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Thermal Axions: What's Next?

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Scatterings and decays of thermal bath particles in the early universe can release new relativistic degrees of freedom into the primordial plasma. We focus on the QCD axion, emphasizing recent advancements in the predictions of its contribution to cosmological observables. We provide a comprehensive review of the state-of-the-art calculations for the axion production rate across different energy scales during the cosmic expansion and outline the key developments required for future progress in this area. Subsequently, we present a phase-space approach to refine predictions for the axion dark radiation abundance. This methodology allows for the examination of light particles that never reach thermal equilibrium throughout the evolution of the universe, facilitating a detailed analysis of their decoupling processes. We show how spectral distortions arise from non-instantaneous decoupling and quantify their observable effects.

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1. Introduction

Axion-like particles (ALPs) are ubiquitous in motivated extensions of the standard model of particle physics. They are naturally light because the mass term is protected by a shift symmetry inherited from the microscopic theory, valid at high energies. Furthermore, their Pseudo-Nambu-Goldstone Boson (PNGB) nature ensures also that they are weakly coupled to visible sector particles via contact interactions suppressed by powers of the scale of spontaneous symmetry breaking. These two properties set the stage for potentially observable cosmological effects.

There are multiple origins for a given ALP low-energy theory. Oftentimes, it is common to misuse the language and refer to any possible explicit construction as an ALP. However, there is a specific realization that deserves a separate presentation and discussion. As is well known, strong interactions are observed to respect time invariance and the bound on the allowed amount of violation is quite severe [1, 2]. Everything is reconciled with the theory of Quantum ChromoDynamics (QCD) only at the price of setting the dimensionless θ parameter to an insanely small value, $\theta \lesssim 10^{-10}$. This is the infamous strong CP problem. Among several attempts, the Peccei-Quinn (PQ) framework [3, 4] is undoubtedly the most elegant solution. The θ parameter is promoted to a dynamical field a dubbed as the *QCD axion* [5, 6] whose origin is the Nambu-Goldstone boson from the spontaneous breaking of an Abelian global symmetry known as the PQ symmetry. A key feature of the PQ symmetry is the color anomaly that induces the dimension 5 contact interaction with gluons

$$\mathcal{L}_{\rm PQ} = \frac{\alpha_s}{8\pi f_a} \, a \, G^A_{\mu\nu} \widetilde{G}^{A\mu\nu} \, . \tag{1}$$

We take the above interaction as the definition of the axion decay constant f_a. Once strong interactions confine, this term induces a non-vanishing axion potential whose leading effects can be captured by the harmonic approximation leading to an axion mass

$$m_a \simeq 5.7 \, \text{meV} \left(\frac{10^9 \, \text{GeV}}{f_a} \right) \, . \tag{2}$$

Furthermore, the minimum of the axion potential is for $a = 0$ and therefore at a CP conserving point. Thus the force acting on the axion field is generated by QCD dynamics itself and it is responsible for relaxing the axion to the minimum of its potential which is CP conserving. The QCD axion kills two birds with one stone. Besides solving the strong CP problem dynamically, the energy density stored in the axion field oscillation red-shift with the Hubble expansion as a pressureless fluid and provides an irreducible contribution to the cold dark matter density [7–9]. In particular, the QCD axion can both solve the strong CP problem and provide a microscopic description of the observed dark matter abundance. These two remarkable properties make the QCD axion one of the strongest motivated particles for physics beyond the standard model.

The production of cold dark matter is not the only potential observable cosmological manifestation of the PQ framework. Axions can also be produced with kinetic energy much larger than their mass. The requirement is satisfied for the QCD axion if the kinetic energy is larger than the quantity in Eq. (2) whereas for a generic ALP, the condition has to be checked with the mass as an additional free parameter. Once these *hot axions* are produced, they can either be thermalized with the surrounding thermal bath or not. Eventually, the universe will be so cold and diluted that it becomes very hard for the axions to interact and therefore they will move along the geodesics of

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the expanding universe and lose their kinetic energy. If they survive until late times, and this is likely to be the case for the QCD axion given its lifetime much longer than the age of the universe, we can attempt to unveil their presence in cosmological observables.

There are several mechanisms through which hot axions could be produced in the early Universe. Here, we will focus on one specific mechanism: *thermal production*. What is the reasoning behind this choice? Besides the interaction in Eq. (1), the axion has model-dependent couplings to the other standard model fields. Because of this, thermal production is essentially unavoidable unless the early universe deviates from the standard thermal history we typically assume. Thus thermal production is both inevitable and computable for each axion model. But what happens to these axions after they are produced? As mentioned above, they can either thermalize with the other bath particles or propagate undisturbed. Eventually, they decouple from the thermal plasma. As a general rule of thumb, the phase-space distribution of these axions remains thermal with a temperature that is not significantly different from that of the surrounding radiation bath, due to the thermal origin of their production. We report the predicted thermal production rates in Sec. 2 for several axion scenarios. The observable effects are quantified in Secs. 3 and 4 for situations where the axion mass can and cannot be neglected, respectively. The current situation is summarized in Sec. 5 where we discuss the status of the axion production rates and possible future developments. A recent improvement in this kind of analysis based on a phase space formalism is presented in Sec. 6. Conclusions can be found in Sec. 7.

2. Thermal production

How can we estimate the amount of thermally produced axions? The simplest approach is to assume that axions were in thermal equilibrium with the bath at early times and decoupled at a temperature T_D. The residual axion population can be quantified in terms of an effective number of additional neutrino species $\Delta N_{\rm eff}$ which is also the observable effect for negligible axion mass values. It is straightforward to connect these two quantities via $\Delta N_{\rm eff} \simeq 13.67\, g_{*s}(T_D)^{-4/3}$ with g_{*s} the number of effective relativistic degrees of freedom contributing to the entropy density s. While this method provides an estimate of the effect, it's not ideal, particularly if T_D occurs at a temperature where g_{*s} is changing (i.e., below the weak scale where the observable effect is more pronounced). It is also true that we would like to explore parameter space regions where axions never reach thermal equilibrium. A more refined approach is necessary, especially since upcoming CMB observations will measure $\Delta N_{\rm eff}$ with exquisite precision.

An improved approach is based upon a Boltzmann equation tracking the axion number density evolution across the expansion history

$$\frac{dn_a}{dt} + 3Hn_a = \gamma_a \left(1 - \frac{n_a}{n_a^{\rm eq}}\right) . \tag{3}$$

Here, the axion production rate γ_a is what we want to compute, and $n_a^{\rm eq}$ is the equilibrium number density. This approach is analogous to solving the Boltzmann equation for WIMP dark matter and exactly as in that case the asymptotic solution is characterized by a constant comoving number density $Y_a = n_a/s$. The resulting contribution to the effective number of additional neutrino species reads in this case $\Delta N_{\rm eff} \simeq 75.6\, Y_a^{4/3}$. This procedure also has clear limitations, as the final step —

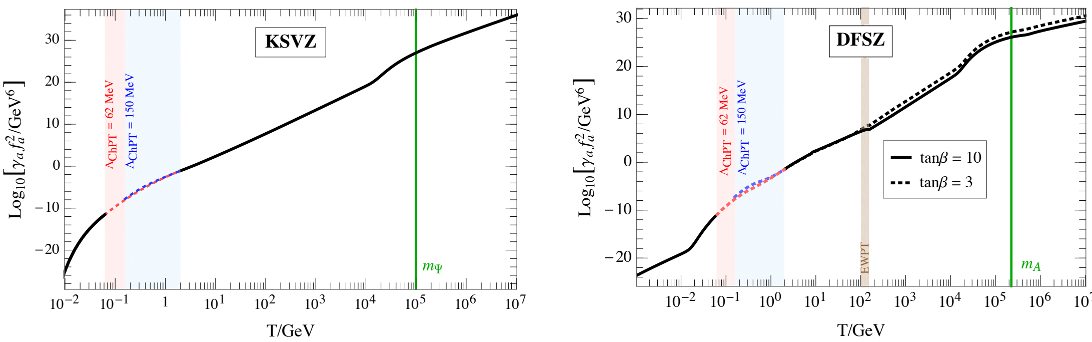

Figure 1: Axion production rate γ_a (multiplied by the squared axion decay constant f_a^2) for the KSVZ (left panel) and DFSZ (right panel) axion. Figures from Ref. [21].

transitioning from a number density to an energy density — assumes that the axions are in chemical equilibrium, which is not always the case. However, this has been the prevailing approach in the literature until recent times, and we will now present the results obtained using this method. In Sec. 6, we will discuss recent developments based on phase space evolution that goes beyond the limitations of this approach.

The collision rate calculation can be performed for each operator and this allows us to determine the number of axions produced when each single axion interaction is switched on. Several studies of this type have been carried out in the literature. For practical applications, we focus on decays and scatterings that produce only one axion particle in the final state given the large axion decay constant suppressing the rates. Axion production rate calculations due to couplings to standard model fermions and valid in the phase where electroweak symmetry is broken, both due to flavor-diagonal and flavor-violating couplings, can be found in Refs. [10–16]. The top quark case is different because its mass is of the size of the Fermi scale. Ref. [17] evaluated the production rates both above and below the weak scale. Other examples include rates due to axion couplings to pions [18, 19], KSVZ fermions [20, 21], and heavy DFSZ Higgs bosons [21, 22]. The most challenging calculations are for axion production rates due to coupling to massless standard model gauge bosons (i.e., gluons, photons, and weak bosons in the unbroken electroweak phase) due to the unpleasant IR behavior of long-range interactions mediated by them. The gluon case is also the most interesting one given the connection to the strong CP problem, and axion production via this interaction has been addressed by Refs. [23–27].

Considering a single coupling is not realistic as explicit models typically involve multiple interactions that are most efficient for production at different temperatures. A proper analysis in such models requires the axion production rate across the full cosmological expansion history. We conclude this section by presenting the results from Ref. [21], where the axion production rate was calculated for two of the most widely studied UV completions: the KSVZ axion, which features a color anomaly induced by a new colored fermion, and the DFSZ axion, which does not require new fermions and with a color anomaly arising from quarks. A key characteristic of both of these cases — and indeed of all UV completions (though the specific details may vary) — is the existence of multiple mass thresholds, where the axion production rate undergoes significant changes as the temperature evolves. The resulting production rates are shown in Fig. 1.

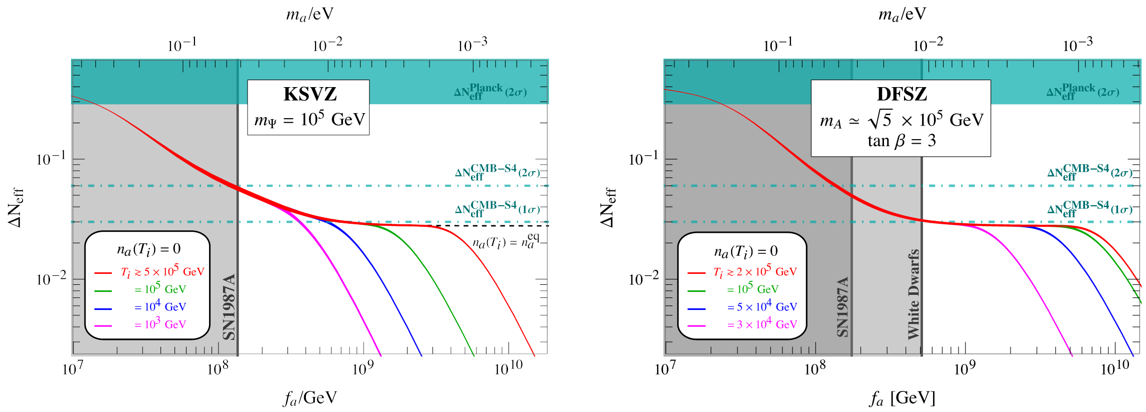

Figure 2: Predicted values for $\Delta N_{\rm eff}$ as a function of the axion decay constant f_a for the KSVZ (left panel) and DFSZ (right panel) axion. Figures from Ref. [21].

3. Current bounds I: massless axions

In this section, we discuss the bounds in the case where the axion mass can be neglected. It is important to clarify one point: whether or not the mass can be neglected is not determined only by the mass value itself, but also by the cosmological epoch we are investigating. For example, during the era of Big Bang Nucleosynthesis (BBN), the QCD axion mass in Eq. (2) can certainly be neglected since the temperatures involved are very high. The amount of additional *dark radiation* at BBN is quite constrained [28–30]. In the case of Cosmic Microwave Background (CMB) formation, the situation is less straightforward, as for low values of the axion decay constant we could obtain mass values close to the recombination temperature. For a generic ALP, there is no connection between the couplings and the mass and therefore the situation must be analyzed on a case-by-case basis. If we can treat the axions as relativistic at recombination, CMB data provide us with a complementary and competitive bound on $\Delta N_{\rm eff}$ [31–33]. Both of these bounds can be approximately written as $\Delta N_{\rm eff} \lesssim 0.2$. Exciting developments are on the horizon for CMB measurements of $\Delta N_{\rm eff}$ [34–38]. Searching for dark radiation has already become a powerful method for probing new physics. As we move forward, these next-generation experiments will refine our constraints on the presence of additional relativistic degrees of freedom in the early universe, potentially revealing new aspects of the dark sector and providing definitive evidence for $\Delta N_{\rm eff} \neq 0$.

We focus here on the observable effects of thermally produced QCD axions within the KSVZ and DFSZ frameworks. The corresponding production rates can be found in Fig. 1. We feed the Boltzmann equation in Eq. (3) with these rates and track the axion number density across the cosmic expansion. The asymptotic axion number density is ultimately translated into an effective number of additional neutrino species. The predictions for $\Delta N_{\rm eff}$ based on this relation, with all the caveats discussed in the previous section, are presented in the left panel of Fig. 2. It is worth noting that the region not excluded by astrophysical constraints is potentially accessible with upcoming missions. Furthermore, the region currently probed by Planck is for axion mass values (upper horizontal axis) comparable to the recombination temperatures. This requires analyzing finite axion mass effects, and this deserves a separate discussion.

4. Current bounds II: finite axion mass effects

Axion mass effects have to be accounted for if one wants to exploit current data. If the mass is non-negligible, axions contribute a hot dark matter component, with cosmological signatures akin to those of massive neutrinos: they suppress cosmological perturbations on scales smaller than their free-streaming length and also affect the CMB temperature anisotropy spectrum through the early integrated Sachs-Wolfe effect. This scenario leads to a mixed hot dark matter model where the axion and the total neutrino masses are anti-correlated. Several studies have explored the consequences of the axion mass in different contexts. Here, we list some of the significant results in the recent literature. Ref. [39] investigated the DFSZ axion within the QCD confined phase. Ref. [40] examined the effects of finite axion mass on the interactions with gluons and photons. In Ref. [41], a comprehensive study was conducted for both the KSVZ and DFSZ frameworks, incorporating all finite mass effects of the axion, utilizing the interpolated rates from Ref. [21]. Additionally, Refs. [42, 43] analyzed the finite mass effects of axions by solving the Boltzmann equation in momentum space, focusing specifically on pion scatterings in the QCD confined phase. The axion mass bound identified by these studies is $m_a \lesssim 0.2\,\mathrm{eV}$.

5. What is next?

The plot in Fig. 3 provides an overview of the current status for the predictions of the axion production rates via the interaction with gluons. We remind how this interaction is particularly important given the strong CP problem, and we observe how the temperature-dependent production rate has distinct behaviors in various regions. Above the confinement scale, perturbative QCD calculations at finite temperatures help to regulate the infrared divergences arising from gluon exchange. In our recent work, we extended these calculations down to the GeV scale offering an improved description in this deconfined phase. On the opposite end, the production rate has been reliably computed only for pion scattering. Recent studies have shown that the perturbative chiral expansion is valid up to temperatures of approximately 100 MeV. This issue is confined to the tiny temperature range of $100\,\mathrm{MeV} \lesssim T \lesssim 150\,\mathrm{MeV}$ because we cannot treat the primordial plasma within the hadron resonance gas (HRG) approximation at larger temperatures. To determine the total number of axions produced, we need to solve the Boltzmann equation in Eq. (3) that requires reliable production rates at all temperatures. But what about the intermediate region between these extremes? Ref. [41] proposed a smooth interpolation that is reasonable for two key reasons: first, all resonant production processes are naturally smoothed out by thermal averaging; second, the production rate is not directly the cosmological observable, but rather the input for the Boltzmann equation. As can be found explicitly in Ref. [21], the effects of this interpolation for the QCD axion are within the sensitivity range of future CMB-S4 surveys. Notably, the QCD scale coincides with the region where the current Planck bound resides. A thorough investigation of the axion production rate across the QCD crossover region is a key priority. There are intriguing open questions on both sides of this critical threshold, and calculations are absent in the intermediate temperature range, $150\,\mathrm{MeV} \lesssim T \lesssim 2\,\mathrm{GeV}$, with the smooth interpolation proposed in Ref. [26] being the sole attempt to address this gap.

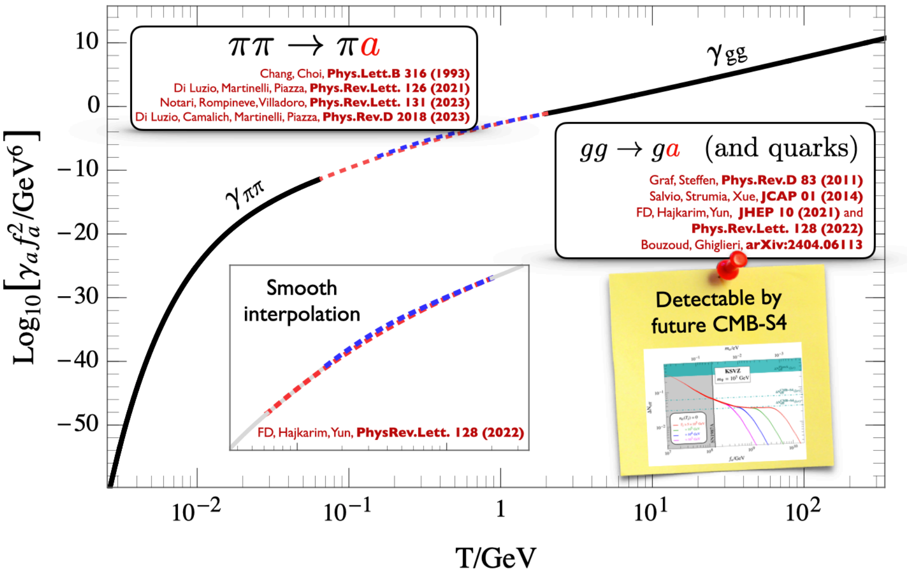

Figure 3: Axion production rate as a function of the thermal bath temperature. We show both the pion and the quark/gluon phases, the only attempt to quantify the rate in the intermediate region is the smooth interpolation of Ref. [41]. Slide shown during the 2nd General Meeting of the COST Action COSMIC WISPers (CA21106), 5 September 2024, Istanbul (Turkey).

6. Back to the phase space

Recently, we revisited the reliability of the method based on axion number density and uncovered some intriguing findings. We developed in Ref. [44] a general formalism to track the phase-space distribution of a generic dark radiation candidate. This approach takes into account both quantum statistical effects and the energy exchange between the thermal bath and the dark sector. For the specific case of the axion, the full cosmological evolution is governed by an integrodifferential Boltzmann equation that tracks the evolution of the axion distribution function $f_a(k,t)$ across different momentum bins as a function of the cosmic time t. This equation takes the schematic form

$$\omega \frac{df_a(k,t)}{dt} = C[f_a(k,t)] \ . \tag{4}$$

Here, the axion energy ω and spatial momentum k are connected via the dispersion relation $\omega = \sqrt{k^2 + m_a^2}$. The collision operator $C[f_a(k,t)]$ on the right-hand side accounts for processes that change the axion number such as scatterings and decays. Ref. [44] conducted a model-independent study focusing on dark radiation produced via two-body decays or scatterings. We predicted the expected amount of dark radiation and compared our results with the ones from an analysis based solely on number density. The difference between the two approaches may exceed the experimental sensitivity of future surveys. This suggests that the effect is significant. What is the origin of this discrepancy? The answer lies in the fact that decoupling is not instantaneous, and different momentum modes decouple at different times. If the number of relativistic degrees of freedom in the thermal bath changes during this epoch, non-negligible spectral distortions arise.

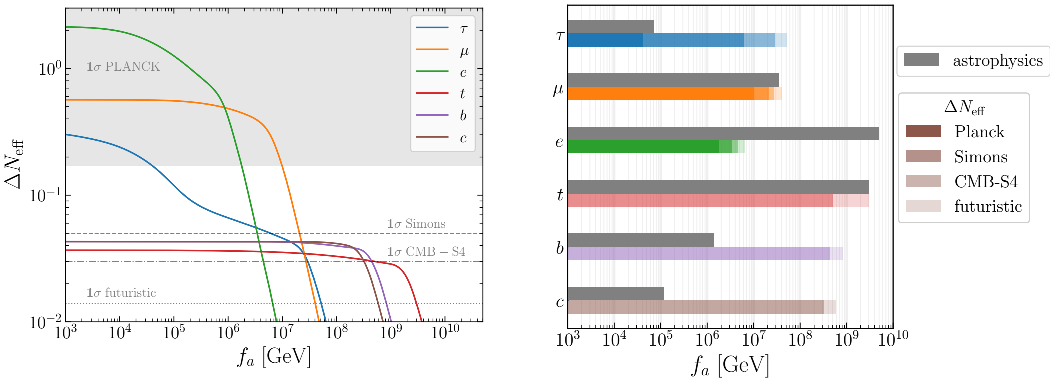

Figure 4: $\Delta N_{\rm eff}$ for axion couplings to fermions (left panel) and comparison between astrophysical and cosmological bounds (right panel). Figures from Ref. [45].

We have applied this formalism to the axion couplings to fermions [45]. All previous analyses were based on the number density formalism, and the key question now is: What are the consequences of an exact calculation? We identified spectral distortions resulting from the non-instantaneous axion decoupling that exceed the sensitivity of upcoming surveys, emphasizing the increasing importance of phase space analyses as the resolution of future experiments improves. Ref. [45] examined axion production via scatterings of charged leptons and heavy quarks. Uncertainties related to the proximity of the QCD crossover were discussed. Ref. [46] examined flavor-conserving and flavor-violating couplings to second and third-generation charged leptons. The predicted values of $\Delta N_{\rm eff}$ are shown in Fig. 4 (left panel), alongside a comparison of astrophysical and cosmological bounds (right panel). As illustrated in the figure, astrophysical constraints are difficult to surpass for couplings to electrons, but the future bounds from CMB observations for all other fermions will be either comparable to or even surpass the astrophysical limits.

7. Conclusions

Axions are strongly motivated candidates for physics beyond the standard model. Beyond these solid theoretical foundations, a crucial aspect is their experimental testability, making them an exciting area of focus for both current and future research in particle physics and cosmology. Due to their light mass and weak couplings, axions offer a wide range of possible cosmological effects. As we have explored, they are not only viable candidates for cold dark matter but also play a pivotal role in a broader cosmological framework, providing new avenues for understanding the universe's evolution across both large and small scales.

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