PROCEEDINGS OF SCIENCE

Natural Metric-Affine Inflation

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We study natural inflation in the low energy (two-derivative) metric-affine theory containing only the minimal degrees of freedom in the inflationary sector, i.e. the pseudo-Nambu-Goldstone boson (PNGB) and the massless graviton. This theory contains the Ricci-like and parity-odd Holst invariants together with non-minimal couplings between the PNGB and the aforementioned invariants. We find regions of the parameter space where the inflationary predictions agree with the most recent constraints at the 2σ level.

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1. Introduction

Observations of the cosmic microwave background radiation (CMB) support the Universe being flat and homogeneous at large distances. Such properties can be explained by assuming inflation i.e. an accelerated expansion during the very early Universe. Natural inflation [1] (a.k.a. axion inflation) was among the most popular inflationary models. Unfortunately now it is strongly disfavored by the most recent data [2]. We make natural inflation again compatible with data by embedding it in a non-minimal metric-affine gravity. This proceeding is based on our article [3].

2. Model

Let us consider a real scalar non-minimally coupled to a metric-affine gravity with action[1]:

$$S_{\rm NI} = \int d^4x\sqrt{-g}\left[\alpha(\phi)\mathcal{R} + \beta(\phi)\tilde{\mathcal{R}} - \frac{\partial_\mu\phi\,\partial^\mu\phi}{2} - V(\phi)\right], \tag{1}$$

where ϕ is the PNGB field (i.e. the axion),

$$V(\phi) \equiv \Lambda^4\left[1 + \cos\left(\frac{\phi}{f}\right)\right] \tag{2}$$

is the natural-inflaton potential [1], Λ and f are two mass scales. $\alpha(\phi)$ and $\beta(\phi)$ are respectively

$$\alpha(\phi) = \frac{M_P^2}{2}\left[1 + \xi\left(1 + \cos\left(\frac{\phi}{f}\right)\right)\right], \qquad \beta(\phi) = \beta_0 + \frac{M_P^2}{2}\tilde{\xi}\left(1 + \cos\left(\frac{\phi}{f}\right)\right) \tag{3}$$

so that the shift symmetry of the potential (2) is preserved. M_P is the reduced Planck mass. $\mathcal{R}$ and $\tilde{\mathcal{R}}$ are, respectively, a scalar and pseudoscalar contraction of the curvature,

$$\mathcal{R} \equiv \mathcal{F}_{\mu\nu}{}^{\mu\nu}, \quad \tilde{\mathcal{R}} \equiv \frac{1}{\sqrt{-g}}\epsilon^{\mu\nu\rho\sigma}\mathcal{F}_{\mu\nu\rho\sigma}, \quad \mathcal{F}_{\mu\nu}{}^{\rho}{}_{\sigma} \equiv \partial_\mu\mathcal{A}_\nu{}^\rho{}_\sigma - \partial_\nu\mathcal{A}_\mu{}^\rho{}_\sigma + \mathcal{A}_\mu{}^\rho{}_\lambda\mathcal{A}_\nu{}^\lambda{}_\sigma - \mathcal{A}_\nu{}^\rho{}_\lambda\mathcal{A}_\mu{}^\lambda{}_\sigma, \tag{4}$$

where $\mathcal{F}_{\mu\nu}{}^{\rho}{}_{\sigma}$ is the curvature constructed from the connection $\mathcal{A}_\mu{}^\rho{}_\sigma$ and $\epsilon^{\mu\nu\rho\sigma}$ is the totally antisymmetric Levi-Civita symbol with $\epsilon^{0123} = 1$. We remind that $\tilde{\mathcal{R}}$ (a.k.a. the Holst invariant) vanishes when the connection is the Levi-Civita one ($\Gamma_\mu{}^\rho{}_\sigma$). It has been proven [3] that action (1) can be cast as

$$S_{\rm NI} = \int d^4x\sqrt{-g}\left[\alpha R - \frac{\partial_\mu\phi\,\partial^\mu\phi}{2} + \frac{\frac{\alpha}{4}\partial_\mu\alpha\partial^\mu\alpha - \alpha\partial_\mu\beta\partial^\mu\beta + 2\beta\partial_\mu\alpha\partial^\mu\beta}{\frac{2}{3}\left(\beta^2 + \frac{\alpha^2}{4}\right)} - V\right], \tag{5}$$

where R is the Ricci scalar (note that $R = \mathcal{R}$ when $\mathcal{A}_\mu{}^\rho{}_\sigma = \Gamma_\mu{}^\rho{}_\sigma$). The non-minimal coupling $\alpha(\phi)R$ can be removed by performing the Weyl rescaling $g_{\mu\nu} \to \frac{M_P^2}{2\alpha(\phi)}g_{\mu\nu}$, which requires $\alpha > 0$ in order to keep the signature of the metric. After some computations [3], we get

$$S_{\rm NI} = \int d^4x\sqrt{-g}\left\{\frac{1}{2}M_P^2 R - \frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - U(\chi)\right\}, \tag{6}$$

[1] We do not introduce any other additional term in $\mathcal{R}$ or $\tilde{\mathcal{R}}$ in order to avoid additional degrees of freedom but the graviton and the inflaton.

where $F(\phi) \equiv 2\alpha(\phi)/M_P^2$ and we have defined the Einstein frame potential as

$$U(\chi) \equiv \frac{V(\phi(\chi))}{F^2(\phi(\chi))}. \tag{7}$$

The canonically normalized scalar χ in (6) is derived from the solution of

$$\frac{d\chi}{d\phi} \equiv \frac{M_P}{\sqrt{2}}\sqrt{\frac{1}{\alpha} + \frac{12(\alpha'\beta - \alpha\beta')^2}{\alpha^2(\alpha^2+4\beta^2)}}, \qquad \chi(\phi=0)=0. \tag{8}$$

where a prime represents a derivative with respect to ϕ. The solution of (8) always exists because the argument of the square root is always positive.

3. Results

In this section we discuss the inflationary predictions of the model. Before proceeding, we remark that there is a symmetry $\beta(\phi) \to -\beta(\phi)$ (see eq. (8)). Therefore, only the relative sign between β_0 and $\tilde{\xi}$ is relevant. From now on we will use the convention where $\tilde{\xi}$ is positive, while β_0 can change sign. Hence, we can identify three different scenarios: $\xi \neq 0$ with $\beta(\phi) = 0$ (not discussed here, but presented in details in our article [3]), $\xi = 0$ with $\tilde{\xi} > 0$ and finally both $\xi, \tilde{\xi} > 0$.

3.1 $\xi = 0$ and $\tilde{\xi} > 0$

We study now the case[2] where $\xi = 0$, $\tilde{\xi} > 0$ and $\beta_0 < 0$. It is useful to introduce the parameters

$$\delta_\Lambda \equiv \frac{\Lambda}{M_P}, \qquad \delta_f \equiv \frac{f}{M_P}, \tag{9}$$

that allow to measure the natural inflation mass scales Λ and f in terms of M_P. In Fig. 1 we plot the corresponding results for the tensor-to-scalar ratio r and the spectral index n_s versus the parameters $\tilde{\xi}$ and δ_f when $\beta_0 = -6M_P^2$, $N_e = 60$ and δ_Λ is fixed so that the amplitude of the scalar power spectrum (A_s) is in agreement with data [4]. We can see that by increasing $\tilde{\xi}$, agreement with data can be achieved for all the considered values of δ_f. Moreover, at big enough $\tilde{\xi}$, the results in the region $r \lesssim 0.015$ and $n_s \gtrsim 0.958$ overlap, but they are still dependent on f. Such a behaviour is evident in Fig. 1(c). This means that by changing f, we can find a $\tilde{\xi}$ so that the results instead do not change. To clarify this feature, we plot for selected benchmark points $U(\chi)$ centered at the origin in Fig. 2(a). We can see that an inflection point is always generated. Moreover, plotting $U(\chi)$ centered around the corresponding inflection point (see Fig. 2(b)), it is hard to discriminate the different potentials around (and after) the corresponding inflection points. This is reflected in the inflationary predictions as well, where all the lines overlap when $\tilde{\xi} \gg 1$.

3.2 $\xi > 0$ and $\tilde{\xi} > 0$

In the previous scenario agreement with data was obtained only when $\delta_f > 1$, which might be considered *unnatural* when looking for a UV-completion of the natural inflation setup. The issue can be solved by using both $\xi, \tilde{\xi} > 0$. The corresponding results are shown in Fig. 3, where, for numerical convenience we used $10 \lesssim \tilde{\xi} \lesssim 25$. The lines corresponding to $\delta_f = 10^{-2,-1}$ can only be distinguished in the α_s (the running of the spectral index) vs. n_s plot. We can see that agreement at very low δ_f with all the constraints involving r, n_s and α_s is only possible when $\xi > 0$.

[2]The case $\beta_0 > 0$ gives predictions disfavored by data [2]. More details are discussed in our article [3].

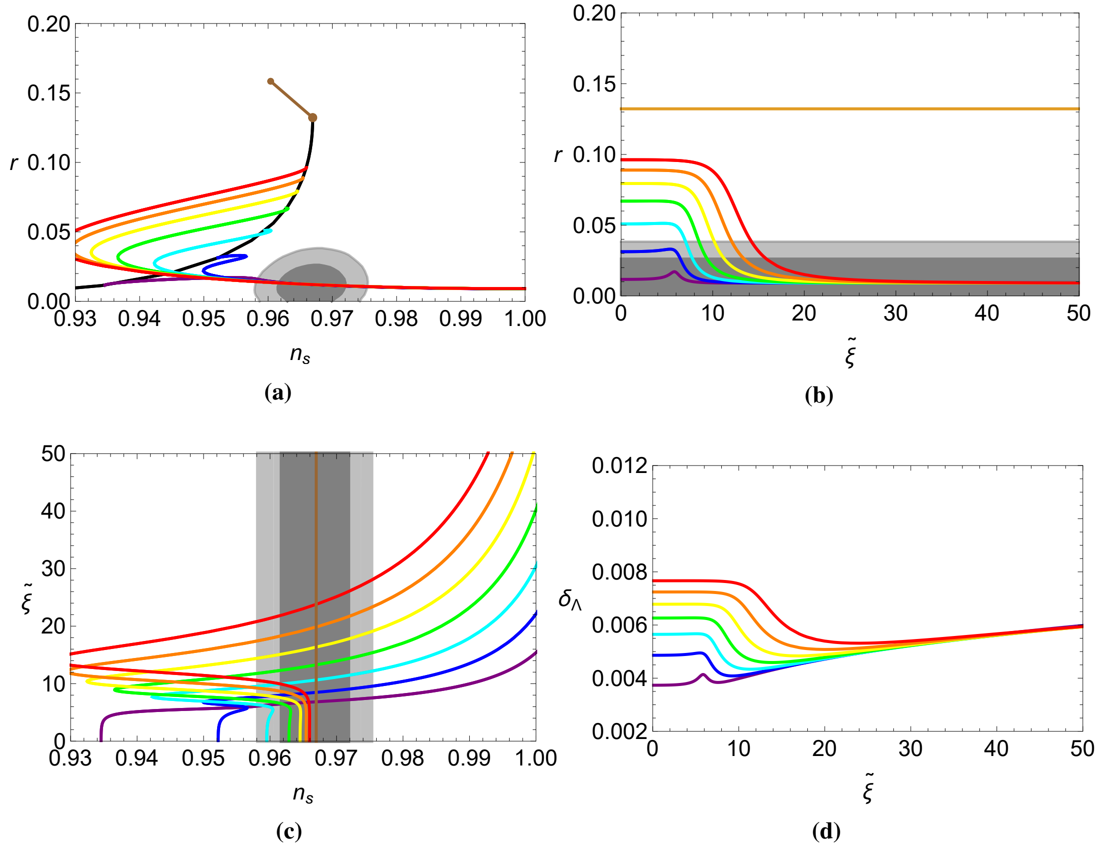


Figure 1: r vs. n_s (a), r vs. $\tilde{\xi}$ (b), $\tilde{\xi}$ vs. n_s (c), δ_Λ vs. $\tilde{\xi}$ (d) for $N_e = 60$ with $\xi = 0$, $\beta_0 = -6M_P^2$ for δ_f ranging from 4 (purple) to 10 (red) with steps of 1, displayed in rainbow colors. The gray areas represent the 1,2σ allowed regions [2]. For reference the predictions of quadratic inflation for $N_e \in [50, 60]$ (brown) and natural inflation for $N_e = 60$ (black).

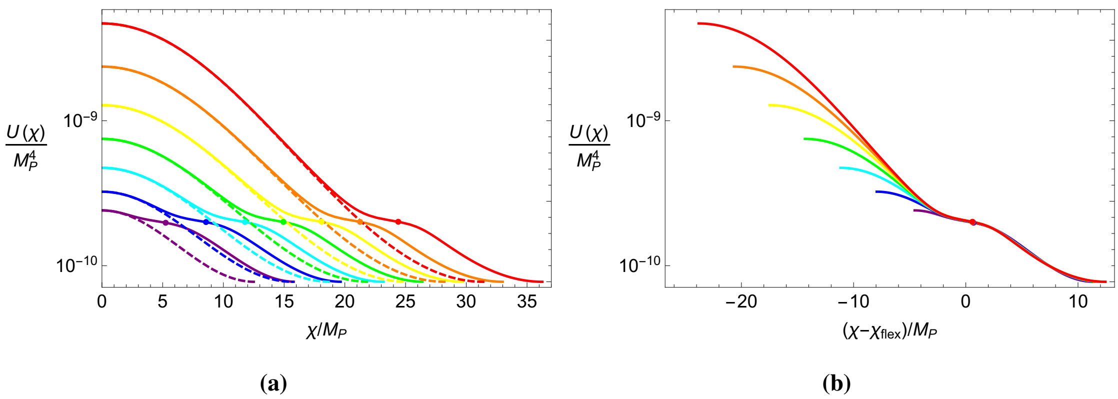
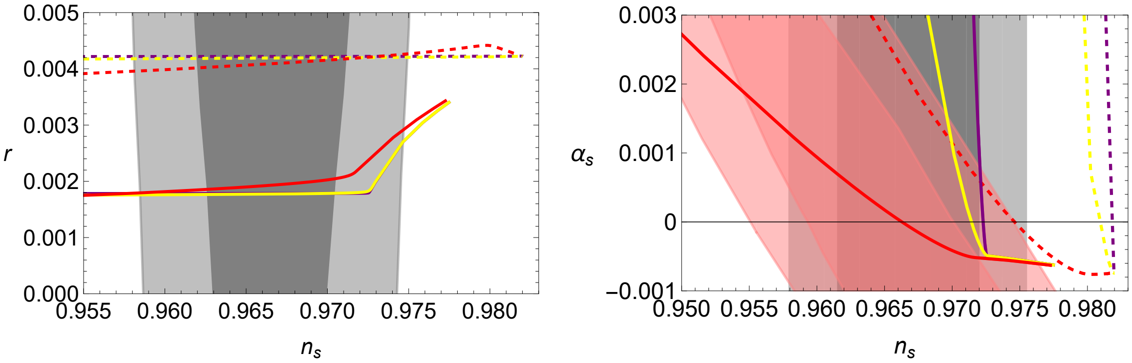


Figure 2: $U(\chi)$ (a) and $U(\chi - \chi_{\rm flex})$ (b) with $\xi = 0$, $\beta_0 = -6M_P^2$ and $n_s \simeq 0.97$ (continuous) and $U(\chi)$ with $\xi = \beta_0 = \tilde{\xi} = 0$ (dashed) for δ_f ranging from 4 (purple) to 10 (red) with steps of 1, displayed in rainbow colors. In the continuous lines $\tilde{\xi}$ varies with f such that $n_s \simeq 0.97$. The bullets represents the corresponding points at χ_N with $N_e = 60$.



(a) **(b)**

Figure 3: r vs. n_s (a) and α_s vs. n_s (b) for $N_e = 60$ with $\beta_0 = -10M_P^2$ for $\delta_f = 10^{-2}$ (purple), $\delta_f = 10^{-1}$ (yellow) and $\delta_f = 1$ (red) when $\xi = \frac{1}{3}$ (continuous) or $\xi = 0$ (dashed). The pink areas represent the 1,2σ allowed regions for α_s vs. n_s coming from the Planck legacy data [4]. The gray color codes are the same as in Fig. 1.

4. Conclusions

We have investigated a metric-affine realization of natural inflation, featuring a PNGB potential non-minimally coupled to the two linear-in-curvature invariants $\mathcal{R}$ and $\tilde{\mathcal{R}}$. We have discovered regions of the parameter space where the inflationary predictions agree with the most recent constraints at the 2σ level for both trans-Planckian and sub-Planckian values of f.

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References

[1] K. Freese, J. A. Frieman and A. V. Olinto, Phys. Rev. Lett. **65** (1990), 3233-3236

[2] P. A. R. Ade *et al.* [BICEP and Keck], Phys. Rev. Lett. **127** (2021) no.15, 151301 [arXiv:2110.00483 [astro-ph.CO]].

[3] A. Racioppi and A. Salvio, JCAP **06** (2024), 033 [arXiv:2403.18004 [hep-ph]].

[4] N. Aghanim *et al.* [Planck], Astron. Astrophys. **641** (2020), A6 [erratum: Astron. Astrophys. **652** (2021), C4] [arXiv:1807.06209 [astro-ph.CO]].