PROCEEDINGS OF SCIENCE

Di-hadron fragmentation in reduced dimensionality and Λ hyperon beam-spin transfer

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In this contribution, we focus on two types of dihadron fragmentation functions (diFF), the "strong-diFF" and the "weak-diFF", where the "strong" and "weak" labels the way the experimentally observed hadrons are formed. The strong-diFF is the more widely used dihadron fragmentation in this community, where in particular the chiral-odd "interference fragmentation function" is employed as a probe for transversity, the main theme of this workshop. We discuss limitations in the analysis of strong-diFF data arising through an incomplete integration of the differential cross section imposed by experimental constraints. In particular, we show that the usual application of momentum requirements together with integration over the polar angle of the hadrons in the di-hadron center-of-mass frame leads to a presently uncontrollable mixture of various partial-wave components of the di-hadron fragmentation function for charged-pion pairs, prohibiting precision extractions of di-hadron fragmentation functions from e^+e^- annihilation, semi-inclusive deep-inelastic scattering, or proton-proton collision data. A prominent example of weak-diFF is the production and subsequent weak decay of Λ hyperons into a pion and a proton. The weak decay reveals the polarization of the parent Λ hyperon, a valuable tool to study spin phenomena. Preliminary HERMES data on the spin transfer from the lepton beam to final-state Λ and $\bar{\Lambda}$ hyperons in semi-inclusive deep-inelastic scattering are presented. No sizable spin transfer is observed.

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1. Introduction

Fragmentation functions have been recognized as powerful tools to probe nucleon structure [1] and have received much increased attention in the past decade. Still, the level of knowledge about those lags behind that about parton distributions. In particular, when studying the transverse-momentum structure of the nucleon through semi-inclusive DIS it will be unavoidable to also know the transverse-momentum structure of hadronization as the transverse momentum of the hadrons in semi-inclusive DIS arises through convolutions of the transverse-momentum dependence of both the parton distribution and fragmentation functions. Spin dependence is an even less constrained field albeit a rich one. Indeed, similar as for the leading-twist parton distributions describing the spin-momentum structure of nucleons, only one out of the eight leading-twist single-hadron fragmentation functions does not involve spin, neither of the fragmenting quark nor of the produced hadron.

Fragmentation functions are mainly constrained through measurements of semi-inclusive DIS [2–5] and of hadron production in electron-positron annihilation, especially the very precise data from the B factories [6–16]. [In case of transverse-momentum integrated fragmentation functions, data on hadron production in proton-proton collision is also used in global analyses.] Most of these studies focused on the inclusive production of unpolarized single hadrons, used in global analyses of collinear unpolarized single-hadron fragmentation functions. Data on dihadron production, by contrast, is sparse, though much needed for global analyses of dihadron fragmentation functions. Fortunately this is part of the ongoing analysis program at the B factories as well as at BESIII. The relative momentum of the two hadrons and its orientation with respect to the momentum sum can be exploited to access the quark polarization of the fragmenting quark. A much discussed example is the chiral-odd di-hadron fragmentation function $H_1^{\sphericalangle,q\to h_1h_2}$ (where the dependence on the kinematic variables has been suppressed here). Unlike $D_1^{q\to h}$, the polarization-averaged counterpart of $H_1^{\sphericalangle,q\to h_1h_2}$, $D_1^{q\to h_1h_2}$, had until recently been unmeasured, but knowledge of it is vital as it enters all spin asymmetries in dihadron production. Its precise determination, however, is hampered by the large number of its kinematic dependences that are difficult to determine simultaneously in statistics-limited measurements. Presented next is a discussion of limitations when exploring dihadron fragmentation in a restricted set of these kinematic variables.

2. Limitations of dihadron fragmentation in reduced dimensions

Dihadron fragmentation has been worked out in detail in Refs. [17–19]. It is conventionally expressed in terms of fragmentation functions that depend on the energy fraction z of the dihadron, the dihadron's invariant mass M_h, the momentum difference ζ, which can be related to $\cos\theta$, where θ represents the polar angle of one hadron in the center-of-mass of the dihadron system, and—in principle—on the transverse momentum of the dihadron system (which will be integrated over in the following).

It is convenient to expand the dependence on ζ in terms of partial waves. Including only s and p waves, as is commonly believed to be a valid approximation for low invariant mass of the dihadron pair, the polarization-averaged dihadron fragmentation functions D_1 can be written as [19]

$$D_1(z,\cos\theta,M_h) \simeq D_{1,oo}(z,M_h) + D_{1,ol}(z,M_h)\cos\theta + D_{1,ll}(z,M_h)\frac{1}{4}\left(3\cos^2\theta - 1\right), \quad (1)$$

$$H_1^{\sphericalangle}(z,\cos\theta,M_h) \simeq H_{1,ot}^{\sphericalangle}(z,M_h) + H_{1,lt}^{\sphericalangle}(z,M_h)\cos\theta\,, \quad (2)$$

where we have suppressed the dependence on the flavor of the fragmenting quark. Furthermore, $D_{1,oo}(z,M_h)$ receives contributions from both s and p waves:

$$D_{1,oo}(z,M_h) = \frac{1}{4}\left(D_{1,oo}^{s}(z,M_h) + 3D_{1,oo}^{p}(z,M_h)\right). \quad (3)$$

It should be noted that while it is experimentally possible to disentangle the various partial-wave contributions in Eqs. (1,2) by angular analysis of the cross section, it is not possible to disentangle the two contributions in Eq. (3) without assuming specific (and differing) invariant-mass dependences of those. Those contributions are, however, useful to impose limits on the partial wave contributions. In particular, the relevant limit in the context of this discussion reads [19]

$$-\frac{3}{2}D_{1,oo}^{p} \le D_{1,ll} \le 3D_{1,oo}^{p}\,. \quad (4)$$

It has been suggested to look at dihadron production integrated over the polar angle (e.g., in Ref. [20]). The obvious advantage of such approach is that most terms in Eqs. (1,2) will drop out. In particular, from Eq. (1) only $D_{1,oo}(z,M_h)$ and from Eq. (2) only the sp interference $H_{1,ot}^{\sphericalangle}(z,M_h)$ would survive, facilitating greatly the interpretation of the resulting cross sections and cross-section asymmetries. Unfortunately, experimental constraints severely limit the θ integration, which will be discussed in the following specific example of dihadron production at B-factories and to which phenomenological precision such data could be used to extract $D_{1,oo}(z,M_h)$.

The Belle data [11] on dihadron production in e^+e^- annihilation is differential in z and M_h but has the experimental requirement of a minimum momentum (and thus energy) for each individual hadron. It should be emphasized here that such momentum requirement is nothing unique to data from e^+e^- annihilation but rather imposed by basically all experimental apparatuses. Furthermore, we will concentrate on the case of pairs of oppositely charged pions, which will further simplify the calculations and which is the one with most experimental data and phenomenological analyses. In that case, θ is commonly defined with respect to the π^+ momentum. As discussed in more detail in Ref. [21], the requirement of $z_{\min} \ge 0.1$ for each individual pion reduces the range in θ that can be accessed:

$$|\cos\theta| \le \frac{z - 2z_{\min}}{\sqrt{[(zE_0)^2 - M_h^2)(M_h^2 - 4m_\pi^2)]}}E_0 M_h \;, \quad (5)$$

where E_0 is the initial energy of the fragmenting quark, e.g., $\sqrt{s}/2$, and m_π the charged-pion mass. Equation (5) translates into a symmetric range about $\theta = \pi/2$. In particular, the accepted range increases with z of the dihadron, which is depicted on the left of Fig. 1. Being symmetric around $\pi/2$ has the advantage that still some terms in Eqs. (1,2) will cancel. Nevertheless, there will be contributions that cannot be disentangled from the one of interest, e.g.,

$$\int_{\cos(\pi-\theta_0)}^{\cos\theta_0} \mathrm{d}\cos\theta D_1^q(z,\cos\theta,M_h) \simeq$$
$$\int_{\cos(\pi-\theta_0)}^{\cos\theta_0} \mathrm{d}\cos\theta D_{1,oo}^q(z,M_h) + \int_{\cos(\pi-\theta_0)}^{\cos\theta_0} \mathrm{d}\cos\theta D_{1,ll}^q(z,M_h)\frac{1}{4}\left(3\cos^2\theta - 1\right), \quad (6)$$

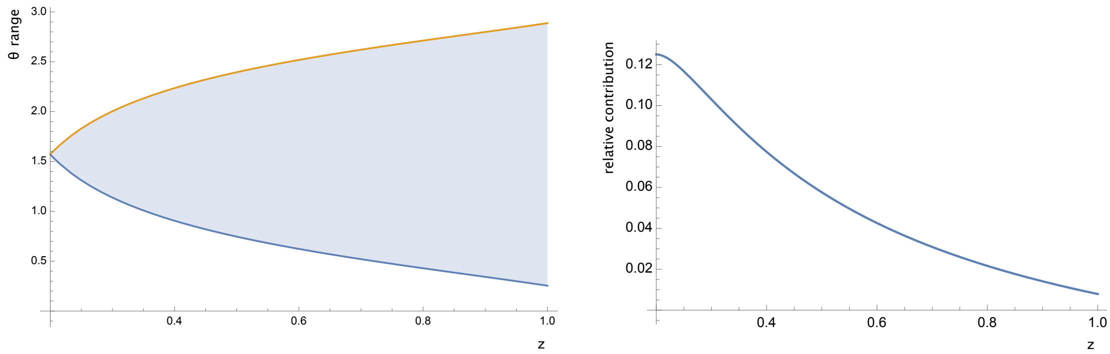

Figure 1: Left: the effect of the requirement of the individual pion's energy on the allowed range of the polar angle θ in the center-of-mass of the two-pion system. The two curves, plotted as a function of the combined z of the the di-pion, delimit the allowed region indicated as shaded area. Right: the strength of the $D_{1,ll}^{q}$ partial-wave contribution to the (partially integrated) cross section relative to the $D_{1,oo}^{q}$ contribution, again as a function of z for an invariant mass of 0.5 GeV, where the model of Ref. [22] yields about 0.5 for the ratio $D_{1,ll}^{q}/D_{1,oo}^{q}$.

It is clear that the surviving contributions are affected differently by the partial integration. In particular, the contribution of $D_{1,oo}^{q}$ increases steadily with opening up the θ range until reaching its full size for $\theta_0 = 0$.

At present it is not possible to quantify the effect of the partial integration. Thus there is room for an educated guess based on the model of Ref. [22]. The relative strength of the $D_{1,ll}^{q}$ contribution can be estimated by relating the two integrals on the r.h.s. of (6). Taking, e.g., $M_h = 0.5$ GeV and as a rough estimate of 0.5 for the size of $D_{1,ll}^{q}$ compared to $D_{1,oo}^{q}$ at that dihadron mass [22] one finds [21] a relative contribution of up order 10% at low values of z, which is the region that dominates the cross section. At larger values of z, where the θ range opens up and thus suppresses the $D_{1,ll}^{q}$ term, the relative contribution of the latter quickly drops as depicted on the right of Fig. 1.

This example demonstrates that a precision extraction of, e.g., transversity, by means of dihadron fragmentation functions might be severely hampered as it requires precision knowledge of not only the chiral-odd $H_1^{\sphericalangle,q}$ but in spin asymmetries also of $D_{1,oo}^{q}$. However, as the elimination of higher partial waves relies on the integration over θ, the minimum fractional-energy requirement in many experimental analyses leads to a surviving contribution from $D_{1,ll}$ in the unpolarized cross section, leaving an ambiguity in the interpretation of the data in absence of precise knowledge about this dihadron fragmentation function. In particular, transversity extracted from spin asymmetries, as performed, e.g., in Ref. [23], using the such-obtained $D_{1,oo}^{q}$ can be easily off by 10%. It should also be mentioned that the partial integration over θ due to the minimum fractional energy requirement leads to a severe underestimation of the true strength of the $D_{1,oo}$ dihadron fragmentation function, especially at low values of z, if the resulting integration range is not properly taken into account.

Last but not least, it is worthwhile to point out that this problem can be avoided in principle by performing a partial-wave analysis. In practice, there is often a limit on how differential the analysis can be performed. Here, dihadron fragmentation poses a challenge due to its dependence on many variables.

3. Beam-helicity dependent production of hyperons in semi-inclusive DIS

A dihadron fragmentation function in a wider context is the production of Λ hyperons and subsequent weak decay into two hadrons, typically a pion and a proton. Again, the polar-angular distribution of the two hadrons in the center of mass of the Λ hyperons reveals the polarization of the latter. This opens the door to the rich world of spin-dependent fragmentation functions that require polarimetry of the final-state hadrons, in this case the Λ hyperon. Indeed, a long history of hyperon polarization measurements (cf. Refs. [8, 24–30]) has been a driving force of both spin physics and the study of transverse-momentum distributions. Longitudinal spin transfer to the Λ hyperon in semi-inclusive deep-inelastic scattering of polarized leptons by unpolarized nucleons provides direct access to the "helicity fragmentation function" $G_1^{q\to\Lambda}$. The process can be pictured as longitudinally polarized lepton emitting a polarized virtual photon that strucks a quark inside the nucleon. The struck quark inherits (part of) the photon polarization, which is then transferred to the final-state Λ hyperon. Sizable polarization has been seen for the production of Λ hyperons from Z decays [25] and also in neutrino-DIS [26].

In electroproduction, the polarization transfer was found to be consistent with zero [27, 28], possibly due to the dominance of up-quark scattering and still large uncertainties. These analyses only used a subset of the large data set taken with polarized beam and unpolarized target at HERMES. Indeed, the final two years of data taking were dedicated to an ambitious program of measuring exclusive reactions with a high-density unpolarized proton target. These data and other data sets not analyzed in previous publications are included in the latest analysis of polarization transfer to both Λ and $\bar{\Lambda}$ hyperons. Besides the longitudinal spin transfer D_{LZ} related to $G_1^{q\to\Lambda}$, also the longitudinal-to-transverse spin transfer D_{LX} is explored. Here, X is the transverse component that lies in the production plane. The transverse component perpendicular to the production plane does not correlate to the beam helicity and is often denoted "spontaneous" polarization (related to the "polarizing" fragmentation function $D_{1T}^{\perp}$).

The preliminary HERMES results for longitudinal spin transfer D_{LZ} and the longitudinal-to-transverse spin transfer D_{LX} are presented in Fig. 2 for both Λ and $\bar{\Lambda}$ hyperons as a function of the Bjorken scaling variable x, z, Feynman-x_F, and the hyperon momentum component $P_{\Lambda\perp}$ transverse to the direction of the virtual photon, while integrating over the other variables. They are consistent with zero for Λ hyperons, as in previous analyses but with much improved uncertainties. They are also consistent with zero for $\bar{\Lambda}$, measured here the first time at this beam energy.

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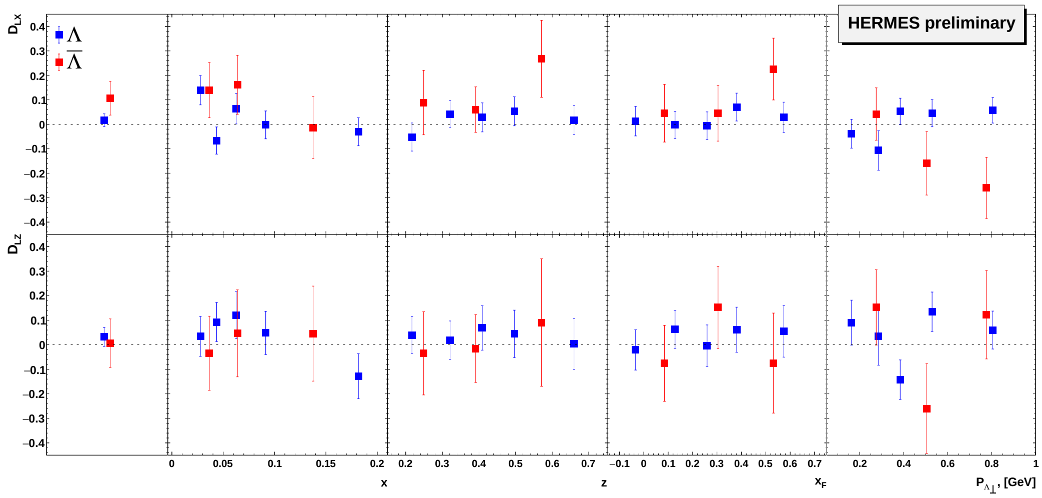

Figure 2: The spin-transfer coefficients D^{Λ}_{LX} (top) and D^{Λ}_{LZ} (bottom) for Λ and $\bar{\Lambda}$ hyperons as a function of x, z, x_F, and $P_{\Lambda\perp}$, while integrating over the other variables. The "overall" results in the left-most column correspond to the entire experimental acceptance. The error bars represent the statistical uncertainties. In addition, for he spin-transfer coefficient D_{LX} (D_{LX}) there is an overall experimental systematic uncertainty of 0.013 (0.008) and 0.015 (0.014) for Λ and $\bar{\Lambda}$ hyperons, respectively, as well as a 4% scale uncertainty arising from the uncertainties in the analyzing power α of the weak Λ and $\bar{\Lambda}$ decays and the beam-polarization measurement.

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