PROCEEDINGS OF SCIENCE

Analysis of (n+1) and n-parton contributions for computing QCD jet cross sections in the local analytic subtraction scheme

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We analyze and implement the Local Analytic Sector Subtraction (LASS) scheme for handling infrared singularities in next-to-next-to-leading order (NNLO) calculations in perturbative QCD. We examine the key aspects of the scheme including sector function construction, singular limit parametrization, subtraction counterterm derivation, and integration techniques. As a proof-of-concept, we numerically implement LASS for the process $e^+e^- \to 3$ jets. In this study we examine the limiting behavior of subtraction terms for real-virtual contribution and explicitly demonstrate the pole cancellation of the double-virtual contribution. Differential cross sections are computed for several event shape observables, showing the stability and efficiency of LASS scheme. This work lays the foundation for developing an automated tool for NNLO QCD calculations using this promising scheme.

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1. Introduction

Precise theoretical predictions are crucial for interpreting collider data and testing the Standard Model. Next-to-next-to-leading order (NNLO) accuracy in QCD is now needed for many key processes. However, at NNLO, handling the infrared divergences from soft and collinear radiation is very challenging. Several approaches have been developed to treat these singularities at NNLO, including slicing methods [1, 2], non-local subtraction schemes [3, 4], and local subtraction methods like antenna subtraction [5, 6], CoLoRFulNNLO [7–9], nested soft-collinear subtraction [10] and the recently proposed Local Analytic Sector Subtraction (LASS) [11–13]. LASS aims to provide a fully local and analytic subtraction framework that can be implemented numerically within any existing code, using sector decomposition of the phase space and flexible parametrizations of singular limits to derive counterterms that can be integrated analytically. In this work, we implement LASS and apply it to the process $e^+e^- \to 3$ jets as a proof-of-concept. We analyze the theoretical framework in depth, perform rigorous checks of the counterterms, and compute differential distributions for event shape observables.

2. Local Analytic Sector Subtraction at NNLO

The NNLO correction to a cross section schematically takes the form:

$$d\sigma^{\mathrm{NNLO}} = \int d\Phi_n VV\,\delta_n(X) + \int d\Phi_{n+1} RV\,\delta_{n+1}(X) \int d\Phi_{n+2} RR\,\delta_{n+2}(X) \tag{1}$$

Here VV, RV and RR denote the double-virtual, real-virtual and double-real squared matrix elements, and $\delta_n(X)$ fixes the observable X at its value for n-body kinematics. To numerically integrate each term, we must either eliminate or regulate all singularities. LASS achieves this by adding and subtracting counterterms: [replace sub with reg for regularized]

$$RR_{\mathrm{sub}}(X) \equiv RR\,\delta_{n+2}(X) - K^{(1)}\,\delta_{n+1}(X) - \left(K^{(2)} - K^{(12)}\right)\delta_n(X)\,, \tag{2}$$

$$RV_{\mathrm{sub}}(X) \equiv \left(RV + I^{(1)}\right)\delta_{n+1}(X) - \left(K^{(\mathbf{RV})} + I^{(12)}\right)\delta_n(X)\,, \tag{3}$$

$$VV_{\mathrm{sub}}(X) \equiv \left(VV + I^{(2)} + I^{(\mathbf{RV})}\right)\delta_n(X)\,. \tag{4}$$

$K^{(1)}$ is a counterterm that accounts for the single unresolved singularities, $K^{(2)}$ accounts for the double unresolved singularities, and $K^{(12)}$ is introduced to avoid double counting by removing the overlap between $K^{(1)}$ and $K^{(2)}$. At RV level there is still a possibility to introduce an additional subtraction term $K^{(\mathbf{RV})}$ which deals with the phase-space singularities of RV contribution. So in total one needs 4 kinds of subtraction terms. Then those subtraction terms are integrated analytically over the factorized singular regions of the radiation phase spaces and the integrated versions of those subtraction terms are then added back. They are defined as:

$$I^{(1)} \equiv \int d\Phi_{\mathrm{rad}}\,K^{(1)}\,, \qquad I^{(2)} \equiv \int d\Phi_{\mathrm{rad},2}\,K^{(2)}\,,$$
$$I^{(12)} \equiv \int d\Phi_{\mathrm{rad}}\,K^{(12)}\,, \qquad I^{(\mathbf{RV})} \equiv \int d\Phi_{\mathrm{rad}}\,K^{(\mathbf{RV})}\,, \tag{5}$$

where Φ_{rad} parametrizes the phase-space of the single unresolved radiation and $\Phi_{\text{rad},2}$ - the phase-space of the double unresolved radiation. To construct the counterterms, LASS partitions the phase space into sectors containing a minimal set of singularities using sector functions W_{abcd} defined in Refs. [11, 13], see also Ref. [14] in these proceedings.

Various singular projectors S_i, C_{ij} etc. are defined to parametrize different unresolved configurations. The counterterms are built by applying these projectors to the matrix element in each sector. Integrating the counterterms over the unresolved phase space is a key aspect of LASS. Flexible mappings of the phase space are used to factorize the integrals into a radiation part and a Born-level part. To this end the Catani-Seymour final-state mapping [15] is being used. This allows the counterterms to be analytically integrated over the radiation variables, yielding expressions that can be added back to the singular contributions in Eqs. (2)- (4).

3. Validation and results for $e^+e^- \to 3$ jets

As a proof-of-concept, we implemented the LASS scheme for the process $e^+e^- \to 3$ jets. At NNLO, this involves the subprocesses:

$$RR: \quad e^+e^- \to q\bar{q}q\bar{q}g\,, \quad e^+e^- \to q\bar{q}r\bar{r}g\,, \quad e^+e^- \to q\bar{q}ggg\,, \tag{6}$$

$$RV: \quad e^+e^- \to q\bar{q}q\bar{q}\,, \quad e^+e^- \to q\bar{q}r\bar{r}\,, \quad e^+e^- \to q\bar{q}gg\,, \tag{7}$$

$$VV: \quad e^+e^- \to q\bar{q}g\,, \tag{8}$$

where the notation q and r is used to distinguish between quarks of different flavors. The processes in Eq. (6) involve only tree-level kinematics and helicity amplitudes, from which the squared matrix elements can be constructed using the results in Refs. [16–18]. The processes in Eq. (7) contain one-loop virtual corrections, and their matrix elements can be constructed using the method described in Ref. [19]. Finally, the process in Eq. (8) involves two-loop virtual corrections, and the corresponding matrix element can be found in Refs. [20, 21].

The integration over physical phase space is managed using an in-house implementation of the VEGAS+ algorithm [22].

3.1 Pole cancellation and limiting behavior

As a first check, we verified the pole structure of the VV contribution by comparing the ϵ expansion of $I^{(2)} + I^{(RV)}$ with the known singularities of the two-loop virtual correction. We found perfect agreement.

Next, we studied the limiting behavior of the RV counterterms in all singular regions of phase space. Since $K^{(\mathbf{RV})}$ is designed to cancel the phase-space singularities of RV, and $I^{(\mathbf{12})}$ cancels the singularities of $I^{(\mathbf{1})}$, it is sufficient to examine their respective ratios in the singular limits appearing in the $K^{(\mathbf{RV})}$ term. This has been done for each individual subprocess listed in Eq. (7). The $e^+e^- \to q\bar{q}q\bar{q}$ process features a total of 4 singular limits, $e^+e^- \to q\bar{q}r\bar{r}$ features 2, and $e^+e^- \to q\bar{q}gg$ features 7 singular limits. For illustration purposes, in this proceedings we present plots obtained for the real virtual and double virtual processes with four equal flavour quarks in the final state. Our results for the other final states are very similar.

The results for the ratios of $I^{(1)}$ and $I^{(12)}$ pair and RV and $K^{(\mathbf{RV})}$ pair for $e^+e^- \to q\bar{q}q\bar{q}$ contribution are plotted in Fig. 1. The precise way of taking these limits together with the definition of the parameter λ that controls the limit is described in the contribution [14] to these proceedings. On these plots, the absolute value of the numbers read from the vertical axis indicates the number of digits of accuracy with which the ratio matches unity and the notation "$\lim_{P}$" means that we are approaching that particular limit. For example, "$\lim_{C_{ijk}}$" corresponds to the case when partons i, j, and k become collinear.

3.2 Technical cut and event shape distributions

Making a step closer towards physical predictions, we computed computed differential distributions for several event shape observables. For this purpose, we choose four event shape observables typically studied in $e^+e^- \to 3$ jet processes [23, 24]. The observables of our choice are: τ-parameter (thrust), C-parameter, Energy-energy correlation, Jet-cone energy fraction. These observables are chosen because they provide complementary information about the event topology and the dynamics of the final-state particles, allowing for a comprehensive study of the underlying QCD processes. In this study we use the definitions taken from [23].

When computing such distributions, due to the finite precision of representing real numbers on the computer, one always have to introduce a technical cut parameter $y_{\min}$ on the phase space to avoid numerical instabilities in the deep IR regions. Similarly to Ref. [25] we are using the following cut to exclude the unstable regions:

$$y_{\min} = \min_{i,j} \frac{s_{ij}}{s}\,, \tag{9}$$

To investigate the impact of this cut we perform the following study: We set the initial value of $y_{\min}$ to 10^{-4} and gradually decrease it in small intervals until it reaches to 10^{-10}. For each value of $y_{\min}$, we perform the Monte Carlo integration by generating 10^7 phase-space points and optimizing the grid 40 times. After every iteration, we calculate a new estimate of the cross-section. Once all 40 iterations are completed, we plot the first moment of the thrust distribution as a function of the technical cut $\langle\tau\rangle(y_{\min})$. To gain further insights, we combine the results of these 40 independent estimates in each bin using two methods. First, we calculate the average of the estimates. Second, we calculate the weighted average, where the inverse values of the mean squared uncertainties are used as weights. This latter method effectively gives preference to more stable integration iterations with smaller uncertainties. The results of this study are depicted in Fig. 2.

We observe that in all studied cases a very good saturation is achieved at the value $y_{\min} = 10^{-8}$. We also considered the different formulations of $y_{\min}$ and for RV contribution no significant effect was found. Hence, for the subsequent studies we fix the value of $y_{\min}$ as defined in Eq. (9) to 10^{-8}.

Next we computed the distributions of the event shape variables by taking into account the regularized real-virtual contributions. To perform this study, we utilized the integration grid obtained in the previous section. To accelerate the speed of the calculation, the integration was performed 200 times in parallel with different random number seeds, each integrating over 10^7 events. This means that in every bin of each event shape observable, we have 200 independent estimates. We combine these estimates using weighted averages described previously, effectively

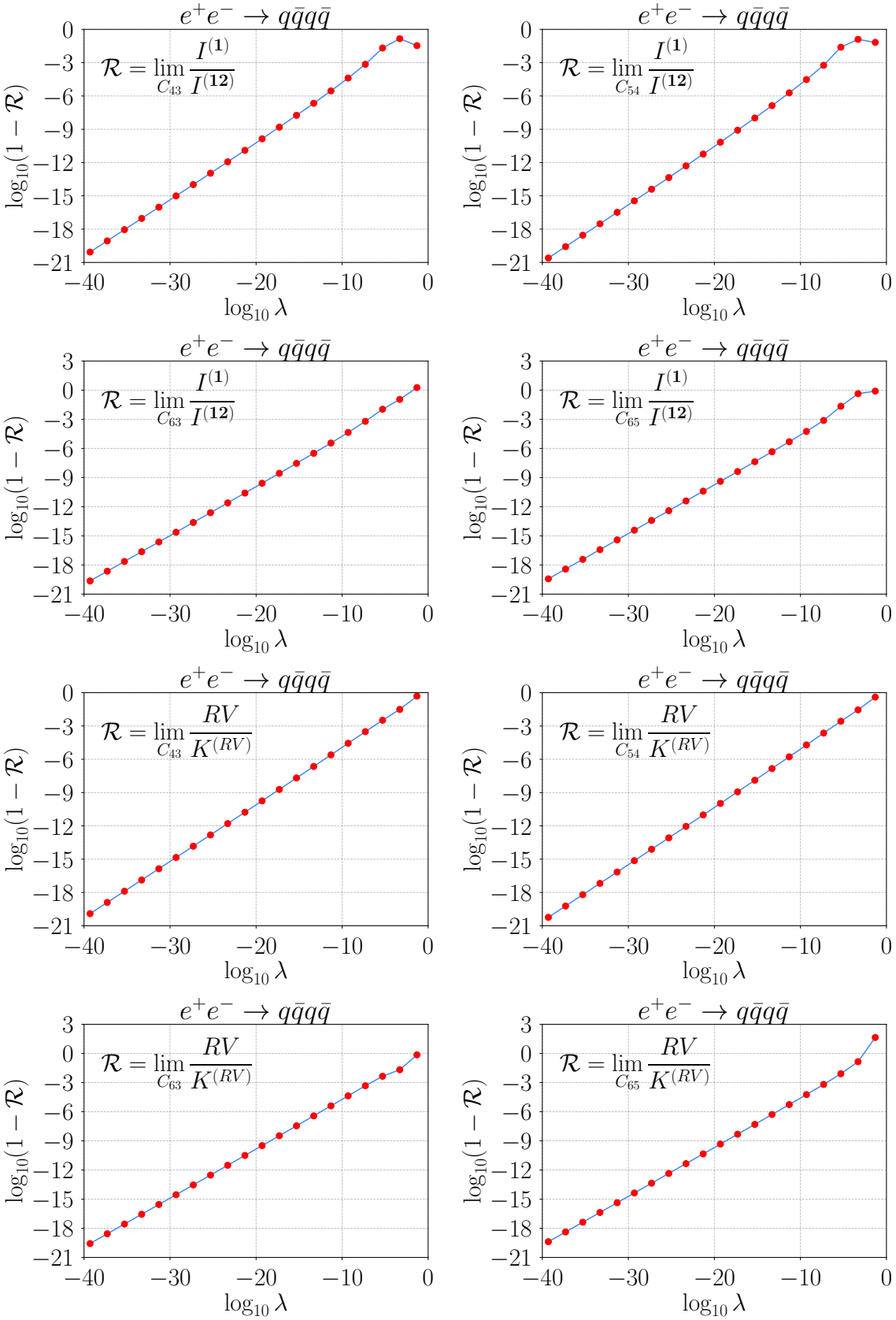

Figure 1: Limiting behavior of the ratio between $I^{(1)}$ and $I^{(12)}$ terms (top row) and RV and $K^{(RV)}$ terms (bottom row) in various singular limits for the one-loop correction to $e^+e^- \to q\bar{q}q\bar{q}$ contribution. Each plot corresponds to a different singular limit, as indicated on the respective plot.

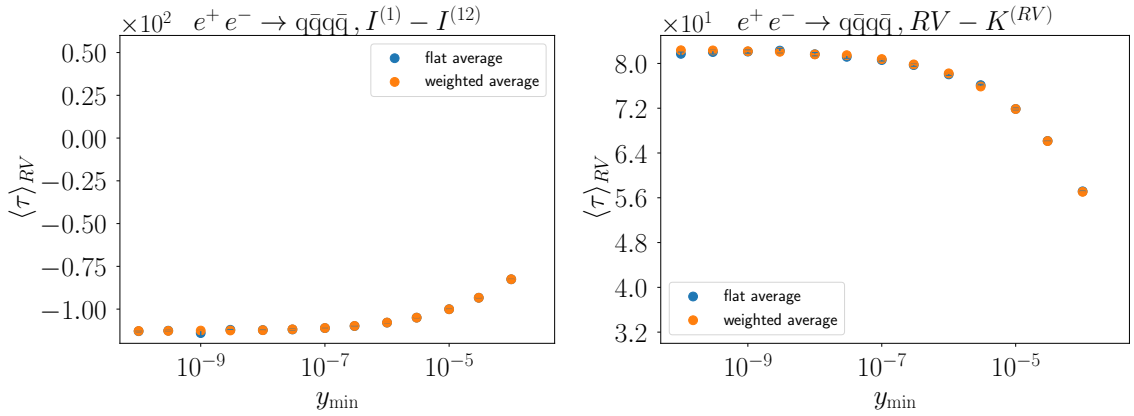

Figure 2: The saturation plots of $I^{(1)} - I^{(12)}$ (to the left) and $RV - K^{(RV)}$ (to the right) contributions to the mean value of the thrust distribution by the process $e^+e^- \to q\bar{q}q\bar{q}$.

accumulating the statistics of 2×10^9 events. The results are depicted in Figs. 3 - 4. As demonstrated in all cases we observe the good numeric stability.

4. Conclusions

We presented a proof-of-concept implementation of the RV and VV contributions regularized according to the LASS scheme for the NNLO corrections to $e^+e^- \to 3$ jets. Explicit pole cancellation was demonstrated for the VV contribution. We have demonstrated that the counterterms correctly approximate the matrix elements in all IR limits. We computed differential distributions of event shape variables and found our predictions stable against variations of the technical cut. This proof-of-concept study lays the groundwork for developing LASS into a general tool for NNLO calculations. Future work will focus on optimizing the phase-space treatment, incorporating initial-state radiation and massive partons, and automating the set-up. The LASS scheme is a promising method that could substantially simplify and speed up precise predictions for collider observables in the coming years.

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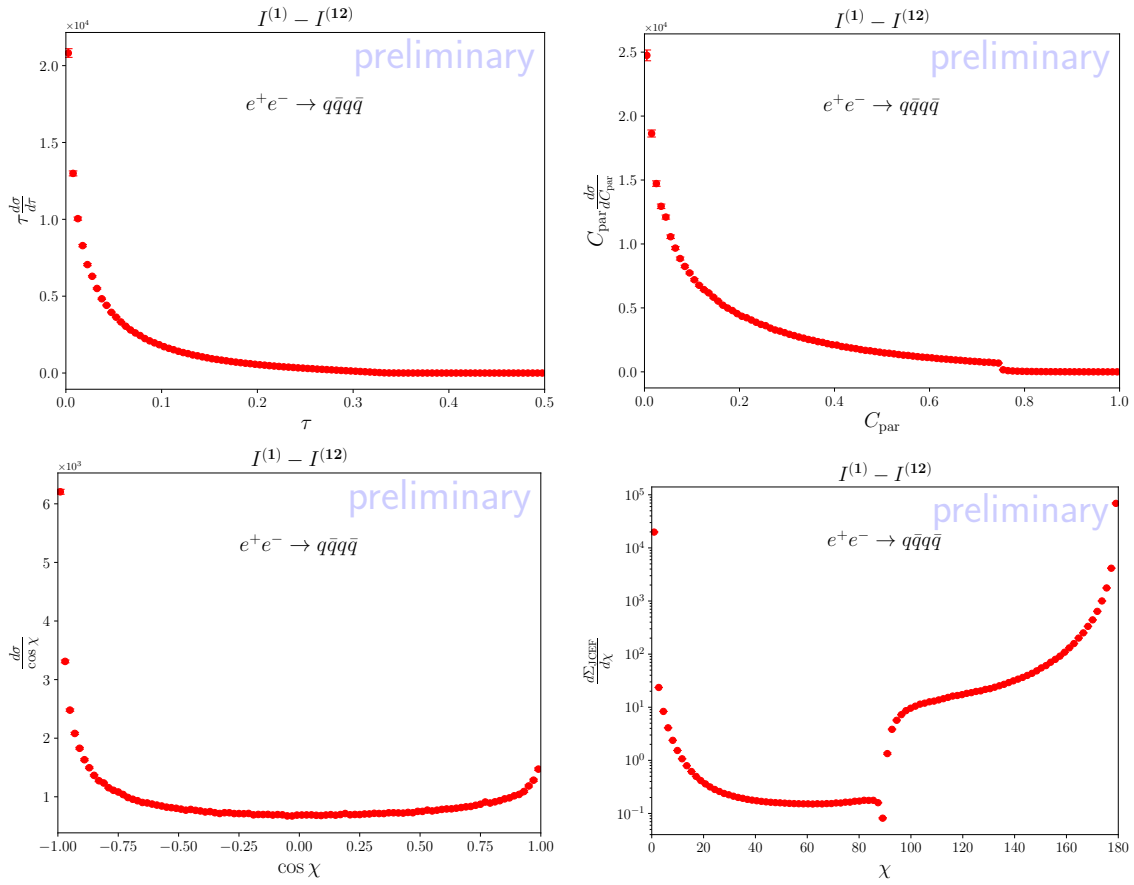

Figure 3: Real-virtual contributions to the (from left to right) τ-parameter, C-parameter, energy-energy correlation and jet-cone energy fraction for $e^+e^- \to q\bar{q}q\bar{q}$ from $I^{(1)} - I^{(12)}$ terms.

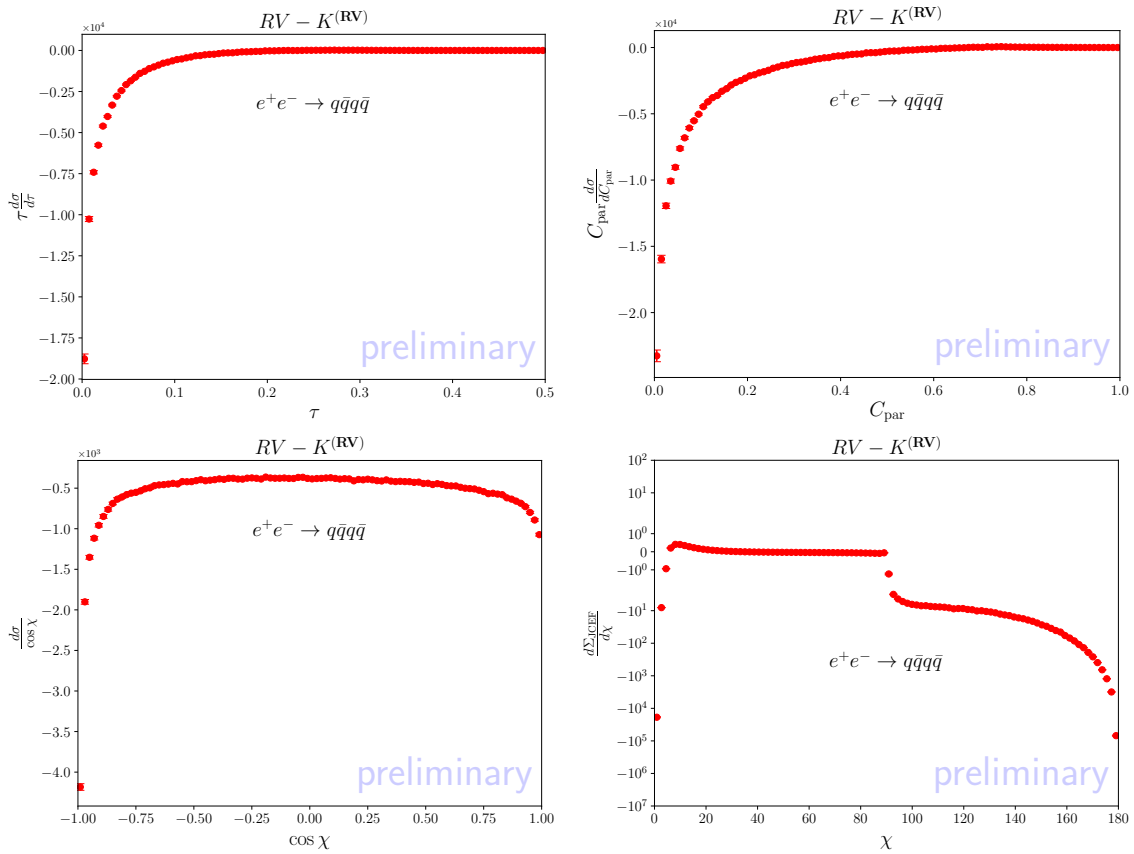

Figure 4: Real-virtual contributions to the (from left to right) τ-parameter, C-parameter, energy-energy correlation and jet-cone energy fraction for $e^+e^- \to q\bar{q}q\bar{q}$ from $RV - K^{(\mathbf{RV})}$ terms.

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