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Anthropic Considerations for Big Bang Nucleosynthesis

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I present recent work on improving constraints for fundamental constants from Big Bang Nucleosynthesis (BBN). After giving an idea of the general usefulness of using BBN as a laboratory to test theories beyond the standard model, the effects of varying the fine-structure constant and the Higgs vacuum expectation value (VEV) on primordial abundances are laid out. Using five publicly available codes, BBN is simulated with different values for these two constants and possible constraints on the variations are derived from experimental data of primordial light element abundances.

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1. Introduction

Fundamental constants like the fine-structure constant, quark masses or the Fermi constant show up not only everywhere in physics but in all sciences and huge efforts go into measuring these constants to highest precision. Looking at the values and uncertainties of these constants in the lists by the Particle Data Group (PDG) [1], one can see that some of them are even known to precisions of one part in a billion. Dirac was one of the firsts to propose that these constants are not, in fact, constant but may have changed over cosmological time scales [2] and many others had similar ideas, e.g. [3, 4]. One way to test these proposals from a theory perspective is using Big Bang Nucleosynthesis (BBN) as a laboratory [5–8]. BBN is the first process in the history of our universe that is well understood. One can predict the primordial abundance for Helium-4 with surprising accuracy even from very simple approximations. In this talk, I present our recent work on the variation of the fine-structure constant α [9, 10] and the Higgs Vacuum Expectation Value (VEV) [11] and its effects on primordial abundances of the lightest elements.

2. Big Bang Nucleosynthesis

An abundance Y_i is defined as an element's number density n_i, $i = \mathrm{H}, d, {}^3\mathrm{He}, {}^4\mathrm{He}, {}^6\mathrm{Li}, \ldots$, divided by the total baryon number density n_b and it depends on the expansion of space, i.e. on the Hubble constant [12], and on the rates of reactions and decays that either create or destroy element i. Plugging in initial conditions from the electron-neutrino plasma and the weak neutron-proton rates, one needs to solve a network of differential equations like

$$\dot{Y}_i = -\sum_{j,k,l} Y_i Y_j \gamma_{i+j\to k+l} + -\sum_{k,l,j} Y_k Y_l \gamma_{k+l\to i+j} - \sum_{j,k,l} Y_i \Gamma_{i\to j+k+l} + \sum_{k,j,l} Y_k \Gamma_{k\to i+j+l}, \quad Y_i = \frac{n_i}{n_b}, \tag{1}$$

where Γ and γ are decay and reaction rates, respectively, and one sums over all possible reactions and decays that involve element i. An example of such a network of reactions is displayed in figure 22 of [8].

The first code with which one could simulate BBN and calculate some primordial abundances as functions of time was written by Wagoner et al. [13], and many others followed. In our work we used five different publicly available codes: `NUC123` [14], `PArthENoPE` [15], `PRIMAT` [8, 16], `AlterBBN` [17] and the very recent `PRyMordial` [18]. Comparing results for the primordial abundances from all of these codes provides an estimate of the systematical errors from choosing a different number of reactions, sources for the rates and numerical methods.

The rough course of BBN is described in the following: at first, weak reactions between neutrons, protons, electrons and neutrinos dominate. The number density ratio of neutrons to protons

$$\frac{n_n}{n_p} = e^{-Q_n/T} \tag{2}$$

depends on the temperature T and the neutron-proton mass difference Q_n (we chose units $k_B = c = \hbar = 1$). At temperatures of roughly 1 MeV, which are reached at time $t \approx 1$ s after the Big Bang, these weak reactions freeze out, i.e., their rates become smaller than the Hubble expansion rate. From then on, the neutron-to-proton number density ratio falls off slowly because of β-decay of free neutrons. After about one minute, the so-called "deuterium bottleneck" is reached that is a result of the small deuteron binding energy: the first nuclear fusion reaction $n+p \to d+\gamma$ can now efficiently produce deuterium. Before this time, the universe was still too hot and the produced photons were so energetic that they immediately dissolved the deuteron again. Then, with deuterium as the next important building block, nuclear fusion effectively starts. After about three minutes, the element abundances are more or less constant until very much later when more and heavier elements are fused in stars. In BBN, elements up to ^{7}Be are considered to be the most relevant.

3. Variation of the fine-structure constant

First, I presented results I derived together with my collaborators Ulf-G. Meißner and Bernard Metsch during the course of my Master's thesis, which were published in [9]. We studied BBN under the variation of the fine-structure constant α, improving considerations made in earlier works, e.g. in [19–23].

The fine-structure constant α appears everywhere in BBN where electromagnetic interactions are relevant. The most prominent effect is the Coulomb barrier that has to be overcome when two positively charged particles interact. This barrier can be parameterized as an energy-dependent "penetration factor" [24] in the cross section that depends on α exponentially. One needs to consider this Coulomb barrier penetration for the incoming and outgoing particles, given that they are charged. For radiative capture reactions, i.e. for reactions with a photon in the final state, there is no final-state penetration factor, of course, but a factor of α appearing in the cross section from the photon coupling. Additional factors parameterizing effects from external capture processes are described in more detail in [20] and references therein. A smaller effect on primordial abundances have final-state Coulomb interactions in β-decays: the electron or positron that is created interacts with the daughter nucleus, resulting in a linear α-dependence of the decay rate. The same applies also to the free neutron decay, which results in a variation of the neutron lifetime. Apart from direct α dependencies of the rates, there are also indirect effects to consider. First, there is a Coulomb contribution to the nuclear binding energies that changes linearly with α. For heavier nuclei the Bethe-Weizsäcker formula [25] is a good enough approximation, but for light nuclei, which are most relevant for BBN, it is not so precise. Therefore, we used values for the Coulomb contributions that were calculated in the framework of Nuclear Lattice Effective Field Theory (NLEFT) and provided to us by S. Elhatisari [26]. Changing the nuclear binding energies changes their masses and thus the reaction Q-value, i.e. the mass difference of in- and outgoing particles. This Q-value appears also in the Coulomb penetration factor of the cross section. Finally, the neutron-proton mass difference has a QED contribution that was recently updated in [27]. Changing the neutron-proton mass difference has a sizeable effect on the weak $n \leftrightarrow p$ reaction rates and the neutron to proton number density ratio. All these considerations, differences to earlier literature and the individual sizes of different effects were laid out in detail in [9].

After implementing all these changes into the five codes and simulating BBN for α varying by $\pm 10\%$, we find the results displayed in fig. 1. There are reliable experimental values available only for Helium-4, deuterium and Lithium-7 [1]. Lithium-7 is not suited for finding constraints on the α variation because its theoretically predicted value is three times higher than the observed one, a phenomenon called the "Lithium problem" [28]. The limits for $\delta\alpha$, the relative difference in α

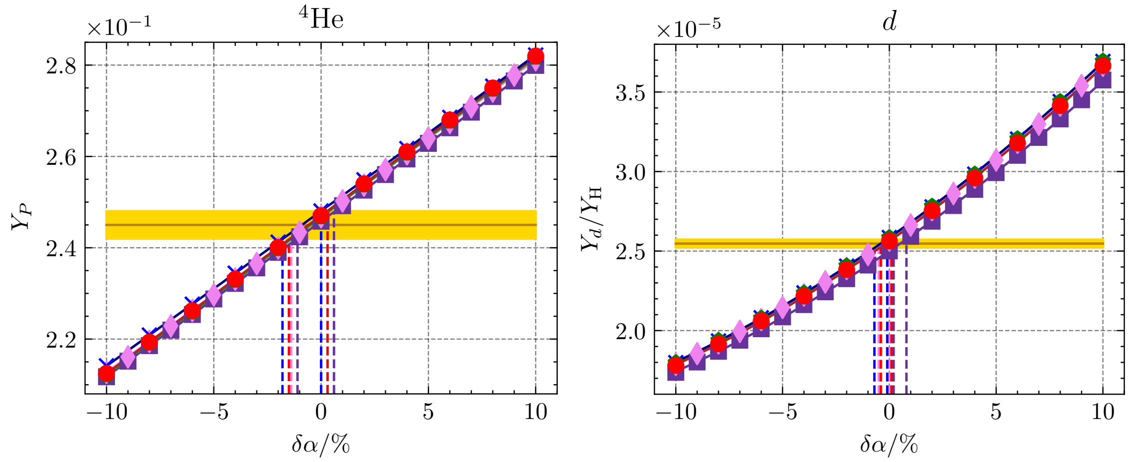

Figure 1: Primordial abundance for Helium-4 (left) and deuterium (right) calculated with five different codes (blue: `AlterBBN`, green: `PRyMordial`, red: `PRIMAT`, pink: `PArthENoPE`, violet: `NUC123`) and compared to available experimental values from the PDG [1] with 1σ error band (yellow band). Here, $\delta\alpha$ is the relative α variation.

can be derived by finding the minimally and maximally possible values for which the calculated abundances are still within uncertainties of the experiment. We could constrain $|\delta\alpha| < 1.8\%$ which is narrower that what was found in earlier works.

The main uncertainty in these considerations comes from the nuclear reaction rates. We could already improve upon earlier works by parameterizing the most relevant cross sections by performing fits to experimental data in [9] so that we could properly include the energy-dependent penetration factor in our calculations. Still, apart from the $n + p \rightarrow d + \gamma$ reaction cross section, for which a pionless effective field theory (EFT) approach was derived in [29], no reliable theoretical predictions for reaction rates were implemented into the codes. This led to our following work on finding the α-dependence of radiative capture rates found in the Halo EFT framework [10] (for a review on Halo EFT see [30]). The relevant rates all included Lithium-7 or Beryllium-7, and they diverged for $\delta\alpha < -5\%$. While this had no noticeable effect on the other abundances, the $^7\text{Li} + {}^7\text{Be}$ abundance also diverges, see fig. 2. We have yet to understand if this result is indeed physically relevant or simply an artefact of the Halo EFT approach that might not be applicable at certain values of α (including changing binding energies).

4. Variation of the Higgs VEV

In the second part of my talk I presented our recent results for varying the Higgs VEV in Big Bang Nucleosynthesis [11]. This work was mainly inspired by [31]. Keeping the Yukawa couplings fixed, all elementary particle masses depend linearly on the Higgs VEV and changing the latter has a number of effects in BBN. Burns et al. [31] were the firsts to consider changes in the QCD scale Λ_{QCD} due to changes in the Higgs VEV. We implemented the following changes into our BBN simulation:

- The QCD-scale scales with $\Lambda_{\text{QCD}} \propto (1 + \delta v)^{0.25}$ [31].

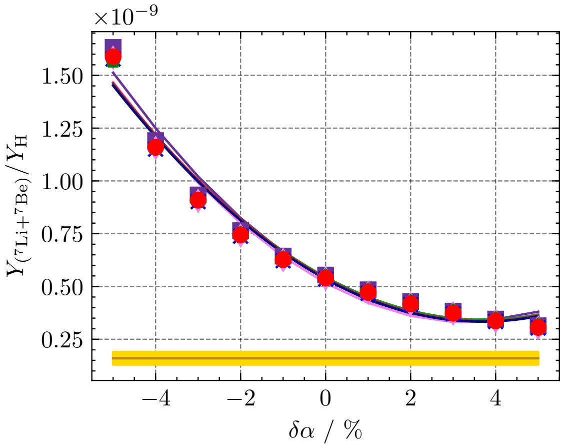

Figure 2: Primordial abundance for Lithium-7 and Beryllium-7 calculated with five different codes (blue: `AlterBBN`, green: `PRyMordial`, red: `PRIMAT`, pink: `PArthENoPE`, violet: `NUC123`)and compared to the experimental values from the PDG [1] with 1σ error band (yellow band).

- Because the W-boson mass also depends on the Higgs VEV, the Fermi constant relevant for weak reactions changes: $G_F \propto (1+\delta v)^{-2}$.
- The electron mass changes in the electron-neutrino plasma and weak interactions.
- The quark masses vary with the Higgs VEV. Through the Gell-Mann Oakes-Renner relation this is equivalent to a change in the pion mass. This affects in turn
  - the QCD part of the neutron-proton mass difference [27],
  - the deuteron binding energy (see fig. 3) and other nucleon-nucleon scattering parameters (we derived the dependencies using lattice QCD data [32–40] and low energy theorems [41, 42]),
  - and the nucleon mass and axial vector coupling (the dependence given by lattice QCD calculations [43–45]).

Plugging all of these changes into pionless EFT cross section for the $n + p \to d + \gamma$ reaction derived in [29], we found that especially changing the deuteron binding energy has a siezable effect on the $n + p \to d + \gamma$ rate, which in turn affects the deuteron abundance immensly. A final effect that we had to add into this calculation in our recent erratum to [11], is that changing the deuteron binding energy affects also the backwards reaction $d + \gamma \to n + p$ and therefore shifts the time when BBN starts (deuterium bottleneck). In fig. 5, these updated results for the Helium-4 and deuterium abundances are shown. This time, we also used the newest observation by the EMPRESS collaboration [46] to find constraints for the Helium-4 abundance.

Especially the deuterium abundance varies drastically with the Higgs VEV and so only a Higgs VEV variation of $-0.07\% \leq \delta v \leq -0.02\%$ is consistent with experiment. These bounds are much stronger than what was found in earlier works.

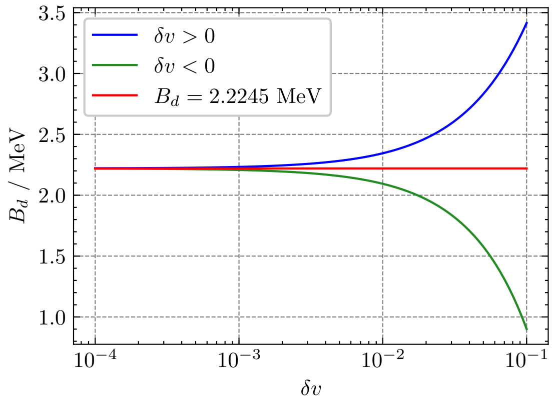

Figure 3: Deuteron binding energy as a function of the Higgs VEV calculated using lattice QCD data and low energy theorems.

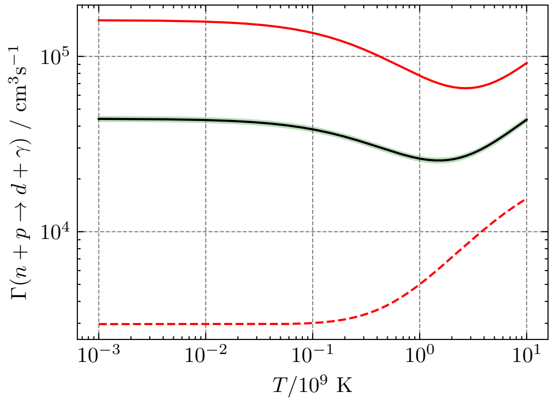

Figure 4: The $n + p \longrightarrow d + \gamma$ reaction rate for a Higgs VEV variation of +10% (red, solid) and −10% (red, dashed) compared to the original rate from [29] (black) and the approximation made in [31] that the rate scales like $\propto (1 + \delta v)^{0.25}$ (green band).

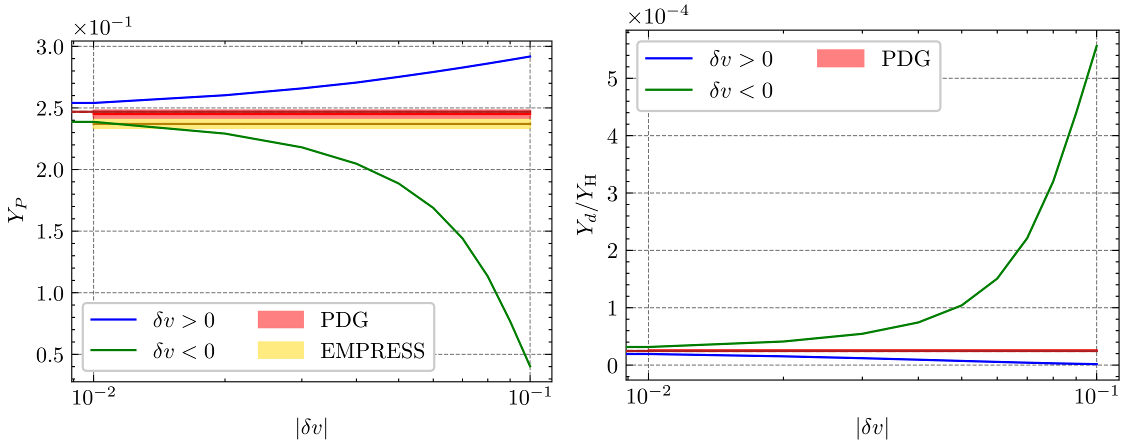

Figure 5: Helium-4 (left) and deuterium abundance (right) calculated with the `PRyMordial` code as a function of the relative difference in the Higgs VEV, compared to experimental values by the PDG [1] and the EMPRESS collaboration [46].

5. Conclusion and Outlook

We used five different publicly available codes to simulate BBN under the variation of both the fine-structure constant and the Higgs VEV and found constraints on these variations from comparing the simulated light element abundances to experimental data. We found that for the fine-structure constant only $|\delta\alpha| < 1.8\%$ and for the Higgs VEV only $-0.07\% \leq \delta v \leq -0.02\%$ would be consistent with measurement.

In future, it would be interesting to do a combined analysis of changing different fundamental constants at the same time or considering BBN under a varying strange quark mass (on-going). We are currently also working on a more quantitative and detailed error analysis of the past calculations. As I mentioned, the main source of uncertainties are reaction cross sections and rates, so it would be important to find more theoretical predictions for these rates to increase precision of these kinds of analysis, especially as cosmological observations become more and more precise. We are therefore currently working on implementing deuteron-deuteron reaction rates in the framework of NLEFT.

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