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A subtraction scheme at NLO exploiting the privilege of k_T-factorization

Andreas van Hameren[a,*]

[a] *Institute of Nuclear Physics Polish Academy of Sciences, Radzikowskiego 152, Kraków, Poland*

E-mail: hameren@ifj.edu.pl

I describe a subtraction scheme for the real radiation contribution to next-to-leading order corrections in k_T-factorization. The main feature is that the momentum recoil for the subtraction terms is subtracted from the initial-state momenta rather than distributed over the final-state momenta.

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*Speaker

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1. Introduction

An important ingredient in the predictive power of quantum chromodynamics (QCD) is that it admits perturbation theory, that is it allows for the systematic increase of precision in exchange for computational cost. For inclusive calculations, the cost is in the integration over unobserved quantities in scattering processes. This can be the explicitly unobservable loop momenta in virtual contributions, or the individually unobserved radiative momenta in real contributions, which can be interpreted as to allow for a more accurate description of jets. Given a computer program for the calculation of scattering processes at a certain precision level in perturbative QCD, it seems at first glance to be the extra required loops that prevent an easy increase of precision. Adding an extra parton to the final state and integrating over its momentum seems easy. However, this integral is plagued with divergences that, while formally cancelling against similar divergences in the loop contributions to a large degree, prevent a straightforward calculation. The subtraction method is an approach to deal with this complication [1–12]. In particular, it uses the resources available for calculations at a given order, the ability to calculate the necessary scattering amplitudes, to calculate a higher order real contribution.

For hadron scattering, the cancellation of divergences is incomplete, and the leftovers need to be absorbed into universal parton density functions (PDFs). The possibility of a consistent procedure to establish this is an essential aspect of *collinear factorization*, in which the initial-state partons of the hard scattering process are collinear to the scattering hadrons. In *k_T-factorization*, the picture of collinear initial-state partons is relaxed and the initial-state partons also carry transverse momentum components. While next-to-leading order (NLO) calculations within collinear factorization have been automated, calculations within k_T-factorization are not near that stage. Any NLO calculation must include a real radiation contribution that corresponds to taking the LO process, and integrating over an extra parton added to the final state. It may be that in a certain factorization approach a certain contribution must be avoided/added/subtracted, but the calculation of the real radiation contribution as described in the previous sentence must be under control. In particular, all divergences must be identified.

Here, we describe the essential points of the subtraction scheme introduced in [13] to achieve this for hybrid k_T-factorization, for which one initial-state parton has transverse components, while the other is collinear to the hadron it is associated with. It is similar to existing schemes for collinear factorization in that it employs known singular limits of matrix elements to build the various subtraction terms. However, since all four initial-state momentum components are non-vanishing in hybrid k_T-factorization, there is the possibility to subtract the momentum recoil associated with the subtraction terms from the initial-state momenta.

2. Exposition

In hybrid k_T-factorization, one of the partonic initial states, here denoted χ, has transverse momentum components, while the other, denoted $\overline{\chi}$ does not. Their momenta are

$$k_\chi^\mu = xP^\mu + k_\perp^\mu \quad , \quad P^\mu = (E,0,0,E) \quad , \quad k_\perp^\mu = (0,k_{\perp,1},k_{\perp,2},0) \; , \tag{1}$$

$$k_{\overline{\chi}}^\mu = \bar{x}\bar{P}^\mu \quad , \quad \bar{P}^\mu = (\bar{E},0,0,-\bar{E}) \; , \tag{2}$$

where $P, \bar{P}$ are the momenta of the scattering hadrons. We write the Born-level formula for the cross section as

$$\sigma_{\mathrm{B}} = \frac{1}{\mathcal{S}_n}\int [dQ]\int d\Phi\big(Q;\{p\}_n\big)\,\mathcal{L}\big(Q;\{p\}_n\big)\,\big|\mathcal{M}\big|^2\big(Q;\{p\}_n\big)\,J_{\mathrm{B}}\big(\{p\}_n\big) \tag{3}$$

where $[dQ]$ refers to the integration over initial-state variables

$$\int [dQ] = \int_0^1 dx\int_0^1 d\bar{x}\int d^2k_\perp \quad , \quad Q^\mu = k_\chi^\mu + k_{\bar{\chi}}^\mu\,, \tag{4}$$

and $\mathcal{L}\big(Q;\{p\}_n\big)$ includes the $k_\perp$-dependent PDF, the collinear PDF, and flux factor:

$$\mathcal{L}\big(Q;\{p\}_n\big) = \frac{F_\chi\big(x,k_\perp,\mu_F(\{p\}_n)\big)\,f_{\bar{\chi}}\big(\bar{x},\mu_F(\{p\}_n)\big)}{8x\bar{x}E\bar{E}}\,. \tag{5}$$

This function implicitly depends on the final-state momenta $\{p\}_n$ via the factorization scale. The other components of the Born formula are the differential final-state phase space

$$d\Phi\big(Q;\{p\}_n\big) = \left(\prod_{l=1}^{n}\frac{d^4p_l}{(2\pi)^3}\delta_+(p_l^2-m_l^2)\right)\frac{1}{(2\pi)^4}\,\delta\!\left(Q-\sum_{l=1}^{n}p_l\right), \tag{6}$$

and the tree-level matrix element $\big|\mathcal{M}\big|^2\big(Q;\{p\}_n\big)$. It involves a space-like gluon and can be calculated using Lipatov's effective action [14, 15], or the auxiliary parton method [16] as is done in the program KaTie [17]. The matrix element does not include symmetry factors and averaging factors, and those are captured by $\mathcal{S}_n$. Finally $J_{\mathrm{B}}\big(\{p\}_n\big)$ denotes the jet function that vanishes if it constructs fewer jets than there are final-state partons, and is assumed to include p_T and rapidity cuts for the jets. The real radiation integral

$$\sigma_{\mathrm{R}}(\epsilon) = \frac{1}{\mathcal{S}_{n+1}}\int [dQ]\int d\Phi_\epsilon\big(Q;\{p\}_{n+1}\big)\mathcal{L}\big(Q;\{p\}_{n+1})\big)\,\big|\mathcal{M}\big|^2\big(Q;\{p\}_{n+1}\big)\,J_{\mathrm{R}}\big(\{p\}_{n+1}\big) \tag{7}$$

still involves tree-level matrix elements, but has one more final-state parton, and has a jet function $J_{\mathrm{R}}\big(\{p\}_{n+1}\big)$ that allows for one jet fewer than the number of final-state partons. This integral is divergent because, contrary to the Born case, now the jet function allows one momentum to become arbitrarily soft, and a pair of momenta to become arbitrarily collinear. The divergences must be treated within dimensional regularization to match the divergences of the virtual contribution stemming from the one-loop amplitude. This means that the coefficients $\sigma_{\mathrm{R}}^{(i)}$ in

$$\sigma_{\mathrm{R}}(\epsilon) = \frac{1}{\epsilon^2}\sigma_{\mathrm{R}}^{(-2)} + \frac{1}{\epsilon}\sigma_{\mathrm{R}}^{(-1)} + \sigma_{\mathrm{R}}^{(0)} + \mathcal{O}(\epsilon)\,. \tag{8}$$

must be determined, where $\epsilon = (\mathrm{dim}-4)/2 \to 0$ is the regularizing parameter. In order to achieve this, the divergent behavior of the real radiation integral is cured with subtraction terms, such that this *subtracted real integral* $\sigma_{\mathrm{R}}^{\mathrm{fin}}$ can be performed numerically. The subtraction terms must be such that, when integrated on their own, assume a form that manifestly includes the divergent terms cancelling against the virtual divergences, so their sum $\sigma_{\mathrm{R}}^{\mathrm{div}}(\epsilon)$ has the same divergent coefficients as $\sigma_{\mathrm{R}}(\epsilon)$,

$$\sigma_{\mathrm{R}}^{\mathrm{div}}(\epsilon) = \frac{1}{\epsilon^2}\sigma_{\mathrm{R}}^{(-2)} + \frac{1}{\epsilon}\sigma_{\mathrm{R}}^{(-1)} + \sigma_{\mathrm{R}}^{\mathrm{div},(0)} + \mathcal{O}(\epsilon)\,, \tag{9}$$

while the final desired finite quantity is given by

$$\sigma_{\mathrm{R}}^{(0)} = \sigma_{\mathrm{R}}^{\mathrm{div},(0)} + \sigma_{\mathrm{R}}^{\mathrm{fin}} \ . \tag{10}$$

In practice this means that it must be possible for each subtraction term to "integrate the radiation out" in order to arrive at expressions that involve only Born-level phase space integrals, just like the divergent part of the virtual contribution. Different subtraction schemes will produce different values for $\sigma_{\mathrm{R}}^{\mathrm{div},(0)}$ and $\sigma_{\mathrm{R}}^{\mathrm{fin}}$ individually, but their sum must be the same. In particular, one can introduce adjustable parameters the individual quantities depend on, and independence of the sum can serve as a check for the correctness of the implementation of the subtraction scheme.

2.1 Definition of the subtraction terms

The real contribution involves a momentum set $\{p\}_{n+1}$ with one more final-state parton than the Born contribution, and we need to establish notation for the relation between the two. We write

$$\{p\}_n^{\not{r}} \text{ is obtained from } \{p\}_{n+1} \text{ by removing momentum } p_r \ , \tag{11}$$

$$\{p\}_n^{\not{r};i} \text{ is obtained by additionally replacing } p_i \text{ with } (1+z_{ri})p_i, \quad z_{ri} = E_r/E_i \tag{12}$$

where E_i is the energy of momentum p_i, and where we assume the momenta to be light-like. The limits that the jet function allows and represent singularities for the radiative matrix element are

$$J_{\mathrm{R}}\big(\{p\}_{n+1}\big) \xrightarrow{p_r\to\text{soft} \Leftrightarrow E_r\to 0} J_{\mathrm{B}}\big(\{p\}_n^{\not{r}}\big) \quad , \tag{13}$$

$$J_{\mathrm{R}}\big(\{p\}_{n+1}\big) \xrightarrow{p_r\| p_i \Leftrightarrow \vec{n}_r-\vec{n}_i\to\vec{0}} J_{\mathrm{B}}\big(\{p\}_n^{\not{r};i}\big) \quad , \tag{14}$$

$$J_{\mathrm{R}}\big(\{p\}_{n+1}\big) \xrightarrow{\quad p_r\| P,\bar{P} \quad} J_{\mathrm{B}}\big(\{p\}_n^{\not{r}}\big) \quad . \tag{15}$$

We define the finite subtracted-real integral $\sigma_{\mathrm{R}}^{\mathrm{fin}}$ as

$$\begin{aligned}\sigma_{\mathrm{R}}^{\mathrm{fin}} = \frac{1}{\mathcal{S}_{n+1}}\int [dQ]\int d\Phi\big(Q;\{p\}_{n+1}\big)\Big\{&\mathcal{L}\big(Q;\{p\}_{n+1}\big)\big)\,\big|\mathcal{M}\big|^2\big(Q;\{p\}_{n+1}\big)\,J_{\mathrm{R}}\big(\{p\}_{n+1}\big)\\ &-\sum_r \mathrm{Subt}_r\big(Q;\{p\}_{n+1}\big)\Big\}\ ,\end{aligned} \tag{16}$$

where the r-sum is over all final-state partons, and where $\mathrm{Subt}_r\big(Q;\{p\}_{n+1}\big)$ is given by

$$\begin{aligned}
&\sum_i \mathcal{L}\big(Q-p_r+z_{ri}p_i;\{p\}_n^{\not{r};i}\big)\quad \mathcal{R}_{i,r}^{\mathrm{F}}(p_r)\otimes\mathcal{A}_{i,r}^{\mathrm{F}}\big(Q-p_r+z_{ri}p_i;\{p\}_n^{\not{r};i}\big)\,J_{\mathrm{B}}\big(\{p\}_n^{\not{r};i}\big)\\
+&\sum_{a\in\{\chi,\overline{\chi}\}}\mathcal{L}\big(Q-p_r;\{p\}_n^{\not{r}}\big)\,\mathcal{R}_a^{\mathrm{I,soft}}(p_r)\otimes\mathcal{A}_a^{\mathrm{I,soft}}\big(Q-p_r;\{p\}_n^{\not{r}}\big)\,J_{\mathrm{B}}\big(\{p\}_n^{\not{r}}\big)\\
+&\sum_{a\in\{\chi,\overline{\chi}\}}\mathcal{L}\big(Q-p_r;\{p\}_n^{\not{r}}\big)\,\mathcal{R}_a^{\mathrm{I,soco}}(p_r)\otimes\mathcal{A}_a^{\mathrm{I,soco}}\big(Q-p_r;\{p\}_n^{\not{r}}\big)\,J_{\mathrm{B}}\big(\{p\}_n^{\not{r}}\big)\\
+&\quad\mathcal{L}\big(Q-\bar{x}_r\bar{P}-p_{\perp r};\{p\}_n^{\not{r}}\big)\,\mathcal{R}_{\chi,r}^{\mathrm{I,col}}(p_r)\otimes\mathcal{A}_{\chi,r}^{\mathrm{I,col}}\big(Q-p_r;\{p\}_n^{\not{r}}\big)\,J_{\mathrm{B}}\big(\{p\}_n^{\not{r}}\big)\\
+&\quad\mathcal{L}\big(Q-x_rP-p_{\perp r};\{p\}_n^{\not{r}}\big)\,\mathcal{R}_{\overline{\chi},r}^{\mathrm{I,col}}(p_r)\otimes\mathcal{A}_{\overline{\chi},r}^{\mathrm{I,col}}\big(Q-p_r;\{p\}_n^{\not{r}}\big)\,J_{\mathrm{B}}\big(\{p\}_n^{\not{r}}\big)
\end{aligned} \tag{17}$$

where also the i-sum is over all final-state partons with $\mathcal{R}^{\mathrm{F}}_{rr}(p_r) \equiv 0$. The subtraction terms are constructed analogously to [3]. The various combinations $\mathcal{R} \otimes \mathcal{A}$ are based on the well-known singular limits of the matrix element $|\mathcal{M}|^2(Q;\{p\}_{n+1})$, where $\mathcal{R}$ represents the singular behavior as function of the radiative momentum p_r, where $\mathcal{A}$ represents a spin-or color-correlated matrix element for the process with the radiative parton removed, and $\otimes$ abbreviates the possible contraction involved with those correlations.

The main difference with [3] lies in the treatment of the momentum recoil. For the "initial-state" subtraction terms, labelled I, the radiative momentum removed from the final state is simply subtracted from the initial state momenta to ensure momentum conservation: the components

$$\bar{x}_r = \frac{p_r\cdot P}{P\cdot\bar{P}} \quad , \quad x_r = \frac{p_r\cdot\bar{P}}{P\cdot\bar{P}} \quad , \quad p^{\mu}_{r,\perp} = p^{\mu}_r - x_r P^{\mu} - \bar{x}_r\bar{P}^{\mu} \tag{18}$$

are subtracted from $\bar{x}$, x, $k^{\mu}_{\perp}$ respectively. For the "final-state" terms labelled with F, the remnant recoil $p^{\mu}_r + p^{\mu}_i - (1+z_{ri})p^{\mu}_i = p^{\mu}_r - z_{ri}p^{\mu}_i$ is subtracted instead. These final-state terms include the collinear terms, soft terms, and soft-collinear counter terms, just like listed explicitly for the initial-state terms. To avoid double counting in the sums over radiation and radiators, a selector function $\theta(E_i > E_r)$ is included for the collinear terms, and singular factors from the soft limits are split up following

$$\frac{(p_i\cdot p_j)}{(p_i\cdot p_r)(p_r\cdot p_j)} = \frac{(p_i\cdot p_j)}{(p_i\cdot p_r)[(p_r\cdot p_j)+(p_i\cdot p_r)]} + \frac{(p_i\cdot p_j)}{(p_j\cdot p_r)[(p_r\cdot p_j)+(p_i\cdot p_r)]} \; . \tag{19}$$

Another difference with [3], and most other subtraction methods, is that for those the equivalent of the $\mathcal{L}$ function is not touched, whereas here the subtraction of the recoil from the initial-state momenta also happens inside the $\mathcal{L}$ function. This is allowed, as long as what is subtracted vanishes at the singular limit. For the initial-state collinear terms, this cannot be the whole radiative momentum p_r itself, since it becomes $x_r P$ or $\bar{x}_r\bar{P}$ at either limit. Only the vanishing components are subtracted in the $\mathcal{L}$ function.

Finally, we mention that all terms are restricted to regions of phase space where they actually matter. The soft and soft-collinear subtraction terms are non-vanishing only if

$$E_r < E_0 \tag{20}$$

for some fixed small energy E_0, and the collinear and soft-collinear terms are non-vanishing only if

$$\angle(\vec{p}_r, \vec{p}_i) < \varphi_0 \tag{21}$$

for some fixed small angle φ_0. The individual contribution $\sigma^{\mathrm{div}}_{\mathrm{R}}(\epsilon)$ and $\sigma^{\mathrm{fin}}_{\mathrm{R}}$ depend on the value of these parameters, but their sum should not. Consequently, the poles in ϵ in $\sigma^{\mathrm{div}}_{\mathrm{R}}(\epsilon)$ should not depend on them, only the finite pieces. As mentioned before, these requirements constitute a powerful check on the correctness of the subtraction method.

We do not present further details about the subtraction terms, and they can be found in [13]. We only mention that, despite one initial-state gluon is not light-like, there is still a collinear singularity associated with it when the radiation becomes collinear to the momentum P, and a subtraction term with splitting function

$$\mathcal{P}_{\chi}(z) = \frac{2N_{\mathrm{c}}}{z(1-z)} \; . \tag{22}$$

has to be included to cure it.

2.2 Integration of the subtraction terms

Thanks to the momentum recoil being subtracted both from the initial-state momenta in the matrix elements and in the $\mathcal{L}$-function, the integration momentum Q can simply be shifted for the calculation of the integrated subtraction terms. After some more manipulations, one finds

$$\sigma_{\mathrm{R}}^{\mathrm{div}}(\epsilon) = \frac{1}{\mathcal{S}_{n+1}}\sum_r\int[dQ]\int d\Phi\big(Q;\{p\}_n^{\not r}\big)\,\mathcal{L}\big(Q;\{p\}_n^{\not r}\big)\,J_{\mathrm{B}}\big(\{p\}_n^{\not r}\big) \tag{23}$$
$$\times\left\{\sum_i \mathcal{I}_{ir}^{\mathrm{F}}\big(\epsilon,Q,\{p\}_n^{\not r}\big)\otimes\mathcal{A}_{ir}^{\mathrm{F}}\big(Q;\{p\}_n^{\not r}\big)+\sum_{a\in\{\chi,\bar\chi\}}\mathcal{I}_{ar}^{I}\big(\epsilon,Q,\{p\}_n^{\not r}\big)\otimes\mathcal{A}_{ar}^{I}\big(Q;\{p\}_n^{\not r}\big)\right\},$$

where

$$\mathcal{I}_{ar}^{I}\otimes\mathcal{A}_{ar}^{I} = \mathcal{I}_{ar}^{\mathrm{I,col}}\otimes\mathcal{A}_{ar}^{\mathrm{I,col}}+\mathcal{I}_{a}^{\mathrm{I,soft}}\otimes\mathcal{A}_{a}^{\mathrm{I,soft}}+\mathcal{I}_{a}^{\mathrm{I,soco}}\otimes\mathcal{A}_{a}^{\mathrm{I,soco}}\,, \tag{24}$$

with

$$\mathcal{I}_{ir}^{\mathrm{F}}\big(\epsilon,Q,\{p\}_n^{\not r}\big) = \int\frac{d^{4-2\epsilon}p_r}{(2\pi)^{3-2\epsilon}}\,\delta_+(p_r^2)\,(1-z_{ri})\,\mathcal{R}_{ir}^{\mathrm{F}}(p_r)\,\Theta(p_r-z_{ri}p_i)\,, \tag{25}$$

$$\mathcal{I}_{a}^{\mathrm{I,soft/soco}}\big(\epsilon,Q,\{p\}_n^{\not r}\big) = \int\frac{d^{4-2\epsilon}p_r}{(2\pi)^{3-2\epsilon}}\,\delta_+(p_r^2)\,\mathcal{R}_{a}^{\mathrm{I,soft/soco}}(p_r)\,\Theta(p_r)\,, \tag{26}$$

$$\mathcal{I}_{\chi r}^{\mathrm{I,col}}\big(\epsilon,Q,\{p\}_n^{\not r}\big) = \int\frac{d^{4-2\epsilon}p_r}{(2\pi)^{3-2\epsilon}}\,\delta_+(p_r^2)\,\mathcal{R}_{\chi r}^{\mathrm{I,col}}(p_r)\,\Theta(p_r)\,\frac{x\,F(x+x_r,k_\perp)}{(x+x_r)F(x,k_\perp)}\,, \tag{27}$$

$$\mathcal{I}_{\bar\chi r}^{\mathrm{I,col}}\big(\epsilon,Q,\{p\}_n^{\not r}\big) = \int\frac{d^{4-2\epsilon}p_r}{(2\pi)^{3-2\epsilon}}\,\delta_+(p_r^2)\,\mathcal{R}_{\bar\chi r}^{\mathrm{I,col}}(p_r)\,\Theta(p_r)\,\frac{\bar x\,f(\bar x+\bar x_r)}{(\bar x+\bar x_r)f(\bar x)}\,. \tag{28}$$

and

$$\Theta(q) = \theta(x_q+x)\,\theta(1-x-x_q)\,\theta(\bar x_q+\bar x)\,\theta(1-\bar x-\bar x_q)\,. \tag{29}$$

We see that the integrated subtraction terms $\mathcal{I}_{\chi/\bar\chi,r}^{\mathrm{I,col}}$ involve the PDFs (their argument $\mu_F\big(\{p\}_n^{\not r}\big)$ is omitted), and cannot be calculated analytically. They are the equivalent of what in [2] is called the "P-operator". The PDFs in the denominator appear because we insisted on having $\mathcal{L}\big(Q;\{p\}_n^{\not r}\big)$ explicitly in Eq. (23), and the ratios $x/(x+x_r)$ and $\bar x/(\bar x+\bar x_r)$ appear because the $\mathcal{L}$-function includes the flux factor. The integration over $\vec p_{r,\perp}$ can be easily performed, and then the variable substitution $x_r = (1-z)x/z$ for Eq. (2.2), and $\bar x_r = (1-z)\bar x/z$ for Eq. (2.2), bring the expressions to the familiar form.

The integrals represented by $\mathcal{I}_{ir}^{\mathrm{F}}$ of Eq. (25) can in principle be performed analytically, but the phase space restrictions dictated by Eq. (29) makes them unnecessarily cumbersome. Observe however that the argument of Θ in $\mathcal{I}_{ir}^{\mathrm{F}}$, the momentum recoil, vanishes both at the collinear and the soft limit, and replacing Θ with $\Theta-1$ makes the integral finite. The integral with Θ replaced by 1 can easily be calculated analytically, and the finite one with $\Theta-1$ can be performed numerically. The latter means in practice that within the Monte Carlo integration of Eq. (23), for each phase space point sampled from $d\Phi\big(Q;\{p\}_n^{\not r}\big)$, one generates three extra random numbers to construct p_r and once evaluates the integrand of Eq. (25) (with $\Theta-1$ instead of Θ).

It turns out that also the phase space restriction of Eq. (20) complicates analytical integration of the soft subtraction terms, which can be resolved by defining an integral with the restriction (and Jacobian) in terms of

$$E_r^{(ij)} = \frac{(p_r\cdot p_i)E_j+(p_r\cdot p_j)E_i}{(p_i\cdot p_j)} \tag{30}$$

instead of E_r. This variable vanishes as E_r vanishes, and becomes E_r if p_r becomes collinear to p_i or p_j. Furthermore, $(p_r\cdot p_i)/(p_i\cdot p_j)$ and $(p_r\cdot p_j)/(p_i\cdot p_j)$ are the natural variables to perform the integrals. The integral of the difference between the original integrand and the one in terms of $E_r^{(ij)}$ can be performed numerically. The integrals represented by $\mathcal{I}_a^{\mathrm{I,soft/soco}}$ of Eq. (26), finally, can be treated with the same approach.

3. Results

All divergent parts of the integrated subtraction terms were calculated in [13], and it is shown that the poles in ϵ indeed do not depend on the phase space restricting parameters. Furthermore, it is shown that these poles have the exact universal form found in literature from other subtraction schemes. This includes the initial-state collinear divergence given by the Born result times

$$-\frac{\alpha_{\mathrm{s}}}{2\pi\epsilon}\int_0^1 dz\,\mathcal{P}_{\overline{\chi}}(z)\,\frac{1}{z}f(\bar{x}/z)\,\theta(\bar{x}<z)\;, \tag{31}$$

where $\mathcal{P}_{\overline{\chi}}(z)$ is the collinear splitting function associated with initial state $\overline{\chi}$, and also

$$-\frac{\alpha_{\mathrm{s}}}{2\pi\epsilon}\int_0^1 dz\left[\frac{2N_{\mathrm{c}}}{[1-z]_+}+\frac{2N_{\mathrm{c}}}{z}\right]\frac{1}{z}F(x/z,k_\perp)\,\theta(x<z)\;, \tag{32}$$

associated with the space-like initial state χ, and coming from the subtraction term with the splitting function of Eq. (22). Also, it is numerically confirmed for the explicit example of dijet production that the subtraction terms correctly perform the task of cancelling the singularities and that $\sigma_{\mathrm{R}}^{\mathrm{fin}}$ of Eq. (16) is finite. Finally, it is also numerically confirmed that the finite pieces of the integrated subtraction terms added to the subtracted integral, Eq (10), is independent of the phase space restricting parameters.

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References

[1] S. Frixione, Z. Kunszt, and A. Signer, *Three jet cross-sections to next-to-leading order*, *Nucl.Phys.* **B467** (1996) 399–442, [hep-ph/9512328].

[2] S. Catani and M. Seymour, *A General algorithm for calculating jet cross-sections in NLO QCD*, *Nucl.Phys.* **B485** (1997) 291–419, [hep-ph/9605323].

[3] G. Somogyi and Z. Trocsanyi, *A New subtraction scheme for computing QCD jet cross sections at next-to-leading order accuracy*, hep-ph/0609041.

[4] G. Somogyi, *Subtraction with hadronic initial states at NLO: An NNLO-compatible scheme*, *JHEP* **05** (2009) 016, [0903.1218].

[5] T. Robens, *Nagy-Soper Subtraction: A Review*, *Mod. Phys. Lett. A* **28** (2013) 1330020, [1306.1946].

[6] G. Bevilacqua, M. Czakon, M. Kubocz, and M. Worek, *Complete Nagy-Soper subtraction for next-to-leading order calculations in QCD*, *JHEP* **10** (2013) 204, [1308.5605].

[7] E. Fox and N. Glover, *Initial-final and initial-initial antenna functions for real radiation at next-to-leading order*, *JHEP* **12** (2023) 171, [2308.10829].

[8] A. Gehrmann-De Ridder, T. Gehrmann, and E. W. N. Glover, *Antenna subtraction at NNLO*, *JHEP* **09** (2005) 056, [hep-ph/0505111].

[9] G. Somogyi, Z. Trocsanyi, and V. Del Duca, *A Subtraction scheme for computing QCD jet cross sections at NNLO: Regularization of doubly-real emissions*, *JHEP* **01** (2007) 070, [hep-ph/0609042].

[10] M. Czakon, *A novel subtraction scheme for double-real radiation at NNLO*, *Phys. Lett. B* **693** (2010) 259–268, [1005.0274].

[11] F. Caola, K. Melnikov, and R. Röntsch, *Nested soft-collinear subtractions in NNLO QCD computations*, *Eur. Phys. J. C* **77** (2017), no. 4 248, [1702.01352].

[12] L. Magnea, E. Maina, G. Pelliccioli, C. Signorile-Signorile, P. Torrielli, and S. Uccirati, *Local analytic sector subtraction at NNLO*, *JHEP* **12** (2018) 107, [1806.09570]. [Erratum: JHEP 06, 013 (2019)].

[13] A. Giachino, A. van Hameren, and G. Ziarko, *A new subtraction scheme at NLO exploiting the privilege of k_T-factorization*, *JHEP* **06** (2024) 167, [2312.02808].

[14] L. Lipatov, *Gauge invariant effective action for high-energy processes in QCD*, *Nucl.Phys.* **B452** (1995) 369–400, [hep-ph/9502308].

[15] E. Antonov, L. Lipatov, E. Kuraev, and I. Cherednikov, *Feynman rules for effective Regge action*, *Nucl.Phys.* **B721** (2005) 111–135, [hep-ph/0411185].

[16] A. van Hameren, P. Kotko, and K. Kutak, *Helicity amplitudes for high-energy scattering*, *JHEP* **1301** (2013) 078, [1211.0961].

[17] A. van Hameren, *KaTie : For parton-level event generation with k_T-dependent initial states*, *Comput. Phys. Commun.* **224** (2018) 371–380, [1611.00680].