PROCEEDINGS OF SCIENCE

SIDIS and DIS beyond eikonal order: quark background field contribution

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Deep inelastic scattering (DIS) is one of the clean processes to study observables in Color Glass Condensate effective field theory. For semi-inclusive deep inelastic scattering (SIDIS) contributions coming due to dipole approximation at eikonal order have been studied till now. In this proceeding, We will present computations of a new contribution to the cross-section of SIDIS at next-to-eikonal accuracy including t-channel quark exchange through quark background field, and obtain its relation to Transverse momentum dependent (TMD) calculations in small-x limit. We calculate similar contributions for inclusive DIS.

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1. Introduction

Generally, in Color Glass Condensate (CGC) effective field theory to study saturation physics two approximations are adopted: semi-classical approximation and eikonal approximation. In the semi-classical approximation, in the case of dilute-dense scattering, a dense target is given by infinitely boosted semi-classical gluon field $A_\mu(x)$ $1/g >> 1$; and later one, eikonal approximation, is the limit of an infinite boost of gluon background field $A_\mu(x)$. In the eikonal approximation only leading power in terms of the high energy of the system is considered ([1],[2]). To increase the precision of the theory, we have to go beyond eikonal approximation. For that purpose, specifically at next-to-eikonal order, either we can relax the approximations considered for the gluon background field [1] or we can include the effect of the quark background field [3].

In the Transverse Momentum dependent (TMD) factorization framework, the differential cross-section for semi-inclusive deep inelastic scattering (SIDIS) is the product of the quark TMD and a hard factor. Furthermore, in dipole factorization, sea quark TMD can be recovered from the splitting of low-x gluon into quark-antiquark pair [4]. Hence, in this model contribution coming from valence quark distribution is not included.
Similarly, in the case of inclusive deep inelastic scattering, from dipole approximation, we can write the total cross-section as [6],

$$\sigma_{L,T}^{\gamma^* p}(x, Q^2) = \sum_f \int d^2\mathbf{r} \int_0^1 \frac{dz}{4\pi} |\Psi_{\gamma^* L,T \to q\bar{q}}|^2 \sigma_{q\bar{q}}(x, r) \tag{1}$$

Dipole approximation explains the contribution coming due to sea quark at small-x. When integration over z tends to 1 or 0, this model is not justified. We have to include corrections to explain kinematics near limits 1 and 0. Also, in this model contributions coming due to valence quark are not included. In this work, we compute the contributions to include valence quark distribution. We will compute this contribution by taking into account the effect of the quark background field through t-channel quark exchange at next-to-eikonal order. Then, we obtain their relation to Transverse momentum dependent (TMD) (for SIDIS) and Parton distribution function (PDF) (for inclusive DIS) calculations in small-x limit.

2. Semi-Inclusive Deep Inelastic Scattering (SIDIS)

As mentioned in the previous section, at small-x apart from the contribution studied in [4] at eikonal order, there is another contribution fig.(1) to semi-inclusive deep inelastic scattering coming due to including the effect of quark background field beyond eikonal order. Each of these contributions are expected to be dominant in different kinematic regions.
The quark background field is suppressed compared to the gluon background field in terms of high energy and it contributes directly at the next-to-eikonal order. Hence, for the fig. (1) contribution, there will be no contribution at the eikonal order. To compute the contribution to the cross-section due to t-channel quark exchange, first, we will calculate the corresponding contribution to s-matrix given as :

$$S_{\gamma_\lambda^* \to q}|_{\Psi,\text{NEik}} = \lim_{x^+ \to \infty} \int d^2x_\perp \int dx^- \, e^{i\check{p}\cdot x} \int d^4z \, \epsilon_\mu^\lambda(q) \, e^{-iq\cdot z} \, \overline{u}(p,h) \, \gamma^+ \, S_F(x,z)|_{Eik}^{IA} (-iee_f\gamma^\mu) \Psi^-(z) \tag{2}$$

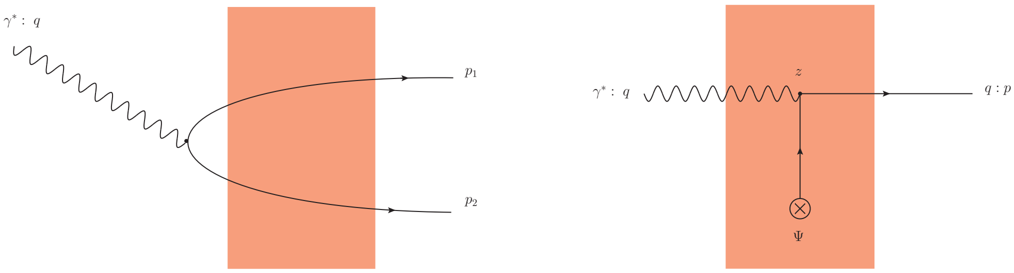

Figure 1: Semi-inclusive deep inelastic scattering(SIDIS): (1) Standard eikonal order contribution: Studied till now; (2) Contribution coming due to t-channel quark exchange: Computed here

where, $S_F(x,z)|^{IA}_{Eik}$ is a quark propagator at eikonal order from inside to after the target [7], $\Psi^-(z) = \frac{\gamma^+\gamma^-}{2}\Psi(z)$ is component of quark background field which is enhanced by the boost of the target and λ is the polarization of photons.
There is no contribution for longitudinal polarization of photons at next-to-eikonal order. For transverse polarization of photons, we obtain cross-section at next-to-eikonal order given as:

$$\frac{d^2\sigma^{\gamma_T^*\to q}}{d^2p_\perp}|_{\Psi,\text{NEik}} = \frac{e^2e_f^2}{(2\pi)^2}\frac{1}{2}\frac{1}{2q^+}\int d^2z'_\perp\int d^2z_\perp\, e^{i(q_\perp-p_\perp)(z_\perp-z'_\perp)}\int dz'^+\int dz^+ \times\left\langle\overline{\Psi}(z')\gamma^-\,\mathcal{U}_F^\dagger(\infty,z'^+,z'_\perp)\;\mathcal{U}_F(\infty,z^+,z_\perp)\;\Psi(z)\right\rangle \tag{3}$$

The physical cross-section is obtained after averaging over the target background field, with suitable probability distribution. But in our work, we are including the effect of the quark background field. Currently, there is no model available that includes a quark background field to generalize the CGC target average. However, one can go back to the quantum expectation value that the CGC target average is supposed to reproduce. We will use this approach here.
For a given color operator O, the CGC-like target average $\langle O\rangle$ should be proportional to the quantum expectation value $\langle P'_{tar}|\hat{O}|P_{tar}\rangle$ in the state $|P_{tar}\rangle$ of the target with momentum P^μ_{tar}. To ensure proper normalization $\langle 1\rangle = 1$ and to avoid ill-defined expression, above relation becomes:

$$\langle O\rangle = \lim_{P'_{tar}\to P_{tar}}\frac{\left\langle P'_{tar}\middle|\hat{O}\middle|P_{tar}\right\rangle}{\left\langle P'_{tar}\middle|P_{tar}\right\rangle} \tag{4}$$

Where we choose the target state normalized as

$$\langle P'_{tar}|P_{tar}\rangle = 2P^-_{tar}(2\pi)^3\delta(P'^-_{tar}-P^-_{tar})\delta^{(2)}(P'_{tar\perp}-P_{tar\perp}) \tag{5}$$

following the usual conventions from Light-Front quantization. We can write equation (3) using this relation.

Also, the unpolarized TMD quark distribution is defined as (up to UV and rapidity regularization issues)[3]

$$f_1^q(x,k_\perp) = \frac{1}{(2\pi)^3}\int_{b_\perp}e^{ik_\perp b_\perp}\int_{z^+}e^{-iz^+xP^-_{tar}}\left\langle P_{tar}\middle|\overline{\Psi}(z^+,b_\perp)\frac{\gamma^-}{2}\mathcal{U}_F^\dagger(\infty,z^+;b_\perp)\mathcal{U}_F(\infty,0;0)\Psi(0,0)\middle|P_{tar}\right\rangle \tag{6}$$

Comparing with unpolarized quark TMD $f_1^q(x, k_\perp)$ and the cross-section, we get,

$$\frac{d^2\sigma^{\gamma_T^*\to q}}{d^2 p_\perp}|_{\Psi,\text{NEik}} = \frac{e^2 e_f^2}{(2\pi)^2}\frac{2\pi}{2q^+}\frac{1}{2P^-_{tar}} f_1^q(0, p_\perp - q_\perp) \tag{7}$$

But due to high energy approximation, we have, $W^2 = (q + P_{tar})^2 \simeq 2q^+ P^-_{tar}$, where W is center of mass energy of the photon-target collision.

$$\frac{d^2\sigma^{\gamma_T^*\to q}}{d^2 p_\perp} = \frac{\alpha_{em} e_f^2}{W^2} f_1^q(x = 0, p_\perp - q_\perp) \tag{8}$$

In the above equation, suppression by the center of mass energy $1/W^2$ is characteristic of a next-to-eikonal contribution, and here it is due to the exchange of quarks in t-channel.

3. Inclusive Deep inelastic Scattering (DIS)

Similar to SIDIS for inclusive deep inelastic, we compute the contributions at next-to-eikonal order by including effect of quark background field. For inclusive DIS there are two contributions given in fig (2) due to t-channel quark exchange: (1) due to the quark propagator from inside to inside the medium and (2) due to antiquark propagator from inside to inside the medium.

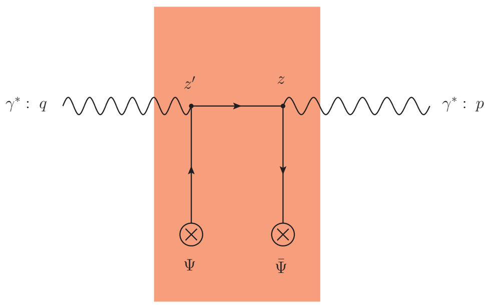

Figure 2: Inclusive deep inelastic scattering (DIS): (1) contribution coming due to quark propagator from inside to inside the medium; (2) Contribution coming due to antiquark propagator from inside to inside the medium

First, similar to SIDIS, we compute the s-matrices for both the contributions at next-to-eikonal order for transverse and longitudinal polarizations of photons, and similar to SIDIS, there is no contribution due to longitudinal polarizations of incoming virtual photons at next-to-eikonal order. For transverse polarization of photons, using optical theorem [5] and equation (4), we get a cross-section due to contribution from the quark propagator from inside to inside the target given as,

$$\sigma_{\text{T}}^{\gamma^*,q}|_{\Psi,\text{NEik}} = \text{Re}\frac{(e^2 e_f^2)}{2q^+ 2P^-_{tar}}\int dz^+ \theta(z^+)\ \langle P_{tar}|\,\overline{\Psi}(z^+,0)\ \mathcal{U}_F(z^+,0,0)\gamma^-\ \Psi(0,0)\,|P_{tar}\rangle \tag{9}$$

We can obtain a similar cross-section for the contribution due to the anti-quark propagator. In equation (9), operator is related to quark PDF at $x = 0$, hence we obtain following contribution to the structure function: $F_2^q = F_T^q = \frac{Q^2}{W^2} \sum_f e_f^2 \, q_f(x)|_{x=0}$, and $F_L^q = 0$. We have, thus, recovered the parton model like expressions from CGC at next-to-eikonal accuracy at low-x.

Correspondingly, we obtain similar relations for antiquark contribution.

4. Conclusion

In this work, beyond eikonal order (next-to-eikonal order) corrections to cross-sections were computed by including quark background field effect through t-channel quark exchange. The t-channel quark contributions to cross-sections in CGC for SIDIS and inclusive DIS were compared with TMD and PDF formalism respectively at small-x. SIDIS cross-section at small-x, where longitudinal momenta of incoming photon and outgoing quark are equal, can be written in terms of a TMD. Quark contribution to PDF at small-x can be written in terms of CGC cross-section. Hence, we recover naive parton model at next-to-eikonal order in small-x limit for inclusive DIS including the contribution of valence quark.

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References

[1] T. Altinoluk and G. Beuf, Phys. Rev. D **105** (2022) no.7, 074026 doi:10.1103/PhysRevD.105.074026 [arXiv:2109.01620 [hep-ph]].

[2] T. Altinoluk, G. Beuf, A. Czajka and A. Tymowska, Phys. Rev. D **104** (2021) no.1, 014019 doi:10.1103/PhysRevD.104.014019 [arXiv:2012.03886 [hep-ph]].

[3] T. Altinoluk, N. Armesto and G. Beuf, Phys. Rev. D **108** (2023) no.7, 074023 doi:10.1103/PhysRevD.108.074023 [arXiv:2303.12691 [hep-ph]].

[4] C. Marquet, B. W. Xiao and F. Yuan, Phys. Lett. B **682** (2009), 207-211 doi:10.1016/j.physletb.2009.10.099 [arXiv:0906.1454 [hep-ph]].

[5] G. Beuf, Phys. Rev. D **96** (2017) no.7, 074033 doi:10.1103/PhysRevD.96.074033 [arXiv:1708.06557 [hep-ph]].

[6] Y. V. Kovchegov and E. Levin, Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. **33** (2012), 1-350 Oxford University Press, 2013, ISBN 978-1-009-29144-6, 978-1-009-29141-5, 978-1-009-29142-2, 978-0-521-11257-4, 978-1-139-55768-9 doi:10.1017/9781009291446

[7] T. Altinoluk, G. Beuf, A. Czajka and A. Tymowska, Phys. Rev. D **107** (2023) no.7, 074016 doi:10.1103/PhysRevD.107.074016 [arXiv:2212.10484 [hep-ph]].