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Using functional methods to study axion-quantum electrodynamics

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Axions, hypothetical particles proposed as an elegant solution to the "strong CP problem" in the Standard Model, have gained attention as promising dark matter candidates. Consequently, there has been strong interest in understanding their interactions with matter and light under specific conditions. As a predecessor of a work aiming at describing the behaviour of axions in spin-1/2 quantum plasmas, this work investigates a new theory of Axion-Quantum Electrodynamics using quantum field theory techniques, such as functional methods, to understand the fundamental interactions of this model through the derivation of Feynman rules.

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1. Introduction

The Standard Model (SM) has provided remarkable insights into the fundamental forces of nature, while predicting phenomena with exceptional accuracy. However, this model is unable to account for several key aspects of our universe, such as the cosmological matter-antimatter asymmetry in the universe, the nature of dark matter, and the incomprehensible small CP-violating term in Quantum Chromodynamics (QCD). Among the most compelling solutions to these problems, we find a hypothetical particle known as the QCD axion. First proposed by R. Peccei and H. Quinn [1, 2] in response to the "strong CP problem", and later on by Weinberg and Wilczek [3, 4], the axion arises from a new $U(1)_{\text{PQ}}$ symmetry that dynamically suppresses CP violation in the strong interaction. This particle's unique properties not only resolve an internal inconsistency in QCD but also make it a well-motivated candidate for dark matter, linking particle physics to cosmological observations.

Axions are expected to couple weakly with ordinary matter and light, and many of the most promising avenues for detecting them are based on astrophysical observations of extreme systems that are able to magnify these feeble interactions. One promising approach is to study the behaviour of axions in plasmas, which could also provide new insights into these astrophysical environments.

In summary, the present work aims to explore the interactions between axions and the elementary particles in a plasma, using quantum field theory techniques and introducing a framework for Axion-Quantum Electrodynamics (Axion-QED).

2. Axion-Quantum Electrodynamics

To do this, we start with the usual lagrangian density of QED, and introduce a pseudoscalar (axion) real field, which couples to the Dirac Field through a Yukawa type interaction, and also interacts with the electromagnetic (EM) field. In this setting, after applying the Faddeev-Popov trick [5] for gauge fixing and integrating by parts, the Lagrangian density that describes the electromagnetic field A_μ interacting with a Dirac fermion ψ of mass m_ψ and charge q, and an axion φ of mass m_φ is given by: (throughout this work we use NU, $\hbar = c = 1$)

$$\begin{aligned}\mathcal{L} = &-\frac{1}{2}\varphi\left(\Box + m_\varphi^2\right)\varphi - \frac{1}{2}A_\mu\left[-\partial^2 g^{\mu\nu} + (1-\frac{1}{\xi})\partial^\mu\partial^\nu\right]A_\nu \\ &+ \bar{\psi}\left(i\partial\!\!\!/ - m_\psi\right)\psi + q\bar{\psi}\gamma^\mu\psi A_\mu - \frac{g_{\varphi\gamma}}{4}\varphi\tilde{F}^{\mu\nu}F_{\mu\nu} - ig_{\varphi\psi}\varphi\bar{\psi}\gamma^5\psi.\end{aligned} \tag{1}$$

This way, we can clearly see the separation between the free-field Lagrangian $\mathcal{L}_0 = \mathcal{L}_\varphi + \mathcal{L}_\gamma + \mathcal{L}_\psi$, and the interacting Lagrangian $\mathcal{L}_{\text{int}} = \mathcal{L}_{\psi\gamma} + \mathcal{L}_{\varphi\gamma} + \mathcal{L}_{\varphi\psi}$.

3. Using functional methods in Axion-QED

In this work, we decided to use functional methods to examine the fundamental interactions of our model, enabling us to derive the Feynman rules for our theory. To achieve this, we need to calculate the relevant correlation functions through the Generating Functional of our theory, $Z[J]$. However, after determining it, we quickly discovered that we cannot use it for our purposes.

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Thus, we resorted to a very useful trick and showed that we can calculate expectation values in the interacting theory by means of expectation values in the free theory:

$$\langle\Omega|\mathcal{T}\{O(x_i)\}|\Omega\rangle = \frac{\left\langle 0|\mathcal{T}\{O(x_i)e^{iS_{\text{int}}}\}|0\right\rangle}{\left\langle 0|\mathcal{T}\{e^{iS_{\text{int}}}\}|0\right\rangle}, \tag{2}$$

where $\langle\Omega|\mathcal{T}\{O(x_i)\}|\Omega\rangle$ refer to expectation values in the interacting theory, while $\langle 0|\mathcal{T}\{O(x_i)\}|0\rangle$ denote expectation values in the free theory. This way, we only need the generating functional for the free theory, $\mathcal{Z}_0$, which is much more simple to compute and is given by:

$$\begin{aligned}\mathcal{Z}_0[\eta,\bar{\eta},\lambda,J_\mu] &= \mathcal{N}_0 \exp\left\{-\frac{1}{2}\int_{wz}\lambda(w)\Delta_F(w-z)\lambda(z)\right\}\times \\ \exp\left\{-\int_{wz}\bar{\eta}(w)S_F(w-z)\eta(z)\right\} &\times \exp\left\{-\frac{1}{2}\int_{wz}J_\theta(w)D^{\theta\phi}(w-z)J_\phi(z)\right\},\end{aligned} \tag{3}$$

Here, $\Delta_F(w-z)$, $S_F(w-z)$ and $D^{\theta\phi}(w-z)$ represent the leading order (LO) propagators of the real scalar field, the Dirac field and of the vector field, respectively. Next, we expand the interaction lagrangian in powers of the three coupling constants, such that up to first order $O(e^1, g^1_{\varphi\gamma}, g^1_{\varphi\psi})$:

$$e^{iS_{\text{int}}} = 1 + i\int_x \mathcal{L}_{\text{int}}(x). \tag{4}$$

This way, we can calculate any correlation function we are interested in, up to first order, according to:

$$\langle O(x_i)\rangle_\Omega = \frac{\langle O(x_i)\rangle_0 + i\int_x \left\langle O(x_i)\left[-e\bar{\psi}\gamma^\mu\psi A_\mu - \frac{1}{4}g_{\varphi\gamma}\varphi\tilde{F}^{\mu\nu}F_{\mu\nu} - ig_{\varphi\psi}\varphi\bar{\psi}\gamma^5\psi\right]\right\rangle_0 + \cdots}{1 + i\int_x\left\langle\left[-e\bar{\psi}\gamma^\mu\psi A_\mu - \frac{1}{4}g_{\varphi\gamma}\varphi\tilde{F}^{\mu\nu}F_{\mu\nu} - ig_{\varphi\psi}\varphi\bar{\psi}\gamma^5\psi\right]\right\rangle_0 + \cdots} \tag{5}$$

3.1 Calculating correlation functions

Now that we have our generating functional for both the free and the interacting theory, we proceed to calculate the correlation functions of interest. We use the usual definition:

$$\begin{aligned}\left\langle 0|\mathcal{T}\{\psi(x_1)\cdots\psi(x_n)\bar{\psi}(y_1)\cdots\bar{\psi}(y_n)\}|0\right\rangle &= \frac{\delta^{2n}\mathcal{Z}_0[\eta,\bar{\eta},\lambda,J_\mu]}{i\delta\eta(y_n)\cdots i\delta\eta(y_1)i\delta\eta(x_n)\cdots i\delta\eta(x_1)} \\ = \frac{1}{\mathcal{Z}_0[0]}\left(-i\frac{\delta}{\delta\eta(y_n)}\right)\cdots\left(-i\frac{\delta}{\delta\eta(y_1)}\right)\left(-i\frac{\delta}{\delta\eta(x_n)}\right)&\cdots\left(-i\frac{\delta}{\delta\eta(x_1)}\right)\mathcal{Z}_0[\eta,\bar{\eta},\lambda,J_\mu]|_{\eta=\bar{\eta}=\lambda=J=0}\end{aligned} \tag{6}$$

where the functional derivatives with respect to the Grassmaman numbers are left-handed.

3.2 Calculating two-point correlation functions

Before we start calculating anything, let's note that correlation functions in the free theory with odd number of photons (A^μ), odd number of axions (φ) or different numbers of ψ's and $\bar{\psi}$'s vanish. This happens, because after taking all functional derivatives, what's left is proportional to powers of (J_μ, λ or $\eta, \bar{\eta}$), which vanish when in the end we take $J_\mu \to 0$, $\lambda \to 0$, and $\eta, \bar{\eta} \to 0$. This will be mentioned as **Rule 1**, and by looking at the denominator in (5), which will always be the same, regardless of the correlation function we are calculating, up to first order, we can see that it is equal to 1, since all the other terms vanish.

In a pedagogical way, we began by determining all the two-point correlation functions of our theory, which correspond to the propagators of the fields present. Starting with the Dirac field propagator, we saw that all terms vanish expect one, meaning that it will be given by:

$$\left\langle \psi_1 \bar{\psi}_2 \right\rangle_\Omega = \left\langle \psi_1 \bar{\psi}_2 \right\rangle_0 = S_F(x_1 - x_2) = \int_p e^{-ip(x_1-x_2)} \frac{(\not{p} + m_\psi)}{p^2 - m_\psi^2 + i\epsilon} = \tag{7}$$

Next, we determine the electromagnetic vector field propagator by calculating the two-point correlation function $\left\langle A_1^\mu A_2^\nu \right\rangle_\Omega$. The only term that does not vanish is $\left\langle A_1^\mu A_2^\nu \right\rangle_0$, so we get the following result:

$$\left\langle A_1^\mu A_2^\nu \right\rangle_\Omega = \left\langle A_1^\mu A_2^\nu \right\rangle_0 = D^{\mu\nu}_{x_1,x_2} = \int_k \frac{-i}{k^2 + i\epsilon} \left[g^{\mu\nu} - (1-\xi) \frac{k^\mu k^\nu}{k^2} \right] e^{-ik(x_1-x_2)} = \tag{8}$$

Lastly, we calculate the two-point correlation function of the scalar field propagator $\langle \varphi_1 \varphi_2 \rangle_\Omega$, corresponding to our axion field. Here, the only non-zero term is $\langle \varphi_1 \varphi_2 \rangle_0$. Thus, we conclude that:

$$\langle \varphi_1 \varphi_2 \rangle_\Omega = \langle \varphi_1 \varphi_2 \rangle_0 = \Delta_F(x_1, x_2) = \int_p \frac{i}{p^2 - m_\varphi^2 + i\epsilon} e^{-ip(x_1-x_2)} = \tag{9}$$

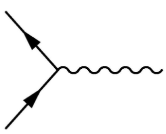
With this, we have calculated all the two-point correlation functions of interest, showing that they correspond to the leading order (LO) of the field propagators, as expected. Lastly, we have also included an illustration of the usual Feynman diagrams that correspond to each one of these propagators.

3.3 Determining the Feynman rules

Now we focus on obtaining the Feynman rules of our model. To do this, we must calculate the three-point correlation functions that correspond to the interactions vertices of our theory. These are: the fermion-photon vertex, $\left\langle \psi_1 \bar{\psi}_2 A_3^\mu \right\rangle_\Omega$, the axion-fermion vertex $\left\langle \psi_1 \bar{\psi}_2 \varphi_3 \right\rangle_\Omega$, and the axion-photon vertex $\left\langle \varphi_1 A_2^\mu A_3^\nu \right\rangle_\Omega$. After this, we performed a Fourier transform on the results and used the LSZ reduction formula [6] to infer the Feynman rules. In the end, it was discovered that up to first order in the coupling constants $O(e^1, g^1_{\varphi\gamma}, g^1_{\varphi\psi})$, the interaction vertices of our theory, are given by the following Feynman rules in momentum space:

QED vertex: $\left\langle \psi_1 \bar{\psi}_2 A_3^\mu \right\rangle_\Omega$

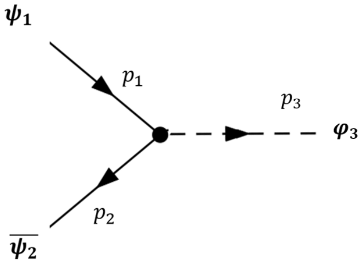
For this case, we see that, at LO, the usual QED vertex remains unchanged when we add the axion field, resulting in the following Feynman rule:



$$\mu \;\; = -ieq\gamma^\mu = ie\gamma^\mu \text{(for electrons)} \tag{10}$$

Fermion-axion vertex: $\langle \psi_1 \bar{\psi}_2 \varphi_3 \rangle_\Omega$

When identifying the three-point vertex that represents the interaction between the fermion and the axion field, it was determined that for one incoming fermion with momentum p_1^μ together with two outgoing anti-fermion and axion with momentum p_2^μ and p_3^μ, respectively, we have:

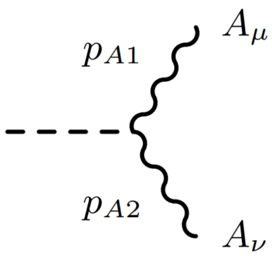


$$= g_{\varphi\psi}\gamma^5 \tag{11}$$

It is important to mention that this interaction only exists at the quantum level, since classically γ^5 is zero.

Axion-photon vertex: $\left\langle \varphi_1 A_2^\theta A_3^\phi \right\rangle_\Omega$

Finally, the last three-point correlation function of interest corresponds to the axion-photon vertex. Thus, it was found that for one incoming axion with momentum p_1^μ and two outgoing photons with momentum p_{A1}^μ and p_{A2}^ν, the corresponding Feynman rule is:



$$= -ig_{\varphi\gamma}\epsilon_{\mu\nu\alpha\beta}p_{A1}^\alpha p_{A2}^\beta \tag{12}$$

With this, we were able to understand the fundamental interactions of our model and could quickly determine the cross-sections for these processes.

4. Conclusion

To conclude, by employing functional methods, we were able to derive the Feynman rules of our model in a clear and straightforward way. This has allowed us to better understand the fundamental interactions of our model and could be particularly useful if we were to pursue an approach discussed in [7], as we would simply need to connect our quantum field theory, named Axion-QED, with a covariant kinetic theory of plasmas. In that framework, we could compute the corrected amplitudes for these processes in a plasma, effectively leading to a form of Thermal Field Theory, which would allow us to better understand the behaviour of axions in astrophysical environments.

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