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Lorentz Violation, CPT Violation, and Spectroscopy Experiments

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This presentation provides a brief overview of tests of Lorentz symmetry in atomic spectroscopy experiments, focusing on exotic atoms and models of Lorentz violation that include nonminimal effects. The talk discusses prevalent signals for Lorentz violation in atomic spectroscopy experiments, such as sidereal variation and CPT violation. It also highlights the importance of testing Lorentz symmetry using exotic atoms like muonium, antihydrogen, and muonic hydrogen. Finally, a brief discussion is provided on experiments used to constrain nonrelativistic coefficients for Lorentz violation, along with prospects for improving these bounds and imposing limits on unconstrained coefficients.

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1. Introduction

The idea that tiny deviations from Lorentz symmetry could signal new physics beyond the Standard Model and General Relativity [1] motivated the creation of a comprehensive framework for systematically testing Lorentz symmetry, known as the Standard-Model Extension (SME) [2]. Closely related to Lorentz symmetry is CPT symmetry since CPT violation implies Lorentz violation in a realistic quantum effective field theory [3]. Thus, the SME also serves as a tool for systematically testing CPT symmetry.

The SME associates each Lorentz-violating operator with a unique coefficient for Lorentz violation, referred to as an SME coefficient. Any indisputable evidence of a nonzero SME coefficient would indicate a breaking of Lorentz symmetry. Approximately half of the Lorentz-violating operators in the SME also violate CPT symmetry. Therefore, the coefficients associated with these operators are coefficients for CPT violation, meaning that evidence for a nonzero value for these coefficients would imply a breaking of CPT symmetry.

Each experiment testing these symmetries is sensitive to a limited number of SME coefficients. Thus, testing Lorentz and CPT symmetry requires considering a wide array of experimental tests and identifying tests sensitive to unconstrained coefficients. Furthermore, as the SME allows us to quantify each experiment's sensitivity to individual coefficients, it permits us to assess the competitiveness, complementarity, or overlap among various tests of Lorentz symmetry.

The initial versions of the SME focused on Lorentz-violating operators with mass dimensions $d \leq 4$, known as minimal operators, to limit introducing nonrenormalizable terms into the framework [2]. However, this restriction was unnecessary because the SME parts from the premise that the Lorentz-violating underlying theory would manifest as an effective field theory at low energies, which does not need to be renormalizable, given that it is only valid within a limited energy range. Therefore, there is no reason to exclude nonminimal operators, those with $d \geq 5$, from the SME. Since then, several expansions of the SME incorporated nonminimal operators [4–8]. One argument for including nonminimal operators is that they offer a compelling explanation for the lack of evidence for Lorentz and CPT violation as their contributions to experimental signals are expected to be suppressed by the ratio of the low-energy experimental scale to the higher-energy scale where the effective field theory breaks down.

The SME distinguishes between similar Lorentz-violating operators that act on different particle types. For instance, an electron Lorentz-violating operator with the same form as a muon operator will be associated with different SME coefficients, and the framework does not assume any *a priori* relationship between these coefficients. Consequently, the SME allows for the possibility that Lorentz symmetry could be broken in the muon sector while remaining intact in the electron sector. Thus, tests of Lorentz symmetry involving exotic matter are highly relevant in the SME context, as these tests can provide unique insights into potential sector-specific Lorentz violations.

An example of an exotic atom used to test Lorentz symmetry is muonium, which provides access to the muon SME coefficients. Not long after the introduction of the SME, the development of models for testing Lorentz symmetry with muonium [9] resulted in bounds on SME minimal coefficients [10]. More recently, models for muonium spectroscopy incorporating nonminimal coefficients were derived, leading to bounds on nonminimal muon coefficients for Lorentz violation [11]. Beyond muonium, models for testing Lorentz symmetry with muonic atoms [11] and positronium have also been proposed [12].

As mentioned, the SME is a framework for the systematic study of CPT symmetry, and for that reason, SME models for antimatter experiments have been prevalent in the literature. These models include early ones that explore the potential of antihydrogen spectroscopy for testing Lorentz and CPT symmetry [13], as well as more recent models that incorporate nonminimal operators [12]. Similarly, models for Penning trap experiments with antiparticles [14] and for studying the anomalous free-fall motion of antimatter [15] were initially derived from the minimal SME and later extended to include nonminimal operators [8, 16].

This overview focuses on nonminimal models for testing Lorentz and CPT symmetry using atomic and molecular spectroscopy, particularly those outlined in [11, 12, 17, 18]. Due to this narrow scope, some notable topics are excluded from this discussion. For example, we will not cover the significant advancements made in antiproton Penning trap experiments [19] or the current models for Lorentz and CPT violation that have been or could potentially be tested in these experiments [14, 16, 20]. Another topic omitted here is free-fall antimatter tests, including the recent measurement of antihydrogen free-fall acceleration [21] and other proposed experiments [22, 23]. High-precision spectroscopy experiments that have tested models for Lorentz and CPT symmetry limited to minimal SME coefficients [24] are also outside the scope of this overview.

In this work, we will present the nonrelativistic (NR) coefficients that serve as the effective SME coefficients in the models discussed here. We will also examine signals for Lorentz violation proposed or used to impose constraints on these NR coefficients. Additionally, we will explore the prospects for establishing new constraints or improving existing ones using spectroscopy experiments with exotic atoms.

2. Main

The Lorentz-violation perturbations for the atomic or molecular systems considered in [11, 12, 17, 18] were obtained by adding a single-particle perturbation $\delta h_w^{\rm NR}$ for each fermion in the atom, where w is an index specifying the fermion type. The single-particle perturbation $\delta h_w^{\rm NR}$ is the nonrelativistic perturbation derived in [6] that captures the dominant effect of Lorentz violation on the free motion of the fermion. As shown in [12], there are terms in the single-particle perturbation that do not contribute to the energy of atomic or molecular levels. Excluding these terms, the Lorentz-violating single-particle perturbation takes the form

$$\begin{aligned} \delta h_w^{\rm NR} &= -\sum_{kjm} |\boldsymbol{p}|^k {}_0Y_{jm}(\hat{\boldsymbol{p}}) \left(\mathcal{V}_w{}^{\rm NR}_{kjm} + \boldsymbol{\sigma}\cdot\hat{\boldsymbol{\epsilon}}_r \mathcal{T}_w{}^{\rm NR(0B)}_{kjm} \right) \\ &\quad + \sum_{kjm} |\boldsymbol{p}|^k \boldsymbol{\sigma}\cdot \left({}_{+1}Y_{jm}(\hat{\boldsymbol{p}})\hat{\boldsymbol{\epsilon}}_- - {}_{-1}Y_{jm}(\hat{\boldsymbol{p}})\hat{\boldsymbol{\epsilon}}_+ \right) \mathcal{T}_w{}^{\rm NR(1B)}_{kjm} . \end{aligned} \tag{1}$$

Here, the summation index k is limited to the values 0, 2, and 4; j ranges from 0 to 5; and m takes values in the range $-j \leq m \leq j$.

For this presentation, a detailed understanding of this perturbation is not required. Therefore, we will focus only on the essential aspects needed to advance the discussion. In particular, we will use the general form of δh_w to clarify some of the indices and conventions used to describe the nonrelativistic spherical coefficients $\mathcal{V}_{w\,kjm}^{\rm NR}$, $\mathcal{T}_{w\,kjm}^{\rm NR(0B)}$ and $\mathcal{T}_{w\,kjm}^{\rm NR(1B)}$.

First, the NR coefficients in $\delta h_w^{\rm NR}$ are linear combinations of SME coefficients that couple to operators with a definite CPT sign. These linear combinations are defined as

$$\begin{aligned} \mathcal{V}_{w\,kjm}^{\rm NR} &= c_{w\,kjm}^{\rm NR} - a_{w\,kjm}^{\rm NR}, \\ \mathcal{T}_w{}^{{\rm NR}(qP)}{}_{kjm} &= g_w{}^{{\rm NR}(qP)}{}_{kjm} - H_{w\,kjm}^{{\rm NR}(qP)}, \end{aligned} \tag{2}$$

where the a- and g-type coefficients are associated with CPT-odd operators, while the c- and H-type coefficients correspond to CPT-even ones. The $\mathcal{V}$-type and $\mathcal{T}$-type coefficients are convenient notations as any nonrelativistic matter experiment is sensitive to the specific linear combinations in (2). However, this is not the case for nonrelativistic antimatter experiments, which require a different set of linear combinations. To distinguish these cases, we use the index $\overline{w}$ to indicate the antiparticle case, as opposed to w, which indicates the particle one. The combinations relevant to antimatter experiments are given by

$$\begin{aligned} \mathcal{V}_{\overline{w}\,kjm}^{\rm NR} &= c_{w\,kjm}^{\rm NR} + a_{w\,kjm}^{\rm NR}, \\ \mathcal{T}_{\overline{w}}{}^{{\rm NR}(qP)}{}_{kjm} &= -g_w{}^{{\rm NR}(qP)}{}_{kjm} - H_{w\,kjm}^{{\rm NR}(qP)}, \end{aligned} \tag{3}$$

where the distinction between (2) and (3) lies in the sign of the coefficients associated with CPT-odd operators. Note that the coefficients with a definite CPT sign are always identified with the particle rather than the antiparticle, which aligns with common conventions in the literature.

The NR coefficients are independent linear combinations of the SME coefficients that appear in the SME Lagrange density [6]. It is convenient to define these linear combinations as effective SME coefficients, as they are prevalent in Lorentz-violation corrections to observables in nonrelativistic experiments. The relationship between the NR coefficients and the standard coefficients for Lorentz and CPT violation is given in Eqs. (111) and (112) of [6]. A notable feature of the nonrelativistic coefficients is that they receive contributions from SME coefficients associated with Lorentz-violating operators of different mass dimensions. Generally, all nonrelativistic coefficients receive contributions from nonminimal SME coefficients, while some also receive contributions from minimal ones.

The notation used to specify the NR coefficients and their indices characterizes the Lorentz-violating operators coupled to these coefficients in the perturbation (1) as explained in detail in [6]. The $\mathcal{T}$-type coefficients are classified as the spin-dependent coefficients because they couple with operators that depend on the fermion's spin state through the Pauli vector $\boldsymbol{\sigma}$. In contrast, the $\mathcal{V}$-type coefficients are the spin-independent coefficients. The indices j and m of the NR coefficients are the same ones as for the spin-weighted spherical harmonics ${}_sY_{jm}(\hat{\boldsymbol{p}})$ in (1) and the index k denotes the power of the fermion's momentum magnitude $|\boldsymbol{p}|$.

The models for Lorentz and CPT violation in atomic and molecular systems, based on the single-particle perturbation (1), contain 178 NR coefficients accessible in atomic spectroscopy experiments by particle type. For instance, hydrogen spectroscopy experiments would be sensitive to 178 electron coefficients and an equal number of proton coefficients. Not all the 178 coefficients

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contribute to the Lorentz-violating energy shift for any arbitrary atomic level, as discussed in [12]. For example, in two-fermion atoms such as hydrogen, antihydrogen, and muonium, only coefficients with index j that satisfy the conditions $j \leq 2J$ and $j \leq 2F$ can contribute to an atomic level with quantum number J for the total angular momentum of the lighter fermion and F for the total atomic angular momentum [12]. For example, transitions between nS states in two-fermion atoms with quantum numbers $J = 1/2$ and $F \leq 1$ are only sensitive to 40 coefficients for each particle type.

Probably the most common signal for Lorentz violation in experiments conducted on the surface of the Earth is a sidereal variation of the resonant frequency. The basic idea is that the spectrum of an atom might depend on the atom's orientation, defined by its total angular momentum $\vec{F}$, relative to a fixed reference frame. We can rotate the atom and its angular momentum $\vec{F}$ by applying a magnetic field that is rotated adiabatically relative to a fixed frame. In an experiment with a fixed magnetic field on the Earth's surface, the planet's rotation causes the magnetic field—and consequently the atom—to rotate adiabatically relative to a fixed reference frame. This motion can result in an oscillation of the resonance frequency with a period of a sidereal day.

Constraints on the SME coefficients are commonly reported in the same reference, called the Sun-centered frame [25, 26], as they are frame-dependent. The indices j and m of the NR coefficients in the Sun-centered frame that contribute to the Lorentz-violating frequency shift provide insight into the type of sidereal variation of the resonance frequency expected in the experiment. Suppose we decompose the time variation of the transition frequency into harmonics of the sidereal frequency, $\omega_\oplus \simeq 2\pi/(1436\,\mathrm{min})$. An NR Sun-centered-frame coefficient with index m can only contribute to the amplitude of the $|m|$-th harmonic of $\omega_\oplus$ [12]. Since $|m| \leq j$, coefficients with index j cannot contribute to the amplitudes of harmonics higher than the j-th harmonic of the sidereal frequency.

Sidereal variation studies of resonance frequencies within the ground state of muonium [10] and hydrogen[27], as well as with a Xe-He co-magnetometer [28], have been used to impose bounds on the $j = 1$ $\mathcal{T}$-type NR coefficients with $|m| = 1$ in the electron, proton, muon, and neutron sectors of the SME [11, 12, 17]. These experiments are only sensitive to $j = 1$ coefficients because the highest total angular momentum for the transitions considered in the experiments is $F = 1$. A sidereal variation study with a Ne-Rb-K co-magnetometer [29], which investigated sidereal variations with the first two harmonics of the sidereal frequency, was used to impose constraints on the $j = 2$ $\mathcal{V}$-type neutron NR coefficients with $|m| = 1$ and $|m| = 2$ [17].

The Sun-centered frame NR coefficients with $m = 0$ do not contribute to the sidereal variation of the resonance frequency in Earth-based experiments with a fixed magnetic field [11, 12, 17]. The constant frequency shift these coefficients produce can be constrained in various ways. For instance, some $m = 0$ coefficients depend on the relative orientation between Earth's rotation axis and the applied magnetic field. An experiment that considered transitions within the ground state of hydrogen with different magnetic field orientations imposed bounds on the proton and electron $\mathcal{T}$-type coefficients with $j = 1$ and $m = 0$ [30]. Another approach to constrain the coefficients with $m = 0$ involves testing CPT symmetry. For example, bounds on the a-type electron and proton coefficients with $j = 0$ resulted from comparing the $1S$-$2S$ transition in hydrogen with that in antihydrogen [32]. Finally, another method to impose bounds on these coefficients involves comparing the predicted resonance frequency from the standard model with experimental results. Bounds on a-type and c-type coefficients with $m = 0$ were obtained in the electron, proton, and muon sectors through comparisons of the $1S$-$2S$ transition in positronium [12], the $1S$-$2S$ transition in muonium [11], the classical Lamb shift in muonium [11, 31], and the $1S$-$2P$ transition in antihydrogen [33, 34].

The terms in the perturbation (1) depend on the momentum's magnitude. Therefore, a system might be significantly more sensitive to some coefficients than other systems, particularly those with $k = 4$, because the momentum of their constituents is considerably higher. For instance, the proton's momentum is higher in deuterium than in hydrogen due to its motion within the deuteron. For that reason, deuterium can be significantly more sensitive to proton coefficients than hydrogen, even if the sensitivity of hydrogen to the Lorentz-violating frequency shift is considerably better [12, 18]. Another example is muonic hydrogen, where the negative muon has a greater momentum than the positive muon in muonium. Therefore, even if the proposed improved measurement of the hyperfine splitting within the ground state of muonium [35] is the best candidate to improve the bounds on the $k = 0$ muon coefficients, the best candidates to improve the bounds on the $k = 2$ and $k = 4$ coefficients are the corresponding measurements in muonic hydrogen currently pursued [36].

NR coefficients with index $j = 3$ or higher remain unconstrained [26] as all the transitions discussed so far have been between states with total angular momentum smaller than or equal to $F = 1$. Therefore, Lorentz and CPT tests involving transitions to higher angular momentum states could be sensitive to unconstrained NR coefficients. For instance, the proposed measurement of the $2S$-$2P_{3/2}$ transition in muonium might lead to the first bounds on $j = 3$ muon NR coefficients [37]. Another possibility currently being studied by this author is the potential for testing Lorentz and CPT symmetry with molecular hydrogen ions such as H_2^+ and HD^+. The rovibrational transitions within the ground state of these molecules might involve states with $F > 1$ due to the molecular rotation and can be determined to a high precision [38, 39]. The possibility of molecular antihydrogen spectroscopy has also been proposed [38, 40]. A clear advantage of molecular antihydrogen spectroscopy for testing CPT symmetry is the possibility of high-precision spectroscopy involving higher angular momentum states, enabling sensitivity to NR coefficients with $j \geq 3$, which are currently inaccessible in the transitions investigated in antihydrogen [12].

3. Final Remarks

Of the 178 NR coefficients, per particle type, presented in the models in [11, 12, 17, 18, 34], only 28 have been constrained in the muon sector and 44 in the electron, proton, and neutron sectors. This leaves significant room to explore potential violations of CPT and Lorentz symmetry. The primary reason for such a large proportion of unconstrained NR coefficients is the lack of Lorentz and CPT tests involving transitions that involve high angular momentum states. While it is crucial to continue tightening bounds on the NR coefficients—many of which can be improved significantly—it is relevant to develop and conduct tests of Lorentz and CPT symmetry that specifically target these unconstrained coefficients.

References

[1] V.A. Kostelecký and S. Samuel, Phys. Rev. D **39**, 683 (1989); V.A. Kostelecký and R. Potting, Nucl. Phys. B **359**, 545 (1991); Phys. Rev. D **51**, 3923 (1995).

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[2] D. Colladay and V.A. Kostelecký, Phys. Rev. D **55**, 6760 (1997); Phys. Rev. D **58**, 116002 (1998).

[3] O.W. Greenberg, Phys. Rev. Lett. **89**, 231602 (2002).

[4] V.A. Kostelecký and M. Mewes, Phys. Rev. D **80**, 015020 (2009).

[5] V.A. Kostelecký and M. Mewes, Phys. Rev. D **85**, 096005 (2012); J.S. Díaz, V.A. Kostelecký, and M. Mewes, Phys. Rev. D **89**, 043005 (2014); V.A. Kostelecký and Z. Li, Phys. Rev. D **99**, 056016 (2019).

[6] V.A. Kostelecký and M. Mewes, Phys. Rev. D **88**, 096006 (2013).

[7] V.A. Kostelecký and M. Mewes, Phys. Lett. B **757**, 510 (2016); Q.G. Bailey, V.A. Kostelecký, and R. Xu, Phys. Rev. D **91**, 022006 (2015); V.A. Kostelecký and J.D. Tasson, Phys. Lett. B **749**, 551 (2015).

[8] V.A. Kostelecký and Z. Li, Phys. Rev. D **103**, 024059 (2021).

[9] R. Bluhm, V.A. Kostelecký, and C.D. Lane, Phys. Rev. Lett. **84**, 1098 (2000).

[10] V.W. Hughes *et al.*, Phys. Rev. Lett. **87**, 111804 (2001).

[11] A.H. Gomes, V.A. Kostelecký, and A.J. Vargas, Phys. Rev. D **90**, 076009 (2014).

[12] V.A. Kostelecký and A.J. Vargas, Phys. Rev. D **92**, 056002 (2015).

[13] R. Bluhm, V.A. Kostelecký, and N. Russell, Phys. Rev. Lett. **82**, 2254 (1999).

[14] R. Bluhm, V.A. Kostelecký, and N. Russell, Phys. Rev. Lett. **79**, 1432 (1997); R. Bluhm, V.A. Kostelecký, and N. Russell, Phys. Rev. D **57**, 3932 (1998); R.K. Mittleman *et al.*, Phys. Rev. Lett. **83**, 2116 (1999).

[15] V.A. Kostelecký and J.D. Tasson, Phys. Rev. D **83**, 016013 (2011); Phys. Rev. Lett. **102**, 010402 (2009).

[16] Y. Ding and M.F. Rawnak, Phys. Rev. D **102**, 056009 (2020); Y. Ding and V.A. Kostelecký, Phys. Rev. D **94**, 056008 (2016).

[17] V.A. Kostelecký and A.J. Vargas, Phys. Rev. D **98**, 036003 (2018).

[18] A.J. Vargas, Phys. Rev. D **109**, 055001 (2024).

[19] M.J. Borchert *et al.*, Nature **601**, 53 (2022); C. Smorra *et al.*, Nature **575**, 310 (2019); S. Ulmer *et al.*, Nature **524**,196 (2015).

[20] A.J. Acevedo-López *et al.*, Phys. Rev. D 108, 076014 (2023).

[21] E K. Anderson *et al.*, Nature **621**, 716 (2023).

[22] P. Pérez *et al.*, Hyperfine Interact. **233**, 21 (2015).

[23] A. Kellerbauer *et al.*., Nucl. Instr. Meth. B 266, 351-356 (2008).

[24] M.A. Hohensee *et al.*, Phys. Rev. Lett. **111**, 050401 (2013); T. Pruttivarasin *et al.*, Nature **517**, 592 (2015); C. Sanner *et al.*, Nature **567**, 204 (2019); Eli Megidish *et al.*, Phys. Rev. Lett. **122**, 123605 (2019); L.S. Dreissen *et al.*, Nat Commun **13**, 7314 (2022).

[25] V.A. Kostelecký and M. Mewes, Phys. Rev. D **66**, 056005 (2002).

[26] V.A. Kostelecký and N. Russell, Rev. Mod. Phys. **83**, 11 (2011); 2024 edition arXiv:0801.0287v17.

[27] D.F. Phillips *et al.*, Phys. Rev. D **63**, 111101(R) (2001); M.A. Humphrey *et al.*, Phys. Rev. A **68**, 063807 (2003).

[28] F. Allmendinger *et al.*, Phys. Rev. Lett. **112**, 110801 (2014).

[29] M. Smiciklas *et al.*, Phys. Rev. Lett. **107**, 171604 (2011).

[30] L. Nowak *et al.*, Phys. Lett. B **854**, 139012 (2024).

[31] B. Ohayon *et al.*, Phys. Rev. Lett. 128, 011802 (2022).

[32] M. Ahmadi *et al.*, Nature **557**, 71 (2018); Nature **541**, 506 (2017).

[33] M. Ahmadi *et al.*, Nature **578**, 375 (2020).

[34] A.J. Vargas, arXiv:2411.10944 [hep-ph].

[35] P. Strasser *et al.*, EPJ Web of Conferences **198**, 00003 (2019).

[36] C. Pizzolotto *et al.*, Eur. Phys. J. A **56**, 185 (2020); P. Amaro *et al.*, arXiv:2112.00138 [physics.atom-ph].

[37] P. Blumer *et al.*, arXiv:2412.19580 [hep-ph].

[38] M.R. Schenkel, S. Alighanbari, and S. Schiller, Nat. Phys. **20**, 383 (2024).

[39] S. Alighanbari *et al.*, Nature **581**, 152 (2020); S. Patra *et al.*, Science **369**, 1238 (2020).

[40] H. Dehmelt, Phys. Scr. **T59**, 423 (1995); E.G. Myers, Phys. Rev. A **98**, 010101 (2018); M.C. Zammit *et al.*, Phys. Rev. A **100**, 042709 (2019).

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