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Manifestly Lorentz invariant formulation of chiral effective field theory for neutrinoless double beta decay

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The neutrinoless double beta decay process of two neutrons, $nn \to ppee$, is studied in the manifestly Lorentz invariant formulation of chiral effective field theory. Due to the better ultraviolet behavior of the relativistic scattering equation, the $nn \to ppee$ contact term is not needed for renormalization at leading order, in contrast to the nonrelativistic case. The predicted $nn \to ppee$ amplitude is consistent with the previous result from the generalized Cottingham formula at 10% level. The present approach is validated by reproducing the charge symmetry breaking and charge independence breaking in the nucleon-nucleon (NN) scattering length.

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1. Introduction

Neutrinoless double beta decay ($0\nu\beta\beta$) is a second-order weak process, in which a nucleus decays to its neighboring nucleus by turning two neutrons into two protons, emitting two electrons but no corresponding antineutrinos [1]. Its observation would signal lepton number violation and other implications of new physics beyond the standard model. Therefore, it becomes one of the top priorities in the field of nuclear and particle physics and stimulates worldwide experimental searches; see Ref. [2] for a recent review.

Nuclear matrix element provides the bridge between the observable $0\nu\beta\beta$ half-life and the beyond-standard-model parameters. Within the standard picture of $0\nu\beta\beta$ decay that involves long-range light neutrino exchange [3], as we consider here, current knowledge of the nuclear matrix element is not satisfactory [4], as various nuclear models lead to discrepancies as large as a factor of around 3.

Chiral effective field theory (EFT) plays an important role in addressing such uncertainties. It can provide the nuclear Hamiltonian and weak currents in a consistent and systematically improvable manner; see [5] for a review. Recently, *ab initio* calculations of nuclear matrix elements using chiral nuclear Hamiltonians were made available [6–8].

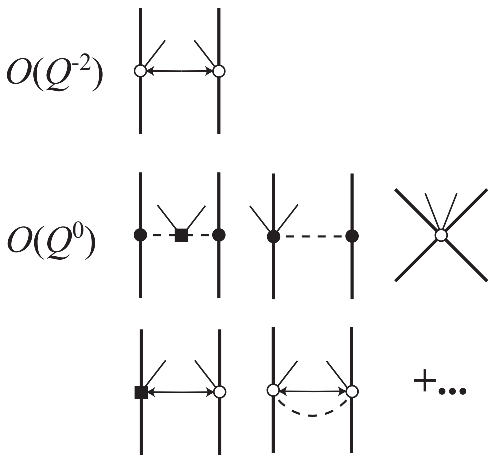

Figure 1: Hierarchy of neutrinoless double beta decay operator in chiral effective field theory, in the Weinberg power counting. The thick solid lines, thin solid lines, and dashed lines represent nucleon, lepton, and pion fields, respectively. A double arrow denotes a neutrino exchange. The empty dots, solid dots, and squares denote vertices of $O(Q^0)$, $O(Q^1)$, $O(Q^2)$, respectively, with Q the expansion parameter in chiral EFT.

According to the Weinberg power counting, the only leading-order (LO) contribution to the $0\nu\beta\beta$ decay operator comes from the long-range neutrino exchange [9], as shown in Fig. 1. However, in the nonrelativistic heavy baryon formulation, a renormalization group analysis showed that a $nn \to ppee$ contact term should be promoted to LO to ensure the renormalizability of the $nn \to ppee$ transition amplitude [10, 11]. In principle, the size of this contact term should be determined by matching to first-principles gauge field theory calculations, which, however, are not yet available [12]. This unknown contact term leads to an additional source of uncertainty for the nuclear matrix elements in addition to the nuclear-structure ones. Subsequently, a generalized

Cottingham formula was proposed to estimate the size of this $nn \to ppee$ contact term [13, 14]. However, the estimate has systematic uncertainties from neglecting the inelastic intermediate states and the model-dependent inputs in the intermediate-momentum region. The former uncertainty was alleviated by quantifying the lowest-lying collection of inelastic states, i.e. πNN states, in a subsequent work [15].

In contrast to the nonrelativistic heavy baryon formulation, it was found that the manifestly Lorentz invariant formulation of chiral EFT can naturally ensure the renormalizability of the $nn \to ppee$ transition amplitude [16], without promoting the uncertain contact term. In this paper, the manifestly Lorentz invariant formulation of chiral EFT approach for the $nn \to ppee$ process is introduced. The validation of this approach in the charge symmetry breaking (CSB) and charge independence breaking (CIB) in the nucleon-nucleon (NN) scattering process is also presented.

2. Theoretical framework

2.1 Manifestly Lorentz invariant chiral EFT

We start from a manifestly Lorentz invariant chiral Lagrangian relevant at leading order [17],

$$\begin{aligned}\mathcal{L}_{\Delta L=0} = &\frac{f_\pi^2}{4}\mathrm{tr}[u_\mu u^\mu + m_\pi^2(uu + u^\dagger u^\dagger)]\\ &+\overline{\Psi}(\mathrm{i}\gamma^\mu D_\mu - M + \frac{g_A}{2}\gamma^\mu\gamma_5 u_\mu)\Psi - \sum_\alpha \frac{C_\alpha}{2}(\overline{\Psi}\Gamma_\alpha\Psi)^2,\end{aligned} \tag{1}$$

where $f_\pi = 92.2$ MeV is the pion decay constant, $g_A = 1.27$ is the nucleon axial coupling, M denotes the nucleon mass, and C_α ($\alpha = S, V, AV, T$) are the low-energy constants (LECs). This Lagrangian consists of the pion field $u = \exp[\mathrm{i}\vec{\tau}\cdot\vec{\pi}/(2f_\pi)]$ and the nucleon field $\Psi = (p, n)^T$, which are coupled to the weak current l_μ via the axial vector $u_\mu = \mathrm{i}u^\dagger(\partial_\mu - \mathrm{i}l_\mu)u - \mathrm{i}u\partial_\mu u^\dagger$ and the chirally covariant derivative $D_\mu = \partial_\mu + \frac{1}{2}[u^\dagger(\partial_\mu - \mathrm{i}l_\mu)u + u\partial_\mu u^\dagger]$. The weak current reads $l_\mu = -2\sqrt{2}G_F V_{ud}\tau^+\overline{e}_L\gamma_\mu\nu_{eL}$ + h.c., with the Fermi constant G_F and the V_{ud} element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [18, 19].

In the standard mechanism of $0\nu\beta\beta$ decay, the lepton number violation is induced by the electron-neutrino Majorana mass

$$\mathcal{L}_{\Delta L=2} = -\frac{m_{\beta\beta}}{2}\nu_{eL}^T C\nu_{eL}, \tag{2}$$

where $C = \mathrm{i}\gamma_2\gamma_0$ denotes the charge conjugation matrix, and $m_{\beta\beta}$ the effective neutrino mass.

The LO contribution to the NN scattering amplitude is obtained by solving the relativistic scattering equation

$$T(\boldsymbol{p}', \boldsymbol{p}) = V(\boldsymbol{p}', \boldsymbol{p}) + \int \frac{\mathrm{d}^3k}{(2\pi)^3}\frac{M^2}{\boldsymbol{k}^2 + M^2}\frac{V(\boldsymbol{p}', \boldsymbol{k})T(\boldsymbol{k}, \boldsymbol{p})}{E - 2\sqrt{\boldsymbol{k}^2 + M^2} + \mathrm{i}0^+}, \tag{3}$$

where E is the total energy, and $\boldsymbol{p}'$ and $\boldsymbol{p}$ are the nucleon outgoing and incoming momenta in the center of mass frame, respectively. This equation is consistent with the three-dimensional reduction of the Bethe-Salpeter equation [20] and satisfies the relativistic elastic unitarity. The LO potential $V = \overline{\varphi}_0 \otimes \overline{\varphi}_0 \mathcal{V} \varphi_0 \otimes \varphi_0$ is defined by the LO two-nucleon irreducible diagrams sandwiched between the leading term of the Dirac spinor $\varphi(\boldsymbol{p}, s)$ expanded in powers of small momenta $\boldsymbol{p}$. As a result, the LO NN and neutrino potentials take the form of those in Weinberg's approach,

$$V_{NN}(\boldsymbol{p}', \boldsymbol{p}) = -\frac{g_A^2}{4f_\pi^2}\vec{\tau}_1 \cdot \vec{\tau}_2 \frac{\boldsymbol{\sigma}_1 \cdot \boldsymbol{q}\boldsymbol{\sigma}_2 \cdot \boldsymbol{q}}{\boldsymbol{q}^2 + m_\pi^2} + C_1 + C_2\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2, \tag{4}$$

$$V_\nu(\boldsymbol{p}', \boldsymbol{p}) = \frac{\tau_1^+\tau_2^+}{\boldsymbol{q}^2}\left[1 - g_A^2\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + g_A^2\boldsymbol{\sigma}_1 \cdot \boldsymbol{q}\boldsymbol{\sigma}_2 \cdot \boldsymbol{q}\frac{2m_\pi^2 + \boldsymbol{q}^2}{(\boldsymbol{q}^2 + m_\pi^2)^2}\right], \tag{5}$$

where $\boldsymbol{q} = \boldsymbol{p}' - \boldsymbol{p}$, and $C_1 = C_S + C_V$ and $C_2 = -C_{AV} + 2C_T$ are two independent LECs.

Note that the present derivation is similar to the so-called modified Weinberg approach [21], which was applied to nucleon-nucleon scattering problem. It has been found to improve the renormalizability of nucleon-nucleon scattering [21] and few-body systems [22, 23]. In the heavy baryon approach, the nonrelativistic expansion of the Lagrangian leads, instead of Eq. (3), to the Lippmann-Schwinger equation. It applies the nonrelativistic limit $(1/M \to 0)$ of the two-nucleon propagator,

$$\frac{M^2}{\boldsymbol{k}^2 + M^2}\frac{1}{E - 2\sqrt{\boldsymbol{k}^2 + M^2} + \mathrm{i}0^+} \to \frac{1}{E_{\mathrm{kin}} - \boldsymbol{k}^2/M + \mathrm{i}0^+}. \tag{6}$$

The relativistic propagator has a milder ultraviolet behavior than the nonrelativistic one. As a result of this milder UV behavior, the $0\nu\beta\beta$ amplitude can be renormalized without promoting a contact term to the LO neutrino potential.

2.2 $nn \to ppee$ process

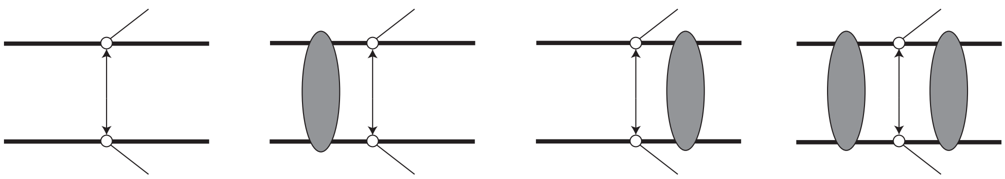

Figure 2: Leading-order contributions to the amplitude of $nn \to ppe^-e^-$. The thick and thin lines denote nucleon and lepton fields, respectively. The two-way arrows denote insertions of neutrino potential V_ν. The circles denote the nucleon axial and vector currents coupled to V_ν. The gray ellipses represent the T matrix generated by iteration of the NN potential.

For the scattering process $nn \to ppee$, the $^1S_0$ channel is the only one that requires a contact term to achieve renormalizability in the heavy baryon approach [10, 11]. Without loss of generality for our arguments, we consider the kinematics $n(\boldsymbol{p}_i)n(-\boldsymbol{p}_i) \to p(\boldsymbol{p}_f)p(-\boldsymbol{p}_f)e(\boldsymbol{p}_{e1} = 0)e(\boldsymbol{p}_{e2} = 0)$ with the emitted electrons at rest. The LO amplitude can be schematically written as

$$\mathcal{A}_\nu^{\mathrm{LO}} = -\rho_{fi}(V_\nu + V_\nu G_0 T_s + T_s G_0 V_\nu + T_s G_0 V_\nu G_0 T_s), \tag{7}$$

where $\rho_{fi} = M^2/\sqrt{(\boldsymbol{p}_f^2 + M^2)(\boldsymbol{p}_i^2 + M^2)}$ is a phase space factor. The four terms in Eq. (7) correspond to the four diagrams depicted in the first row of Fig. 2, and here we denote them as $\mathcal{A}_A$, $\mathcal{A}_B$, $\overline{\mathcal{A}}_B$, and $\mathcal{A}_C$ from left to right. They have different number of insertions of the strong T-matrix in the initial and final states. The strong T-matrix depends only on the LEC $C_{^1S_0} = C_1 - 3C_2$, which is fixed by the experimental scattering length $a_{^1S_0} = -23.74$ fm. Since the long-range neutrino potential does not contain LECs, the calculated LO amplitude comes out as parameter-free predictions.

The renormalizability of the transition amplitude is manifested by an analysis of the degree of ultraviolet divergences within the diagrams. The strong T-matrix does not contain any ultraviolet divergences. Therefore, the ultraviolet divergences appear only in loops that involve long-range neutrino exchange. Counting the powers of loop momenta, one finds the degree of divergence to be $D = L(3+g) - 2$ with L being the number of loops and g the ultraviolet scaling of the two-nucleon propagator, and -2 comes from the $|\boldsymbol{q}|^{-2}$ dependence of V_ν. Since the relativistic propagator has $g = -3$ and the nonrelativistic one has $g = -2$ [Eq. (6)], one finds that all diagrams $\mathcal{A}_A$, $\mathcal{A}_B$, $\overline{\mathcal{A}}_B$, and $\mathcal{A}_C$ have $D = -2$ (no divergence) in the relativistic case, while $\mathcal{A}_C$ has $D = 0$ (logrithmic divergence) in the nonrelativistic case.

2.3 CSB and CIB in the NN scattering process

In the $nn \to ppee$ process, a massless neutrino propagator is coupled to two nucleons' weak current. A similar structure arises in the electromagnetic contributions to the NN scattering process, in which a massless photon propagator is coupled to two nucleons' electromagnetic current. Since the electromagnetic contributions dominate the CSB and CIB in NN scattering at low energies, the available NN scattering data allow us to validate the predictions of the present manifestly Lorentz invariant chiral EFT approach.

To this end, we calculate the CSB and CIB in the NN scattering length,

$$a_{\text{CSB}} = a_{pp}^C - a_{nn}, \quad a_{\text{CIB}} = \frac{a_{pp}^C + a_{nn}}{2} - a_{np}, \tag{8}$$

by taking into account the leading electromagnetic effects–the static one-photon exchange and the mass splitting between neutral and charged pions. In the $^1S_0$ channel, the np, nn, and pp potentials take the form

$$\begin{aligned} V_{np}(\boldsymbol{p}', \boldsymbol{p}) &= -\frac{g_A^2}{4f_\pi^2}\frac{1}{q^2+m_\pi^2} + \tilde{C}_{^1S_0}, \\ V_{nn}(\boldsymbol{p}', \boldsymbol{p}) &= -\frac{g_A^2}{4f_\pi^2}\frac{1}{q^2+m_{\pi^0}^2} + \tilde{C}_{^1S_0}, \\ V_{pp}(\boldsymbol{p}', \boldsymbol{p}) &= -\frac{g_A^2}{4f_\pi^2}\frac{1}{q^2+m_{\pi^0}^2} + \tilde{C}_{^1S_0} + \frac{4\pi\alpha}{q^2}, \end{aligned} \tag{9}$$

with $q = |\boldsymbol{p}' - \boldsymbol{p}|$, $m_\pi = 138.039$ MeV the average pion mass, and $m_{\pi^0} = 134.98$ MeV the neutral pion mass. As charge-dependent contact terms appear at higher orders [5], the LEC $\tilde{C}_{^1S_0} = C_{^1S_0} + \frac{g_A^2}{4f_\pi^2}$ remains the same for np, nn, and pp and has already been determined by the np scattering length $a_{np} = -23.74$ fm. Therefore, the calculated a_{CSB} and a_{CIB} are parametric-free predictions.

In Eq. (8), the scattering lengths a_{nn} and a_{np} are calculated by standard momentum-space methods for short-range potentials. The calculation of the pp scattering length a_{pp}^C needs additional treatment due to the presence of the infinite-range Coulomb potential. We closely follow the momentum-space method used in Refs. [24, 25]. In this method, the Coulomb potential is truncated to a radius R that is much larger than the strong-interaction range m_π^{-1}. In the momentum space, the truncation corresponds to the following replacement,

$$\int_0^R \mathrm{d}^3 r e^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}} \frac{\alpha}{r} = \frac{4\pi\alpha}{q^2}(1-\cos qR). \tag{10}$$

The short-range phase shift δ_s is calculated for the strong potential with the truncated Coulomb potential. Finally, by matching to the asymptotic Coulomb wave function at radius R, the scattering phase shifts in the presence of Coulomb potential is expressed in a Wronskian form,

$$\tan(\delta_{pp}^C) = \frac{\tan\delta_s[F,G_0]+[F,F_0]}{[F_0,G]+\tan\delta_s[G_0,G]}. \tag{11}$$

where $F,\ G\ (F_0,\ G_0)$ are the regular and irregular Coulomb functions (with zero charge), respectively, and

$$[F,G] = \left(G\frac{dF}{dr} - f\frac{dG}{dr}\right)_{r=R}. \tag{12}$$

The pp scattering length is defined by a modified effective-range expansion,

$$C_\eta^2 k\cot\delta_{pp}(k) + h(\eta) = -\frac{1}{a_{pp}^C} + \frac{r_{pp}^C}{2}k^2 + \cdots \tag{13}$$

where $\eta = \alpha M/2k$ is the Sommerfeld parameter, $C_\eta^2 = 2\pi\eta[\exp(2\pi\eta)-1]^{-1}$ is the Sommerfeld factor, and $h(\eta) = \mathrm{Re}[\psi(1+\mathrm{i}\eta)] - \ln\eta$.

3. Results and discussion

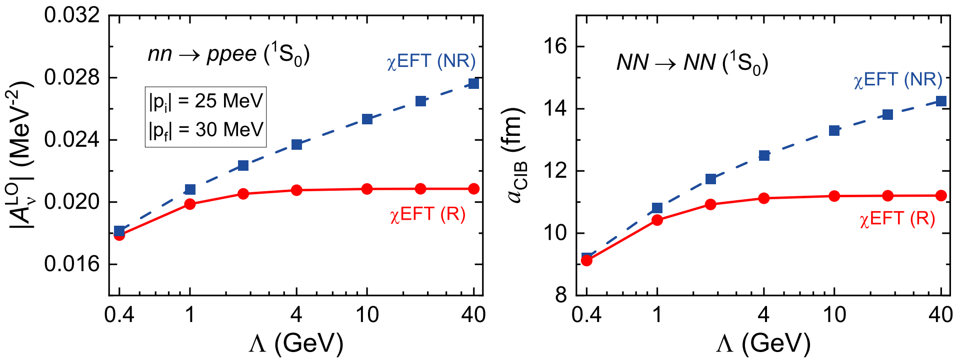

Figure 3: Cutoff dependence of the LO $nn \to ppee$ transition amplitude $|\mathcal{A}_\nu^{\mathrm{LO}}|$ (left panel) and the CIB scattering length a_{CIB} (right panel). The results for the manifestly Lorentz invariant and the nonrelativistic heavy baryon formulations of chiral EFT are denoted by solid and dashed lines, respectively.

Figure 3 depicts the LO $nn \to ppee$ transition amplitude $|\mathcal{A}_\nu^{\mathrm{LO}}|$ as a function of cutoff Λ. As an example, the amplitude at kinematics $|\boldsymbol{p}_i| = 25$ MeV and $|\boldsymbol{p}_f| = 30$ MeV is shown. The strong potential is regulated by $e^{-(p'^4+p^4)/\Lambda^4}$. Consistent with the analysis of degrees of divergences, in the nonrelativistic heavy baryon formulation, $|\mathcal{A}_\nu^{\mathrm{LO}}|$ diverges logarithmically as the cutoff goes to infinity. In contrast, in the manifestly Lorentz invariant formulation, $|\mathcal{A}_\nu^{\rm LO}|$ converges as the cutoff goes to infinity and, thus, is renormalizable. The cutoff dependence of the CIB scattering length $a_{\rm CIB}$ is also shown in Fig. 3. Due to the similarity between neutrino and photon exchange, the cutoff dependence is the same for $a_{\rm CIB}$ (also $a_{\rm CSB}$ and $|\mathcal{A}_\nu^{\rm LO}|$. Consequently, the manifestly Lorentz invariant formulation can predict the $nn \to ppee$ transition amplitude $|\mathcal{A}_\nu^{\rm LO}|$ and the scattering lengths $a_{\rm CIB}$, $a_{\rm CSB}$, after taking the cutoff to infinity $\Lambda \to \infty$.

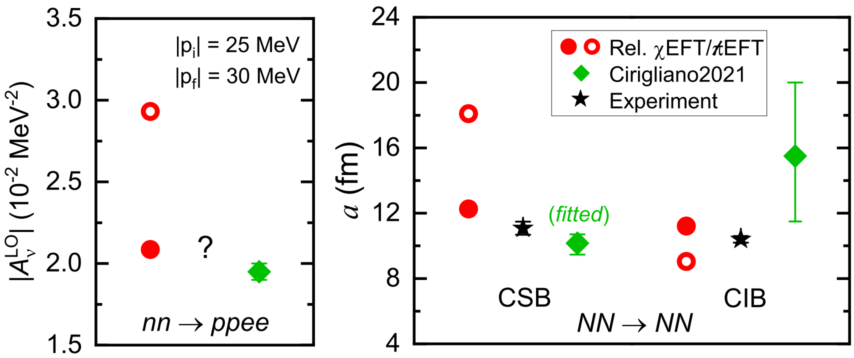

Figure 4: Predictions for the LO $nn \to ppee$ transition amplitude $|\mathcal{A}_\nu^{\rm LO}|$ (left panel) and the CIB and CSB scattering lengths $a_{\rm CIB}$ and $a_{\rm CSB}$. The solid and empty dots denote the predictions from the chiral and pionless EFT in the manifestly Lorentz invariant formulation, respectively. The results from the generalized Cottingham formula in Ref. [13] are shown for comparison, with the error bar denoting the estimated systematic uncertainties. The experimental values of $a_{\rm CIB}$ and $a_{\rm CSB}$ are shown by stars.

In the left panel of Fig. 4, the prediction of $|\mathcal{A}_\nu^{\rm LO}|$ from the manifestly Lorentz invariant formulation of chiral EFT is compared to that from the previous result from the generalized Cottingham formula [13]. The error bar reflects its systematic uncertainties from the neglect of the inelastic intermediate states and the model-dependent inputs in the intermediate-momentum region. The two results are consistent at the 10% level, while the present result avoids the model-dependent inputs beyond the EFT framework. The result of $|\mathcal{A}_\nu^{\rm LO}|$ obtained from pionless EFT in the manifestly Lorentz invariant formulation is also shown. It is significantly larger than the other two results. Such discrepancy should be reduced by taking into account the $nn \to ppee$ contact term that arises from integrating out pions [23].

Since there are no experimental data on the $nn \to ppee$ transition amplitude $|\mathcal{A}_\nu^{\rm LO}|$, it is crucial to test the different approaches by other observables with available data. In the right panel of Fig. 4, the results of the CIB and CSB scattering lengths $a_{\rm CIB}$ and $a_{\rm CSB}$ are shown for the different approaches. The results of chiral EFT in the manifestly Lorentz invariant formulation agree well with the experimental values at the 10% level, an accuracy one would expect at LO. We stress that $a_{\rm CIB}$ and $a_{\rm CSB}$ are parameter-free predictions since the approach only contains one LEC $C_{^1S_0}$ fixed by the np scattering length a_{np}. In comparison, for the generalized Cottingham formula, only $a_{\rm CIB}$ is predicted from the generalized Cottingham model, with its lower limit of estimated uncertainty reaching the experimental value.

4. Summary and outlook

We present the manifestly Lorentz invariant formulation of chiral EFT for the $nn \to ppee$ process. Due to the better ultraviolet behavior of the relativistic scattering equation, the $nn \to ppee$ contact term is not needed for renormalization at leading order, in contrast to the nonrelativistic case. Since there are no unknown LECs, chiral EFT predictions of the $nn \to ppee$ transition amplitude are realized in the manifestly Lorentz invariant formulation. The prediction is consistent with the previous result from the generalized Cottingham formula at the 10% level. As a validation, we show that the present approach can reproduce the charge-symmetry-breaking and charge-independence-breaking contributions to the nucleon-nucleon scattering lengths.

In the future, it would be interesting to benchmark the present approach with first-principles lattice QCD calculations for the $nn \to ppee$ process; see Ref. [26] for the first attempt to benchmark with the lattice QCD calculations of the $nn \to ppee$ process [27]. It would also be interesting to carry out relativistic nuclear-structure calculations [28–30] using the $0\nu\beta\beta$ decay operator as derived in the present approach. In such a calculation, the LO $nn \to ppee$ contact term is not needed.

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