

Precision calculations for groomed event shapes at HERA

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The possibility to reanalyse data taken by the HERA experiments offers the chance to study modern QCD jet and event-shape observables in deep-inelastic scattering production. In this contribution we present resummed and matched predictions for the groomed invariant-mass event shape in neutral-current DIS including the effect of grooming the hadronic final state using the soft-drop technique. Non-perturbative corrections from hadronisation are taken into account through parton-to-hadron level transfer matrices extracted from dedicated Monte Carlo simulations with SHERPA, including uncertainties extracted from replica tunes to data from the HERA experiments.

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1. Introduction

Studying deep-inelastic scattering (DIS) has historically been one of the primary methods to discover and investigate strong-interaction phenomena. With several future projects like the EIC but also proposed projects such as the LHeC and FCC-eh machines, it is of considerable interest to challenge the theoretical and experimental methods currently employed at the LHC with tasks necessary for DIS [1]. This includes the standardisation of next-to-leading order matched event generation, either with the MC@NLO [2] or POWHEG [3] methods and merging techniques at LO and NLO accuracy. Both have been developed in full generality, but have received less attention in the context of DIS over the past years. Further, jet-substructure techniques [4] receive significant attention at the LHC [5], and can find applications in a DIS context as well. One example is the soft-drop grooming method [6], that has been generalised and applied also to jets at lepton colliders [7, 8] and event shapes at both lepton colliders [7, 9], in Higgs boson decays [10] and at hadron colliders [11], and, recently, also to event shapes in DIS [12].

In this context, the H1 collaboration recently made use of this version of soft-drop grooming, to reanalyse data and measure the groomed jet mass as well as the groomed 1-jettiness [13]. Likewise, plain 1-jettiness was measured [14], which is equivalent to thrust but formulated as a manifestly global event shape. We here report on the semi-analytical predictions used in these measurements, that were originally derived in [15].

DIS measurements further offer a wealth of data valuable for tuning non-perturbative model parameters in Monte Carlo event generators. These data are largely complementary to both lepton colliders, without initial-state hadrons, and hadron colliders, where often effects from the underlying event dominate. This was exploited in [15] to produce tunes of SHERPA's newly implemented hadronisation model [16], including replica tunes in the style of [17] for uncertainty estimates.

We will here focus on predictions for the groomed-mass observable, that we compile in the same framework as used in [15] but have not discussed in detail there. In Sec. 2 we introduce the technical details of soft-drop grooming and the observable definitions. We proceed in Sec. 3 to discuss the Monte Carlo simulation of DIS events and in Sec. 4 our framework for resummation of event shapes in DIS. We present our results at the example of the groomed mass in Sec. 5 before concluding.

2. Soft-drop groomed mass in DIS

Soft-drop grooming is a popular jet substructure technique used at the LHC in various contexts, see for example [11, 18–24]. The general idea is to recluster a given object in a collision event with a suitable jet algorithm to identify the branching history, and then drop softer branches while proceeding reversely through this tree. The DIS specific version of [12] is based on the CENTAURO jet algorithm [25]. The distance measure between particles with momenta p_i, p_j in this algorithm is given by

$$d_{ij} = (\Delta\bar{z}_{ij})^2 + 2\bar{z}_i\bar{z}_j(1 - \cos\Delta\phi_{ij}), \quad \text{with } \bar{z}_i = 2\sqrt{1 + \frac{q \cdot p_i}{x_B P \cdot p_i}} \quad \text{and} \quad \Delta\bar{z}_{ij} = \bar{z}_i - \bar{z}_j, \quad (1)$$

where we are using the standard DIS kinematics with P_μ the proton momentum, and the exchanged photon momentum given by the difference between the incoming and outgoing electron momentum

$q_\mu = k_\mu - k'_\mu$. For future reference we also define the usual variables

$$Q^2 = -q^2, \quad x_B = \frac{Q^2}{2P \cdot q}, \quad \text{and} \quad y = \frac{P \cdot q}{P \cdot k}. \quad (2)$$

Two branches in the clustering history are compared using the measure

$$\frac{\min[z_i, z_j]}{z_i + z_j} > z_{\text{cut}}, \quad \text{with} \quad z_i = \frac{P \cdot p_i}{P \cdot q}, \quad (3)$$

where z_{cut} is an adjustable parameter of the grooming algorithm. If the soft-drop condition is not satisfied the branch with smaller z is dropped, and the procedure is repeated with the other branch. The algorithm terminates if either Eq. (3) is satisfied, or if there is only one particle left.

After applying the grooming algorithm, properties of the surviving final state can be calculated as before. We focus on the mass of the groomed final state

$$\rho = \frac{(\sum_i p_i)^2}{Q_0^2}, \quad (4)$$

which has also been studied in [12]. The sum here extends over all final-state hadrons that have not been dropped during the grooming procedure. To be compatible with [12] and the measurement in [13], we normalise to the minimal Q^2 value considered in the measurement, namely $Q_0^2 = 150 \text{ GeV}^2$.

3. SHERPA framework for DIS

We derive hadron-level predictions for the DIS event shapes using a pre-release version of SHERPA-3.0 [26, 27]. To analyse our simulated event samples we employ the RIVET analysis package [28]. For jet clustering we use the CENTAURO plugin [25] within the FASTJET framework [29].

We consider the massless single and dijet production channels in neutral current DIS at next-to-leading order (NLO), and three- and four-jets at leading order (LO). In our simulation we consider u, d, s quarks to be massless, and include single and dijet production at NLO, whereas processes involving massive c, b quarks are added at LO [30]. For all cases, three- and four-jet processes are included at LO. Different multiplicities are consistently merged together according to the MEPS@NLO [31] and MEPS@LO [32] truncated-shower prescriptions using the Catani–Seymour dipole shower [33]. The DIS specific adaptations to the merging formalism have originally been described in [34]. Tree-level matrix elements are provided by COMIX [35] and AMEGIC [36]. As parton density functions we use the NNLO PDF4LHC21_40_pdfas set [37] with $\alpha_S(M_Z^2)=0.118$ obtained from LHAPDF [38]. Beyond the core process, the arguments of the strong-coupling factors are determined by the clustering algorithm [32], and we set the core scale as well as the merging-scale parameter dynamically, thereby following Ref. [34]. The events get hadronised using SHERPA’s new implementation of the cluster hadronisation model [16], tuned to LEP [17] and DIS [15] data, including replica tunes to estimate the uncertainty induced by non-perturbative corrections.

4. Resummed predictions with SHERPA + CAESAR

We derive predictions at NLL accuracy using the implementation of the CAESAR formalism [39] available in the SHERPA framework [40, 41]. The formalism provides a master formula, valid for recursive infrared and collinear (rIRC) safe observables, for the cumulative cross section integrating observable values up to $v = \exp(-L)$. For a 2-jet observable like groomed jet mass in DIS it is written as follows:

$$\Sigma_{\text{res}}(v) = \int d\mathcal{B} \frac{d\sigma}{d\mathcal{B}} \exp \left[- \sum_l R_l^{\mathcal{B}}(L) \right] \mathcal{P}^{\mathcal{B}}(L) \mathcal{F}^{\mathcal{B}}(L) \mathcal{H}(\mathcal{B}), \quad (5)$$

where $\frac{d\sigma}{d\mathcal{B}}$ is the fully differential Born cross section and \mathcal{H} implements the kinematic cuts applied to the Born phase space \mathcal{B} . Since we are dealing with an additive observable, the multiple emission function \mathcal{F} is simply given by $\mathcal{F}(L) = e^{-\gamma_E R'} / \Gamma(1 + R')$, with $R'(L) = \partial R / \partial L$ and $R(L) = \sum_l R_l(L)$. The collinear radiators R_l for the hard legs l were computed in [39]. We match our resummed calculation in the multiplicative matching scheme along the lines of [41]. The extensions made in [11] to accommodate the phase-space constraints implied by soft-drop grooming with general parameters z_{cut} and β , and that have been used to describe groomed jet substructure in [22, 23, 42], are directly applicable here. For the detailed chain of arguments see also [15].

We use the functionality of SHERPA as a matrix element generator and fixed order Monte Carlo to produce $\mathcal{O}(\alpha_s^2)$ accurate differential predictions for normalised event shape distributions. This can be achieved by considering an NLO calculation of the 2-jet production process with a cut requiring a minimal value of the considered observable. This cut can be chosen to be smaller than any bin resolved by the experiment (or otherwise of interest). The missing contributions completely drop out of normalised distributions. Here however, we want to include the total cross-section prediction. We do this by computing the NNLO accurate cross-section using the projection-to-Born method [43] that has been automated in SHERPA for DIS [44] and the Drell-Yan process [45]. The cross section differential in y and Q^2 is sufficient to fix the missing contributions to the differential cross section. Note that while at fixed order that contribution is confined to the bin including a value of the event shape of 0, this is not necessarily true in our multiplicative matching scheme.

While soft-drop grooming has been shown to reduce the impact of non-perturbative corrections in various circumstances, for example in [7, 9, 11, 19, 22, 23, 46, 47], it is typically still necessary to account for a remaining small hadronisation contribution. We here adopt the approach of [23] to extract transfer matrices from SHERPA Monte Carlo simulations. This approach has been shown to be superior to bin-wise ratios between hadron and parton level Monte Carlo, see Refs. [23, 47], and has been connected to the shape function approach [48] in [47].

5. Results for groomed jet mass

We derive predictions for the groomed invariant mass ρ in the phase-space region $0.2 < y < 0.7$ and $150 \text{ GeV}^2 < Q^2 < 20000 \text{ GeV}^2$. To estimate perturbative uncertainties, we consider 7-point variations of the factorisation and renormalisation scales in the matrix element and the parton shower that get evaluated on-the-fly [49]. The resummation scale we keep fixed. We estimate the

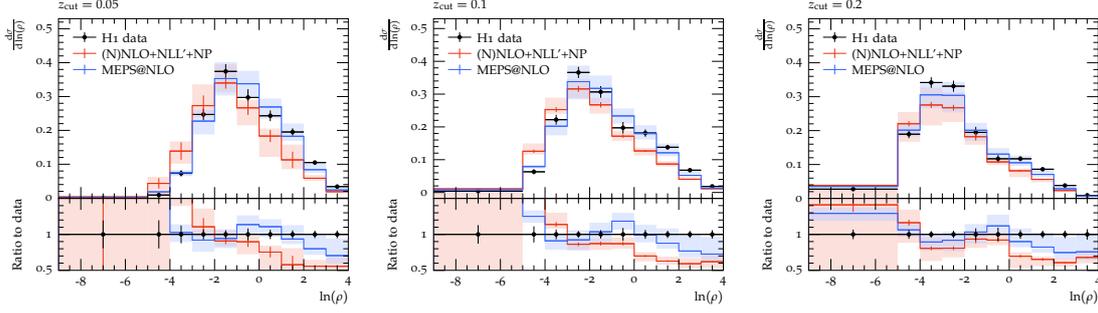


Figure 1: Differential cross section of the natural logarithm of the groomed invariant mass $\ln(\rho)$ in DIS at $\sqrt{s} = 319 \text{ GeV}$ for $z_{\text{cut}} \in \{0.05, 0.1, 0.2\}$ (left to right panel) and $\beta = 0$. The phase space is restricted to $0.2 < y < 0.7$ and $150 \text{ GeV}^2 < Q^2 < 20000 \text{ GeV}^2$. The bars on the data points illustrate the combined statistical and systematic uncertainty. The systematic uncertainties of the theory predictions are shown by the colored envelopes, while the error bars depict the statistical uncertainties.

impact of sub-leading logarithms in the resummation by varying x_L in the form of the logarithm according to

$$L \rightarrow \ln \left(\frac{x_L}{v} - x_L + 1 \right) \xrightarrow{v \rightarrow 0} \ln \left(\frac{x_L}{v} \right), \quad (6)$$

assuming $x_L \in \{0.5, 1, 2\}$, leaving the distribution at the kinematic endpoint unchanged. Furthermore, we consider non-perturbative uncertainties through a set of replica tunes, for that we extract individual transfer matrices. The final systematic uncertainty estimate is derived by forming an envelope of all variations for the MEPS@NLO distribution and of all combinations of the scale, x_L and transfer matrix variations for the resummed distribution.

Fig. 1 compares hadron-level MEPS@NLO simulations from SHERPA and (N)NLO+NLL'+NP predictions to H1 data from [13] for the differential cross section of the groomed invariant mass for soft-drop parameter $z_{\text{cut}} \in \{0.05, 0.1, 0.2\}$. SHERPA provides a good description of the data distributions, independent of the value of z_{cut} , largely consistent within uncertainties. Only the first two bins show a larger deviation between prediction and data, with SHERPA overestimating the data. The resummed results matched to NLO and including non-perturbative corrections predict a lower mass than observed in data. However, the agreement improves for stronger grooming where non-perturbative corrections are reduced.

This is supported when considering the transfer matrices shown in Fig. 2 that are used to account for hadronisation corrections. With higher z_{cut} values the parton-to-hadron level migration matrices become more centered around the diagonal. In particular for $z_{\text{cut}} = 0.05$ for $\text{PL } \ln(\rho) < -2$ the shifts in the observable towards larger values at HL are quite significant. However, in the region where the largest deviations from data are observed, *i.e.* for large values of $\ln(\rho)$, non-perturbative corrections are indeed rather mild. In turn, the discrepancy between data and resummation at large $\ln(\rho)$ presumably originate from the lack of higher-order hard-emission contributions.

6. Conclusion

We have studied the groomed jet-mass observable in DIS as an example of a groomed event shape and supplementing the results of [15] for 1-jettiness. Our predictions are compared to data

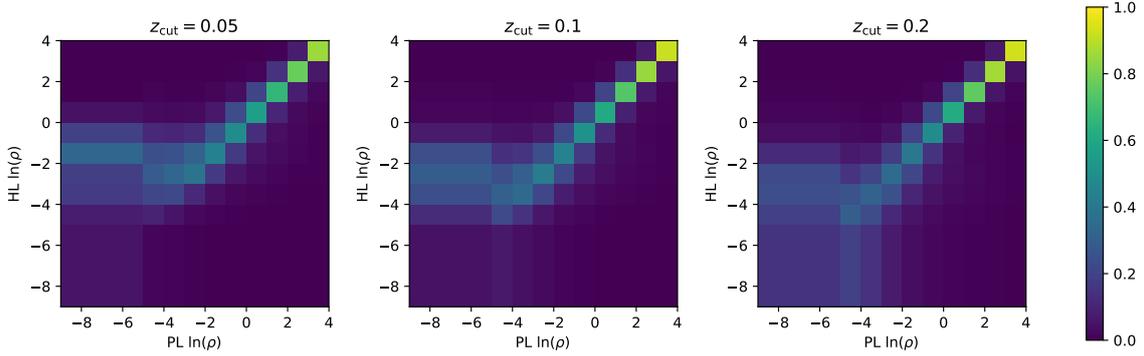


Figure 2: Transfer matrices (corresponding to the default tuning parameter set) used to account for hadronisation corrections to the resummed predictions for the groomed invariant mass for $z_{\text{cut}} \in \{0.05, 0.1, 0.2\}$ (left to right). Each matrix entry T_{ij} describes the probability for an event in parton level (PL) bin j to migrate into the hadron level (HL) bin i .

from the H1 experiment [13] where recently also the 1-jettiness observable was studied [14]. An interesting complementary measurement in this context is the distribution of events with an empty current hemisphere [50]. We achieve NNLO accuracy for the DIS cross section in the event-selection phase space, corresponding to NLO accuracy for the event shapes we consider. We match this to an NLL accurate calculation obtained within the CAESAR formalism, to achieve overall (N)NLO + NLL' accuracy. Hadronisation corrections are included in the transfer-matrix approach based on Monte Carlo simulations with the SHERPA event generator. We also showcase those predictions at MEPS@NLO accuracy, enabling the critically needed comparison between parton showers and analytic resummation [51]. Both the (N)NLO+NLL' result and the SHERPA sample give a fair description of the groomed mass. Notable deficiencies are observable in the small-mass limit, susceptible both to all-orders perturbative as well as non-perturbative corrections. This is consistent with the observations in [13, 14], where excellent agreement for the corresponding predictions for 1-jettiness was found, however not as closely examining the soft limit on a logarithmic scale like the groomed jet-mass measurement does. Note, the predictions for the groomed-mass observable presented here are also discussed in [13], where a more extensive comparison to data differential in y and Q^2 bins can be found.

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