accurately as LY-IV. In contrast, we found

that Chi2 overbounds by several MeV due to the excessive potential depth at the saturation density  $\rho_0 \approx 0.16 \text{ fm}^{-3}$ . Thus, Chi2 can be excluded, yet we need other data to constrain the repulsion of the  $\Lambda$  potential at high densities.

### 3. Evaluating the repulsive $\Lambda$ potential from heavy-ion collision data

Next, we consider the rapidity dependence of the  $\Lambda$  directed flow in heavy-ion collisions [15],

$$v_1 = \langle \cos \phi \rangle = \left\langle \frac{p_x}{\sqrt{p_x^2 + p_y^2}} \right\rangle, \quad (1)$$

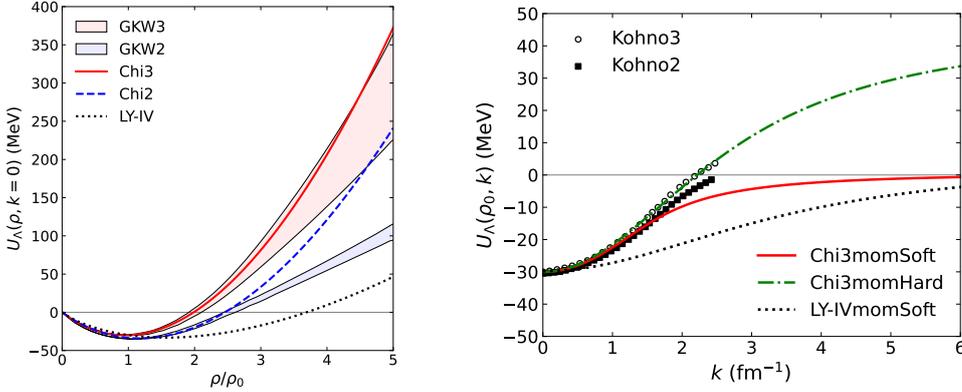
where  $\phi$  is the azimuthal angle measured from the reaction plane and  $p_x$  and  $p_y$  are the transverse momenta of a particle. We use the Lorentz vector version of the relativistic quantum molecular dynamics (RQMdv) model [16] implemented in the JAM2 transport code<sup>1</sup>.

In the heavy-ion collision simulation, the high momentum part of the momentum dependence is important. We construct Chi3momSoft, Chi3momHard, and LY-IVmomSoft by extrapolating the momentum dependence of Chi3 and LY-IV to a high momentum region by assuming the Lorentzian form:

$$U_m(\rho(x), k) = \frac{C}{\rho_0} \int d^3k' \frac{f(x, k')}{1 + [(k - k')/\mu]^2}, \quad (2)$$

where  $C$  and  $\mu$  are fitting parameters and  $f(x, k)$  is the single-particle distribution function. In the actual heavy-ion simulations, we implement the momentum-dependent potential as the Lorentz

<sup>1</sup><https://gitlab.com/transportmodel/jam2>



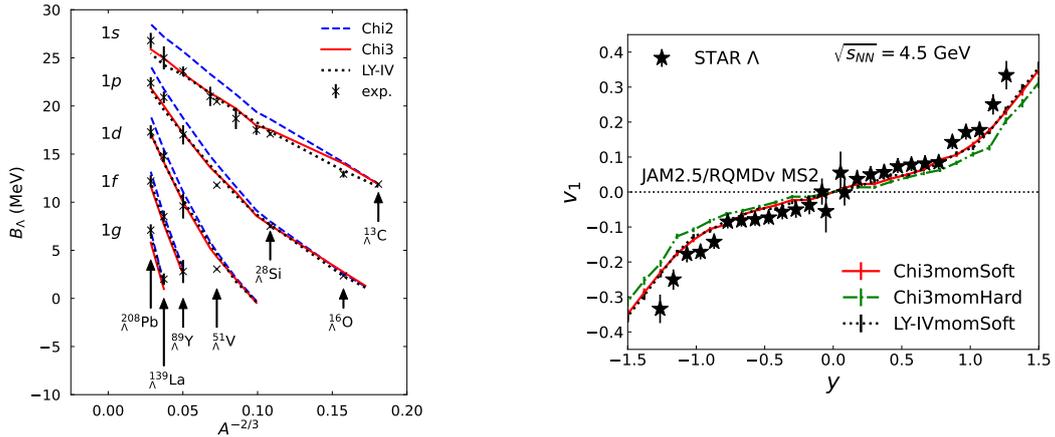
**Figure 1:** (left panel) Density dependence of the  $\Lambda$  potential. GKW3 represents the results from  $\chi$ EFT with  $YN$  and  $YNN$  forces [2]. GKW2 is also from  $\chi$ EFT but without the 3BFs [2]. Chi3 (solid line) and Chi2 (dashed line) are fitted to GKW2 and GKW3 up to  $\rho/\rho_0 < 1.5$ , respectively. LY-IV (dotted line) is a conventional  $\Lambda$  potential [13]. (right panel) Momentum dependence of the  $\Lambda$  potential. Kohno3 represents the result from  $\chi$ EFT with  $YN$  and  $YNN$  forces [12]. Kohno2 is the result from  $\chi$ EFT without the 3BFs [12]. Chi3momSoft (solid line) and Chi3momHard (dash-dotted line) are constructed to reproduce Kohno3 up to  $2.5 \text{ fm}^{-1}$  and  $1.0 \text{ fm}^{-1}$ , respectively. LY-IVmomSoft (dotted line) is fitted to the momentum dependence of LY-IV up to  $1.0 \text{ fm}^{-1}$ .

vector  $U_m^\mu$  [15]. Since  $\chi$ EFT is not entirely reliable above the momentum cutoff of  $550 \text{ MeV} \simeq 2.8 \text{ fm}^{-1}$  [12], we prepared two variations: Chi3momHard and Chi3momSoft are constructed to reproduce the  $\chi$ EFT result [12] up to  $2.5 \text{ fm}^{-1}$  and  $1.0 \text{ fm}^{-1}$ , respectively. LY-IVmomSoft is constructed to reproduce the momentum dependence of LY-IV with Eq. (2) up to  $1.0 \text{ fm}^{-1}$ . The momentum dependence of the  $\Lambda$  potentials is plotted on the right panel of figure 1. We note that the density dependence of Chi3momSoft and Chi3momHard is almost identical to that of Chi3, as is LY-IVmomSoft to LY-IV.

The results of  $v_1$  of  $\Lambda$  in mid-central Au + Au collisions at  $\sqrt{s_{NN}} = 4.5 \text{ GeV}$  are shown on the right panel of figure 2 and compared with the STAR data [14]. One can see that both Chi3momSoft and LY-IV reproduce  $v_1$  of  $\Lambda$  with equal accuracy, which implies that  $v_1$  of  $\Lambda$  is not so sensitive to the density dependence of the  $\Lambda$  potential. On the other hand, Chi3momHard underestimates  $v_1$  of  $\Lambda$ , which indicates that  $v_1$  of  $\Lambda$  is sensitive to the momentum dependence of the  $\Lambda$  potential. Experimental information on the optical potential of  $\Lambda$  may be useful for reducing the model uncertainty.

#### 4. How about the $\Sigma$ potential?

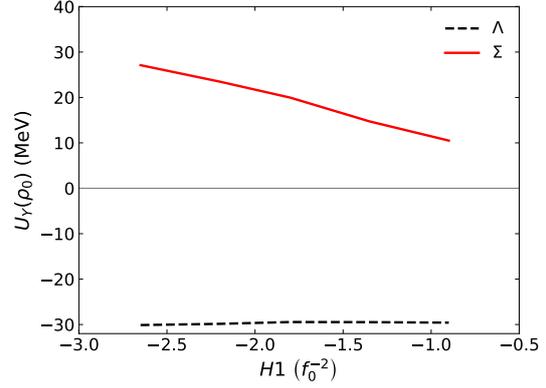
As already mentioned above, in Ref. [2], the 3BFs have been adjusted in such a way that the  $\Lambda$  potential is sufficiently repulsive at high density so that the appearance of  $\Lambda$  hyperons in neutron stars is suppressed. This is possible for different combinations of the low-energy constants (LECs),  $H_1$  and  $H_2$ , that characterize the strength of the 3BF (see the solid lines in figure 6 of Ref. [2]). For other combinations, cf. the dashed lines, the repulsion might not be strong enough to achieve that goal.



**Figure 2:** (left panel)  $\Lambda$  binding energy of the  $\Lambda$  hypernuclei. Experimental data (cross) can be found in Ref. [11]. The figure is adopted from Ref. [11]. (right panel) Directed flow of  $\Lambda$  in mid-central Au+Au collisions at  $\sqrt{s_{NN}} = 4.5 \text{ GeV}$ . The STAR data are taken from Ref. [14]. The figure is updated from Ref. [15] by using the updated version of JAM2.

$H_1$ ( $f^{-2}$ )	$H_2$ ( $f^{-2}$ )
-2.650	0.100
-2.200	0.000
-1.800	-0.100
-1.350	-0.200
-0.900	-0.300

**Table 1:** Considered combinations of LECs of the  $YNN$  3BF that reproduce  $U_\Lambda(\rho_0) = -30$  MeV for NLO13(500). The values are in units of the inverse squared pion-decay constant with  $f \approx 92$  MeV, and correspond to the left line in figure 6 of Ref. [2]. The values are taken from Gerstung’s PhD thesis [21].



**Figure 3:**  $\Sigma$  (red solid) and  $\Lambda$  (black dashed) potentials in symmetric nuclear matter. The horizontal axis corresponds to the three-body LECs  $H_1$  with  $H_2$  listed in table 1.

However, it remains unclear how those 3BFs affect the corresponding  $\Sigma$  potential. The effective two-body forces resulting from the 3BFs considered in Ref. [2] contribute not only to the  $\Lambda N$  and  $\Sigma N$  channels but also to the  $\Lambda N$ - $\Sigma N$  transition potential. A presently accepted constraint on the  $\Sigma$  potential is  $U_\Sigma(\rho_0) = 30 \pm 20$  MeV [17], which is inferred using data on  $\Sigma^-$  atoms and on  $(\pi^+, K^+)$  inclusive spectra. This constraint is fairly well met by the original chiral  $YN$  potentials from 2013 and 2019 without 3BF [18, 19].

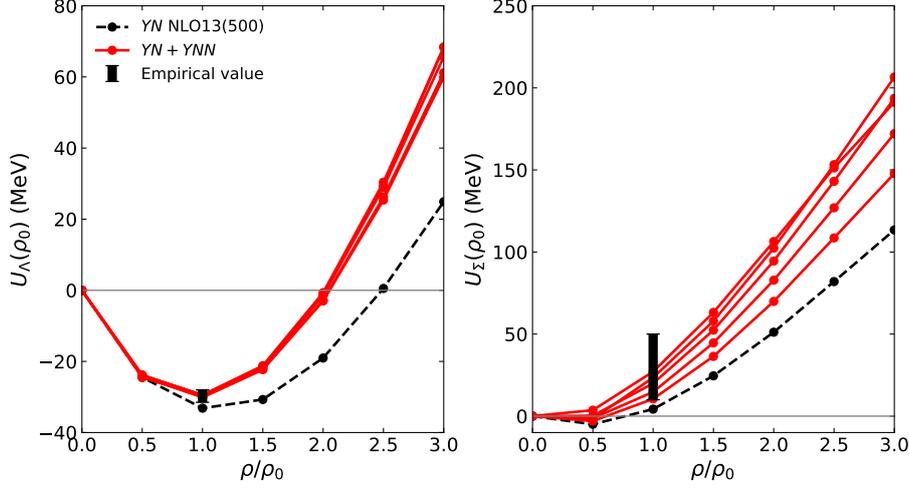
We evaluate the  $\Sigma$  potential in the same way as Gerstung et al. [2] have done for the  $\Lambda$  potential. The Brueckner-Hartree-Fock method with a continuous choice [18] is employed to calculate the hyperon single-particle potential. Regarding the nuclear forces, the  $N^3\text{LO}$   $NN$  potential from Ref. [20] is used, while the  $NNN$  3BF at  $N^2\text{LO}$  is taken into account via a density-dependent two-body force. The cutoff in the regulator function [20] is chosen as 500 MeV. The nuclear saturation properties are reproduced by the nucleon forces [21].

For the hyperonic force, the  $YN$  potential NLO13(500) [4] is employed. The  $YNN$  force is implemented as an effective density-dependent  $YN$  two-body force. The number of LECs involved in the  $YNN$  force is reduced by assuming decuplet dominance approximation [3]. Then, there are only three LECs: one related to the meson-octet-decuplet baryon vertex, and two denoted by  $H_1$  and  $H_2$  characterizing the strengths of the contact vertices with three-octet and one-decuplet baryons. The meson-octet-decuplet coupling is constrained by the decay width  $\Gamma(\Delta \rightarrow N\pi)$ , and its large- $N_c$  value is employed [2]. The two LECs of the contact terms are fixed by requiring the reproduction of the empirical value of the  $\Lambda$  potential [11, 17],

$$U_\Lambda(\rho_0) \simeq -30 \text{ MeV}, \quad (3)$$

inferred by using the  $\Lambda$  hypernuclear spectroscopy, and a strongly repulsive  $U_\Lambda$  at high density, sufficient to resolve the hyperon puzzle [2]. Some combinations of the contact LECs that fulfill these requirements are listed in table 1.

The  $\Lambda$  and  $\Sigma$  potentials in symmetric nuclear matter at  $\rho_0$  are shown in figure 3 for various combinations of  $H_1$  and  $H_2$ . The  $\Lambda$  potential is practically constant by construction, i.e., due to the



**Figure 4:** Density dependence of the  $\Lambda$  (left panel) and  $\Sigma$  (right panel) potentials in symmetric nuclear matter. The red solid lines are calculated by using the 3BF LECs in table 1. The only two-body case with the NLO13(500) parameter set is represented by the dashed line. The black bands show the empirical values of the  $\Lambda$  potential,  $-31.5 < U_\Lambda(\rho_0) < -28.0$  MeV [11], and the  $\Sigma$  potential,  $U_\Sigma(\rho_0) = 30 \pm 20$  MeV [17].

constraint (3). In contrast, the  $\Sigma$  potential varies from about 30 to 10 MeV.

In figure 4, we show the density dependence of the single-particle potentials. One can see that certain sets of the 3BF LECs reproduce the empirical constraint  $U_\Sigma(\rho_0) = 30 \pm 20$  MeV [17]. Interestingly, those 3BFs also yield  $\Lambda$  potentials that are strongly repulsive at high densities, as needed to suppress the  $\Lambda$  hyperons in neutron stars. Thus, the 3BFs can be chosen to solve the hyperon puzzle of neutron stars while at the same time the empirical value of  $U_\Sigma$  is reproduced.

## 5. Summary

To suppress the  $\Lambda$  hyperons in neutron stars, Gerstung et al. [2] calculated the  $\Lambda$  single-particle potential by adding an effective 3BF to the chiral  $YN$  potentials of the Jülich-Bonn group [4], which is one of the state-of-the-art modeling of the potential based on the  $\chi$ EFT. In this contribution, we have examined the consistency of those potentials with the data from experiments and observations of substantially different physics. Specifically, we referenced the data from the  $\Lambda$  hypernuclear spectroscopy, the  $\Lambda$  directed flow created in heavy-ion collisions, and the value of the  $\Sigma$  single-particle potential at the saturation density. Some of the 3BF LEC sets of  $(H_1, H_2)$  from Ref. [2] turned out to be consistent with the empirical information in all three physics.

The results presented here are based on a single chiral  $YN$  potential, NLO13(500). Variants such as NLO19 [19] should be considered in order to provide an estimate of the theoretical uncertainty. Furthermore, recently a  $YN$  interaction up to  $N^2$ LO in the chiral expansion has been presented [22]. Initial studies suggest that it yields more attractive  $\Lambda$  and  $\Sigma$  potentials [22]. Here, a more rigorous investigation of the in-medium properties is required. Such a work will be performed in the future.

In addition, it is desirable to improve the theoretical treatment of the heavy-ion collision simulation. The results shown here have been obtained by using the same  $\Lambda$  potential for all

other hyperons, including their resonance states. As a large number of  $\Sigma$  hyperons and hyperon resonances are produced during the evolution of the heavy-ion collisions, and as the behavior of the  $\Lambda$  and  $\Sigma$  potentials are very different, as seen in figure 4, we intend to include different potentials for different hyperons to explore their effects on  $v_1$  of  $\Lambda$  and  $\Sigma$  in future studies.

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