

Conceptual Basics of CPT Violation

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This work provides an overview of some theoretical and conceptual ideas regarding CPT tests with antimatter. In particular, interpretational challenges of CPT tests outside a proper framework are highlighted. The ensuing need for taking into account the connection between CPT symmetry, translation invariance, and Lorentz symmetry is reviewed. This connection is properly addressed within the SME framework, whose construction is briefly discussed.

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1. Introduction

The prediction of antimatter is widely regarded as a hallmark success of quantum field theory (QFT). Although antimatter companions of the known elementary particles have been observed directly in nature, the physical properties and laws pertaining to antimatter have often only been inferred from the corresponding investigations with ordinary matter. This methodology rests on the idea of CPT invariance, a basic property shared by all realistic Lorentz-symmetric QFTs. CPT invariance is a discrete spacetime symmetry under the combined transformations of charge conjugation (C), parity inversion (P), and time reversal (T). Roughly speaking, its key consequence is that the behaviors of a physical system and its antimatter conjugate are essentially identical.

However, cosmological observations indicate that the amounts of baryonic matter and antimatter are unequal in the Universe. While established CPT-invariant physics in the form of the Standard Model (SM) of particle physics exhibits the qualitative features to generate a baryon asymmetry, it has thus far been unsuccessful in accommodating its observed value. Many approaches to address this tension between theory and experiment involve physics beyond the SM. In particular, it is known that hypothetical Planck-suppressed CPT violation may produce the size of the observed asymmetry bypassing some of the usual Sakharov criteria [1].

In a related development, minuscule violations of CPT invariance can be accommodated in various candidate fundamental theories seeking a unified understanding of quantum and gravitational physics [2–6]. This development has provided impetus for the theoretical study of CPT breakdown at presently accessible energy scales culminating in the construction of the Standard-Model Extension (SME) test framework [7]. The SME framework, in turn, has led to a surge of experimental CPT investigations involving not only baryons, but also many other systems [8–16].

The circumstances recounted above indicate that CPT tests involving antimatter, and especially those with antiprotons and antihydrogen, assume particular urgency. The present work reviews the theoretical foundations underlying CPT symmetry and CPT violation, highlights the relation to Lorentz symmetry, and reviews various ideas behind the SME test framework. Specific SME predictions for antiprotons and antihydrogen are covered in a separate contribution to this volume [17]. Throughout, we work in natural units with $c = \hbar = k_B = 1$, and $\eta^{\mu\nu} = \text{diag}(+1, -1, -1, -1)$.

2. Conceptual Considerations

In the SM, the individual C, P, and T transformations along with any product containing two different transformations from this set fail to be associated with exact symmetries. It might therefore seem reasonable that departures from CPT invariance, if discovered, would represent a simple extrapolation of this observed pattern of discrete-symmetry breaking. However, departures from CPT invariance would be much more profound: as opposed to the above violations that have been observed, they are confined by the boundaries provided by the celebrated CPT theorem [18]. This fact entails various conceptual ramifications that differentiate putative CPT breaking from other discrete-symmetry violations. This section contains a few remarks highlighting some of these complexities.

A natural idea for an experimental test of a discrete symmetry R involves the comparison of a physical system S , which has observables Ω_j , to its R -conjugate system \bar{S} , which has the corre-

sponding observables $\bar{\Omega}_j$. Observations performed on S and \bar{S} that yield

$$\bar{\Omega}_j = R \cdot \Omega_j \quad (1)$$

for some observables labeled by j support R invariance, while measurements that show

$$\bar{\Omega}_k \neq R \cdot \Omega_k \quad (2)$$

for some other observables $k \neq j$ establish violation of R symmetry. Many actual tests of discrete spacetime symmetries R indeed contain elements of this idea.

However, on closer inspection it is usually infeasible to test the relation (1) or establish the violation (2) directly. For practical reasons, it is often necessary to perform the measurements on the system S at times and locations and with orientations of the apparatus that differ from those of the corresponding measurements on the system \bar{S} . Such a situation arises, for example, if antimatter measurements are made at CERN's AD and the equivalent matter versions of these experiments at a laboratory in Vienna. A more accurate description of the relation being tested is then

$$\bar{\Omega}_j \stackrel{?}{=} R \cdot \mathcal{T} \cdot \Lambda \cdot \Omega_j, \quad (3)$$

where the spacetime translation \mathcal{T} accounts for location and time differences between the two conjugate measurements and the Lorentz transformation Λ implements orientation and velocity differences between frames associated with the two experiments.

Suppose now that measurements establish

$$\bar{\Omega}_j \neq R \cdot \mathcal{T} \cdot \Lambda \cdot \Omega_j. \quad (4)$$

Strictly speaking, we can then only conclude that $R \cdot \mathcal{T} \cdot \Lambda$ is not an exact symmetry. In the case that $R \neq CPT$ represents one of the other discrete spacetime transformations $R \in \{C, P, T, PT, CT, CP\}$ one may proceed in a self consistent way under the assumption that both translation and Lorentz symmetry continue to hold and conclude that it is indeed R that is violated. This is a consequence of the fact that there is practically no interdependence between violations of R on the one hand and violations of \mathcal{T} and Λ on the other hand, as is exemplified by the electroweak theory.

If, however, $R = CPT$ an analogous straightforward interpretation is hampered by the CPT theorem. In essence, this theorem states

$$\text{QFT} + \mathcal{T} \text{ symmetry} + \Lambda \text{ symmetry} + \text{mild math. assumptions} \rightarrow \text{CPT symmetry}. \quad (5)$$

If it is assumed that the measurement (4) arises as a result of CPT violation, not all ingredients of the CPT theorem can be correct, i.e., violations of CPT on the one hand and violations of \mathcal{T} and Λ on the other hand are correlated. In particular, it turns out that theoretically viable interpretations of the measurement (4) exist that only violate Lorentz symmetry and leave CPT intact.

3. CPT Theorem and CPT Violation

To obtain further insight into the correlation between CPT, \mathcal{T} , and Λ it is worthwhile to examine the CPT theorem (5) in a little more detail. This section is aimed at sketching the key ideas behind the proof of this theorem.

In general, it is straightforward to identify examples in which a discrete spacetime transformation is intertwined with a continuous spacetime transformation. Consider a 2-dimensional (x, y) space. On this space, we may define a parity transformation P_2 , such that $P_2(x, y) = (-x, -y)$. We may also define a rotation $R_2(\theta)$ by an angle θ : $R_2(\theta)(x, y) = (x \cos \theta + y \sin \theta, y \cos \theta - x \sin \theta)$. The crucial observation is that a parity transformation is equal to a rotation by $\theta = 180^\circ$, i.e.,

$$P_2 = R_2(180^\circ). \quad (6)$$

It follows that parity violation means that the rotation $R_2(180^\circ)$ is not a symmetry either, so there cannot be rotation invariance in this situation: parity breaking implies rotation symmetry violation.

The above shows that a discrete transformation may be an element of a continuous set of transformations, and this insight captures a key idea in the CPT theorem. In that case, CPT represents the discrete transformation and Λ the continuous set of transformations. However, one additional ingredient is required because CPT is not an element of the real-valued Lorentz transformations.

To illustrate this, consider the Lorentz-transformation matrix of the coordinate vector x^μ :

$$\Lambda(w, \theta) = \begin{pmatrix} \cosh w & \sinh w & 0 & 0 \\ \sinh w & \cosh w & 0 & 0 \\ 0 & 0 & \cos \theta & \sin \theta \\ 0 & 0 & -\sin \theta & \cos \theta \end{pmatrix}. \quad (7)$$

This Lorentz transformation consists of a boost along the x^1 direction with speed v , which corresponds to rapidity $w = \operatorname{artanh} v$, and a rotation about the x^1 axis by an angle θ . The CPT transformation acting on x^μ flips its overall sign due to P and T. The C transformation typically involves hermitian conjugation, so that x^μ is expected to remain unaffected here. It is thus apparent that

$$CPT = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad (8)$$

implements a CPT transformation on x^μ . As opposed to the 2-dimensional example discussed above, CPT is not a real Lorentz transformation since $\cosh w > 0$ for physical boosts $w \in \mathbb{R}$.

However, most physics relations remain formally valid for complex numbers and can be analytically continued into the complex plane. With this in mind, note that $\cosh i\pi = -1$, so that

$$\Lambda(w = i\pi, \theta = 180^\circ) = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad (9)$$

i.e., the CPT transformation on x^μ can be implemented by a complex-valued Lorentz transformation on x^μ .

To complete the proof the CPT theorem, conditions must be specified that ensure the required smoothness of the physics relations and thereby allowing analytic continuation, and the reasoning must be extended to all observables, not just x^μ . From a physics perspective, these conditions are

quite general. They primarily consist of features related to spacetime symmetries and quantum mechanics. One of these is the unitary implementation of Lorentz and translation symmetry. The eigenvalues of the momentum operator P^μ , which implements translations, must obey $P^\mu P_\mu \geq 0$ ensuring energy positivity in all frames. There is also a condition that guarantees microcausality.

In essence, CPT may be viewed as a complex Lorentz transformation. The observation of CPT breaking would therefore imply that the complex Lorentz transformations cannot be a symmetry of the theory. In such a case, physical Lorentz symmetry has to be violated or our fundamental laws of physics cannot be smooth. A rigorous version of this reasoning can be found in Refs. [19].

4. Standard-Model Extension

The interrelationship between CPT invariance and translation and Lorentz symmetry discussed above makes the interpretation of CPT tests, such as matter–antimatter comparisons based on Eq. (4), challenging. It is therefore desirable to have a theoretical framework that describes hypothetical departures from these symmetries in a self consistent and general way: CPT experiments can then be analyzed within this framework permitting the determination which physics principle is actually being tested. Such a framework, the aforementioned SME, exists. It has been constructed for the study of CPT and Lorentz breakdown maintaining other desirable physics principles. What follows is an overview of the key ideas behind the construction of the SME framework.

One of these is an approach utilizing effective field theory (EFT). EFT has a long been successfully employed in various subfields of physics including elementary-particle, nuclear, and condensed-matter systems. Moreover, this approach is theoretically well understood and provides a general and flexible theoretical tool for the description of dynamical systems with large numbers of degrees of freedom. It is thus natural to expect that the low-energy effects of putative CPT and Lorentz violation can be captured within EFT. This assumption entails a lagrangian formulation for the SME.

The next step is the specification of the SME lagrangian, which is based on two additional key ideas. One of these concerns the characterization of departures from CPT and Lorentz symmetry. Lorentz breaking can be implemented naturally via preferred direction in the form of non-dynamical external vector or tensor coefficients $b^\mu, c^{\mu\nu}, \dots$. It turns out that a subset of these coefficients also control the type and extent of CPT violation. In the flat-spacetime limit, these SME coefficients may be taken as constant, maintaining energy–momentum conservation. Of note here is the analogy to the spontaneous violation of a conventional gauge symmetry, which also generates a constant background carrying the indices of the appropriate representation of the corresponding gauge group.

The other idea necessary for the construction of the lagrangian is coordinate independence: although CPT and Lorentz symmetry are violated, it should still be possible to choose any suitable reference frame for the mathematical description of physical laws. In other words, coordinate systems are simply a product of human thought and should not acquire physical significance. Coordinate independence is guaranteed if the laws of physics are formulated in terms of geometrical quantities, such as scalars, vectors, tensors, and spinors. This implies that the SME coefficients $b^\mu, c^{\mu\nu}, \dots$ should enter the SME lagrangian with all their indices contracted. Note again the analogy to the spontaneous violation of a conventional gauge symmetry: each term in the lagrangian emerges from a gauge singlet with all group indices properly contracted. This features is then trivially maintained, when one field operator in such a term acquires a vacuum expectation value.

These ideas lead to the following general structure of the SME lagrangian \mathcal{L}_{SME} :

$$\mathcal{L}_{\text{SME}} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{EH}} + \delta\mathcal{L}_{\text{SME}}. \quad (10)$$

Here, \mathcal{L}_{SM} and \mathcal{L}_{EH} are the ordinary SM and Einstein–Hilbert pieces, and $\delta\mathcal{L}_{\text{SME}}$ contains small CPT- and Lorentz-violating corrections constructed according to the reasoning above:

$$\delta\mathcal{L}_{\text{SME}} = -\bar{\psi}b^\mu\gamma_5\gamma_\mu\psi + i\bar{\psi}c^{\mu\nu}\gamma_\mu\partial_\nu\psi + \dots, \quad (11)$$

where b^μ and $c^{\mu\nu}$ are the aforementioned examples of preferred directions. They are usually assumed to be generated by underlying physics. Within the present context, they represent phenomenological coefficients that control Lorentz violation. We note that the b^μ coefficient also governs certain types of CPT breaking, while $c^{\mu\nu}$ is CPT even.

Other desirable physics features, such as the conventional $U(1) \times SU(2) \times SU(3)$ gauge symmetry in the SM, are usually imposed on the SME. The brief overview above has primarily highlighted the flat-spacetime SME. However, the SME construction can be generalized to curved-spacetime situations when gravity is present. We finally note that the SME has provided the basis for numerous theoretical and mathematical studies of Lorentz and CPT violation [20–25], and that the framework has also found applications in adjacent research fields [26].

As an EFT, the SME lends itself to a natural classification of CPT- and Lorentz-breaking perturbations according to their mass dimension d . The restriction to $d = 3, 4$ is often called the minimal SME (mSME), and it is expected that this restriction collects the effects that dominate at low energies. However, for the broadest possible generality, the full SME also contains non-minimal contributions characterized by $d > 4$. For instance, specific underlying models, such as noncommutative field theory, are indeed known to generate Lorentz violation starting at $d = 5$. This classification by mass dimension can be refined in various ways including by considering the C, P, and T transformation properties of a given SME term.

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