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Three loop vertices with massive particles

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Vertex corrections, or form factors, are important building blocks for higher order calculations, since they describe virtual corrections to the tree level process. In flavor physics, especially B meson decays, vertex corrections with flavor changes, and therefore asymmetric mass assignements on the external legs are important. In these processdings I present recent results for these kind of corrections focusing on the generic heavy-to-light form factor and the inclusive decay of $b \to s\gamma$.

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1. Introduction

Form factors are elementary building blocks of scattering amplitudes, where they can describe the fully virtual corrections to many observables. Form factors with two heavy fermions of the same mass have been obtained to three loop [1–21]. In flavor-changing processes also form factors of heavy fermions with different external masses are necessary. In this context I focus on the heavy-to-light form factors which involve one heavy and one massless external fermion. Results up to two loop order have been obtained in Refs. [22–28], while first results in the color-planar limit for the interactions with vector, axialvector, scalar and pseudoscalar currents have been computed in Refs. [29, 30]. In this contribution, I report on new analytic and semi-analytic results for the full heavy-to-light form factors including also interactions with a tensor current [31]. Furthermore, I present new results for the charm mass dependent contributions to the inclusive $b \to s\gamma$ decay, which can also be calculated via three loop heavy-to-light vertices [32, 33].

2. Charm mass dependent contributions to $b \to s\gamma$

Rare decays in the Standard Model (SM) are powerful tools to search for or constraint potential new physics. One of the prime examples is the decay $B \to X_s\gamma$, where X_s describes any charmless final state with strangeness $S = -1$.

The current world average for the CP- and isospin averaged branching ratio is given by [34]

$$\mathcal{B}(B \to X_s\gamma)|_{E_\gamma > 1.6\mathrm{GeV}} = (3.49 \pm 0.19) \times 10^{-4} \,, \tag{1}$$

and the most precise theory prediction is given by [35]

$$\mathcal{B}(B \to X_s\gamma)|_{E_\gamma > 1.6\mathrm{GeV}} = (3.40 \pm 0.17) \times 10^{-4} \,. \tag{2}$$

One observes very good agreement between the experimental and theoretical values and both have an uncertainty of around 5%. In the near future Belle II is expected to reduce the experimental uncertainty by roughly a factor 2, therefore also the theory prediction needs to be refined. As has been shown in Refs. [35, 36] the theory uncertainty is divided into 3% coming from higher order corrections, 2.5% from the input parameters and non-perturbative effects, and an additional 3% coming from charm mass dependent contributions of the interferences of the operators $Q_{1,2}$ and Q_7 for which only an interpolation between the large mass expansion and the zero mass value is known [37–39]. The errors are added in quadrature.

The last source of uncertainty in the theory prediction can be eliminated by computing these interferences at the physical charm mass value. The full calculation has to consider 2-, 3- and 4-particle cut Feynman integrals with the insertion of operators as shown in Fig. 1. Here, I want to present results for the 2-particle cut contributions.

The 2-particle cut contributions can be calculated by considering cut forward scattering diagrams, but also by considering vertex diagrams as shown in Fig. 2. The amplitude for the $b \to s\gamma$ decay then reads

$$A = \frac{4 G_F m_b^2}{\sqrt{2}} V_{ts}^* V_{tb} M^\mu \epsilon_\mu \,, \tag{3}$$

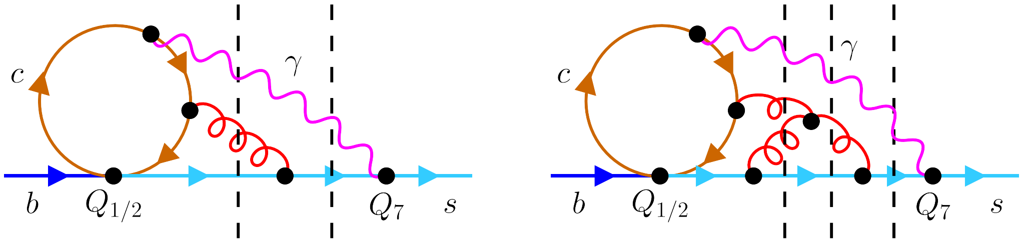

Figure 1: Three- and four-loop sample diagrams for the interference of $Q_{1,2}$ and Q_7. The dashed black lines represent possible cuts through the diagrams. Only cuts through two particles are considered. Taken from Ref. [32].

with the photon polarizaion vector ϵ_μ and the vertex function M^μ can be parametrized as

$$M^\mu = \bar{u}_s(p_s) P_R \left(t_1 \frac{q_\gamma^\mu}{m_b} + t_2 \frac{p_b^\mu}{m_b} + t_3 \gamma^\mu \right) u_b(p_b) \tag{4}$$

with $p_b^2 = m_b^2$, $p_s^2 = q_\gamma^2 = 0$ and all momenta incoming. The scalar coefficient functions t_i depend on the mass ratio $x = m_c/m_b$ and t_1 will not contribute to the physical observables, once the vertex is contracted with the photon polarization.

We use a well tested and automatized set-up to calculate the amplitudes at two and three loop order. We generate the diagrams with `qgraf` [40] and process them with `tapir` [41] and use `exp` [42, 43] to prepare `FORM` [44, 45] code for their evaluation. Afterwards, the scalar Feynman diagrams are reduced to 14 (479) at two loop (three loop) order using the program `Kira` [46, 47] with `Fermat` [48] as back-end. We assure a basis of good master integrals by using an improved version of `ImpoveMasters.m` [49] on a simpler reduction of test integrals. We derive differential equations in the variable x for all master integrals using `LiteRed` [50] and the reduction tables.

Analytic calculation at two loop

The master integrals at two loop order have previously been obtained in terms of a threefold integral representation or by solving the differential equations numerically using boundary conditions in the large charm mass limit. Also analytic expansions around small charm mass were presented (see Ref. [51] and references therein). In the context of the three loop calculation we were able to calculate the two loop master integrals analytically. To do this we utilize the algorithmic way presented in Ref. [16] implemented with the help of `Sigma` [52, 53] and `HarmonicSums` [54–65]. In intermediate steps we find generalized iterated integrals over the alphabet

$$\frac{1}{y}, \quad \frac{1}{1 \pm y}, \quad \frac{1}{2 \pm y}, \quad \sqrt{4 - y^2}\,. \tag{5}$$

After rationalizing the iterated integrals containing the last letter by passing to the new variable

$$x = \frac{\sqrt{w}}{1 + w}, \qquad w = \frac{1 - \sqrt{1 - 4x^2}}{1 + \sqrt{1 + 4x^2}}\,, \tag{6}$$

we observe that the final results for the two loop corrections can be expressed solely in terms of harmonic polylogarithms of argument x and w, which can be easily evaluated using public tools like `ginac` [66].

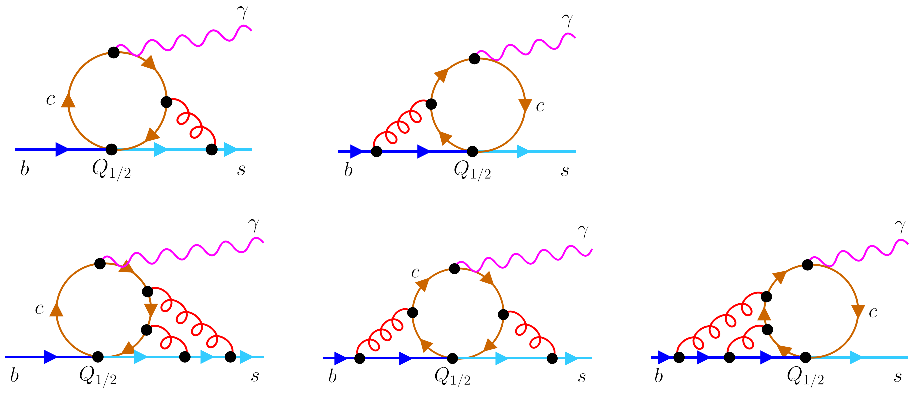

Figure 2: Two- and three loop sample diagrams contributing to the decay vertex $b \to s\gamma$. Taken from Ref. [32].

Calculation at three loop

To evaluate the master integrals at three loop level we use the "expand and match" method developed in Refs. [19–21, 67]. The main idea of the method is to construct symbolic series expansions of the master integrals by inserting a suitable ansatz into the differential equation. The resulting linear system of equations can subsequently be solved in terms of a few boundary constants for which we use `Kira` with its backend to `FireFly` [68, 69]. We fix the first boundary constants at the regular point $x_0 = 1/5$, where we compute the master integrals numerically with an accuracy of 60 digits using `AMFlow` [70]. We furthermore construct an expansion around $x_0 = 1/10$, which we match numerically to the evaluation of the previous series expansion at $x = 0.15$ and at $x_0 = 0$ which we match to the second expansion at $x = 1/15$ in order to cover the full physically interesting parameter range of $0 < x < 0.4$.

The analytic results at two loop order and the series expansion at three loop order are presented in Ref. [32] and can be downloaded in computer readable form [71]. An independent calculation based on cuts of forward scattering amplitudes instead of vertex corrections was achieved in Ref. [33] and found full agreement.

3. Heavy-to-light form factors

The heavy-to-light form factors are important ingredients in many phenomenologically interesting decay modes, such as $t \to bW^*$, $b \to cW^*$ and $b \to uW^*$. Furthermore, the tensor form factors evaluated at vanishing virtuality enter as a hard matching coefficient in the soft-collinear-effective-theory based analysis of $B \to X_s\gamma$ [72–75], where they form one of the last missing ingredients to achieve a full next-to-next-to-next-to-leading order analysis of the photon energy spectrum. Therefore, in Ref. [76] it has been treated as a nuisance parameter in the corresponding analysis. In this section, I will report on the calculation of the heavy-to-light form factors at three loop accuracy and the extraction of the hard matching coefficient for the analysis of the photon energy spectrum in $B \to X_s\gamma$.

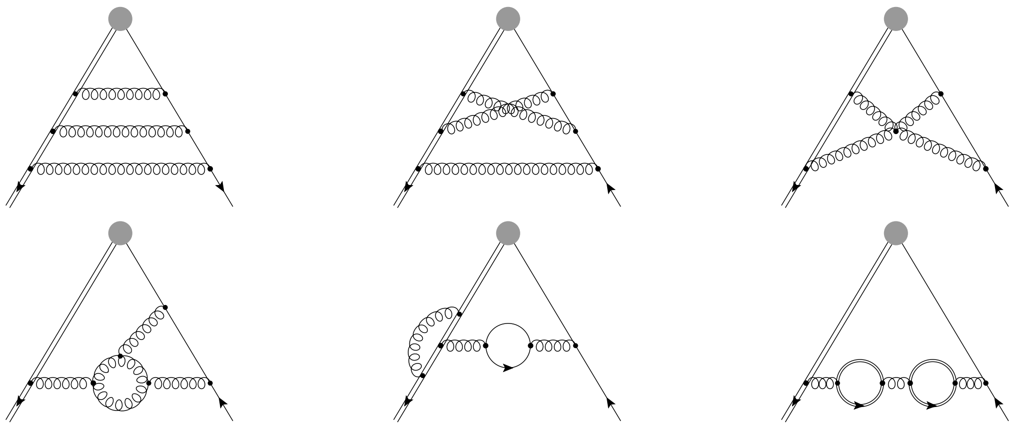

Figure 3: Some sample Feynman diagrams which contribute to the three loop corrections of the heavy-to-light form factors. The double solid, solid and curly lines refer to the massive quark, the massless quarks and gluons, respectively. The gray blob can be either of the external currents. Taken from Ref. [31].

Technical details

We consider the vertex formed by a heavy quark (Q), a light quark (q) and the interaction with a vector, axialvector, scalar, pseudoscalar, or tensor current defined by

$$j_\mu^v = \bar{\psi}_Q \gamma_\mu \psi_q, \quad j_\mu^a = \bar{\psi}_Q \gamma_\mu \gamma_5 \psi_q, \quad j^s = \bar{\psi}_Q \psi_q, \quad j^p = \mathrm{i} \bar{\psi}_Q \gamma_5 \psi_q, \quad j_{\mu\nu}^t = \mathrm{i} \bar{\psi}_Q \sigma_{\mu\nu} \psi_q, \tag{7}$$

where $\sigma^{\mu\nu} = \mathrm{i}[\gamma^\mu, \gamma^\nu]/2$ is anti-symmetric in the indices μ and ν. The wave functions of the heavy and light quark fields are denoted by ψ_Q and ψ_q, respectively. The vertex functions can be parametrized by several scalar form factors as defined in Ref. [31]. We have $p_Q^2 = m^2$, $p_q^2 = 0$ and $(p_Q - p_q)^2 = s$, where p_Q and p_q denote the momenta of the heavy and light quark, respectively. Sample diagrams are shown in Fig. 3.

For the generation of the amplitude we again use the automated chain of tools mentioned in Sec. 2. Notably we again use `Kira` to reduce our amplitude to 3, 18 and 429 master integrals at one-, two- and three loop order, respectively.

Evaluation of the master integrals

We calculate the master integrals up to two loop order analytically, utilizing again the algorithm of Ref. [16]. At three loop order we only calculate the master integrals contributing to the amplitude in the color-planar limit, involving at least one closed light fermion loop, and involving two closed heavy fermion loops analytically. The remaining parts of the amplitude contain master integrals with homogenous solutions related to elliptic curves. The boundary values are computed numerically to high precision at the regular point $s = 0$ with `AMFlow` and are analytically reconstructed using the `PSLQ` [77] algorithm. We observe only constants related to (alternating) harmonic sums and the final result can be written in terms of iterated integrals over the following alphabet:

$$\frac{1}{y}, \quad \frac{1}{1 \pm y}, \quad \frac{1}{2 - y}\,. \tag{8}$$

The same alphabet has been found at two loop order before.

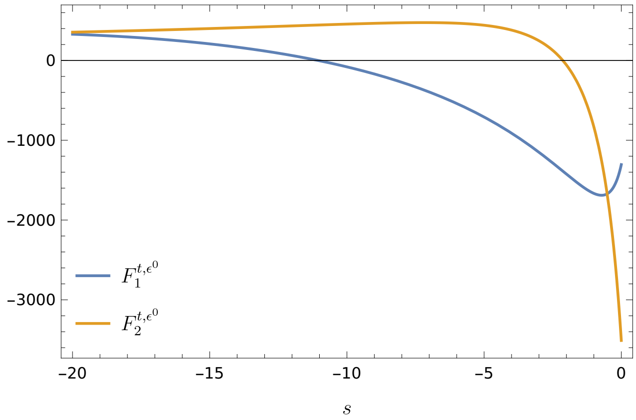

Figure 4: The tensor form factor F_1^t and F_2^t (for a precise definition see Ref. [31]) at $O(\epsilon^0)$ for $s \in (-20, 0)$.

Our results at one- and two loop order agree with the literature [22–28] and the color planar limit at three loop order for all currents except the tensor current agrees with the results presented in Ref. [30]. The results for the other color factors and the tensor current at three loop order are new.

We also calculate the full set of master integrals with the "expand and match" approach described before. In order to cover the physically interesting phase space of $-\infty < s < 1$ we use the numerical initial values at $s = 0$ and match to series expansions around

$$\frac{s}{m^2} = \{-\infty, -60, -40, -30, -20, -15, -10, -8, -7, -6, -5, -4, -3, -2, -1, -\frac{1}{2}, 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{7}{8}, 1\} . \tag{9}$$

This allows the evaluation of the full heavy-to-light form factor at three loop order.

Results

The analytic results of the heavy-to-light form factors can be downloaded from [78]. For the evaluation of the full heavy-to-light form factor at three loop order we provide a `Fortan` package which can be downloaded from `https://gitlab.com/formfactors3l/ffh2l` or Zenodo [79]. It can also be easily linked to `Mathematica`. After installation all form factors, as well as the infrared subtracted finite remainders can be evaluated numerically. As an example we show the terms of $O(\epsilon^0)$ for the first two tensor form factors for $s \in (-20, 0)$ in Fig. 4.

4. Summary and outlook

In these proceedings, I present new results for heavy-to-light form factors, as well as charm mass contributions to inclusive $b \to s\gamma$ decays from diagrams with two particles in the final state. The results have been obtained partially analytically, partially by using a semi-analytic method based on overlapping series expansions and a numerical matching. All results can be retrieved from publicly accessible repositories.

To finalize the calculation for the $b \to s\gamma$ decay also contributions with three and four particles in the final state need to be considered. The calculation is currently under way and will be completed soon. In the future also heavy-to-light form factors with quarks with two different masses will be important, especially for $b \to c$ decays, where power corrections in m_c/m_b are usually not negligible.

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