PROCEEDINGS OF SCIENCE

Third-order QCD predictions for W- and Z-boson production

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The precision of 1% that is reached today in LHC W- and Z-boson measurements is a great achievement, but also a great challenge. The reward of these measurements comes from a wealth of Standard Model precision tests and phenomenology. On the other hand, the achieved precision challenges practical calculational methods for higher order cross-sections, and even stress tests underlying concepts like factorization. In particular, perturbative third-order QCD predictions are necessary to reduce residual truncation uncertainties to the level of $1-2\%$. These developments open up a future focus on many similarly sized effects in non-perturbative proton structure and electroweak effects, for example. In these proceedings we briefly summarize our third-order QCD predictions for W- and Z-boson production, which are publicly implemented in the code MCFM. We put our calculation in the broader context of current calculations and future development needs.

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Experimental status. Measurements of W- and Z- boson observables at the LHC have been limited only by luminosity measurements for a while. For example, measurements of Z production [1–4] and W production by ATLAS, CMS and LHCb [5–9] [10–14] [15–19] typically reach uncertainties at the level of 2-3%. New luminosity measurements with an uncertainty of just 1% [20, 21] pass this precision on to new vector-boson measurements [22, 23]. The precision in vector-boson measurements is generally much greater than many existing Standard Model inputs, and allows for precise W-mass measurements [24–27], charge asymmetries [14, 28–31], parton distribution functions (PDFs) [8, 32–35], as well as the strong coupling α_s [36–38].

Theory status & Drell-Yan at N^{3}LO in QCD. Experimentally, vector-boson production is among the simplest processes to analyze at the LHC with an abundance of statistics. Also for the theoretical description it is among the simplest to model and predict at the LHC. Together with Higgs production and deep-inelastic scattering, it is a process where the highest order in perturbation theory is reached, but also where even today the highest level of precision is needed to allow for interpretation of data, and to model signal and background. Reaching the 1% level will require pushing fixed-order expansions in QCD, QED and electroweak couplings beyond current limits, as well as higher-order resummation and parton showers. Non-perturbative effects in PDFs and TMDs become limiting factors, and effects of event-generator tuning, numerical precision, and even subleading power terms in (collinear) factorization become all equally important aspects. Only a comprehensive combination will allow us to take maximum advantage of the experimental precision. Recent contributions of non-QCD effects include mixed QCD⊗EW corrections at fixed-order, see e.g. refs. [39–48], as well as resummation including non-QCD effects, see e.g. refs. [49, 50].

The first Drell-Yan predictions at N^{3}LO in QCD were total cross-sections at a fully inclusive level [51, 52]. Using q_T subtractions, differential N^{3}LO results were first published in ref. [53] for the Z-boson rapidity distribution. A finding of those calculations were unexpectedly large corrections of about -2.5% due to cancellations between partonic initial-state channels. At the time of these papers no N^{3}LO PDFs were available, and it is meanwhile understood [54, 55] that this effect is counteracted by the inclusion of N^{3}LO PDFs [56, 57]. The current state-of-the-art is at a fiducial and fully differential level and further includes transverse-momentum (q_T) resummation at a similar level in α_s (N^{4}LL) [54, 55, 58–62]. Residual QCD truncation uncertainties are at the level of 1 % to 2 % for sufficiently inclusive quantities and at small transverse momenta $q_T \lesssim m_V$ due to the higher-order q_T resummation.

N^{3}LO subtraction methods. Currently, all fully differential QCD calculations of Drell-Yan production at N^{3}LO rely on the q_T slicing method [63].[1] At this order they are made possible through calculations of the corresponding three-loop beam-functions [68–70], complete three-loop hard function [71–75] and NNLO calculations of V+jet production [76–80].

The extension of N-jettiness slicing [81, 82] [83, 84] to N^{3}LO, as applicable to Drell-Yan production ($N = 0$), is still work in progress. It relies on the corresponding three-loop beam functions [85, 86], while the soft function is still missing certain triple-real emission contributions [87–91].

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[1] Note that the projection-to-Born method [64] has also been applied for N^{3}LO QCD corrections in the cases of Higgs production [65] and DIS [66, 67].

See also ref. [92] for general challenges towards wide-spread N^{3}LO phenomenology.

Power corrections. Perturbative power corrections in q_T factorization play a dual role. In resummed predictions they are important to increase the precision and improve matching with fixed-order predictions. In the form of q_T subtractions they are essential to allow for larger slicing cutoffs. These issues are closely related and have been identified and studied in refs. [93–95]. These insights in recent years have caused a shift from relying on fixed-order calculations for total fiducial cross-sections to resummed calculations. The most prominent case is that of symmetric lepton cuts, causing numerical issues as well as an unphysical sensitivity to low scales in fixed-order cross-sections, which would otherwise be resolved through different cuts [59, 94] instead of resummation. While in q_T factorization fiducial power corrections can be included through a numerically easy to implement recoil prescription [93, 96], they are numerically more difficult for 0-jettiness subtractions [97]. On the other hand, for 0-jettiness there have been significant efforts in analytically computing power corrections up to N^{3}LO [97–100], making 0-jettiness subtractions a promising alternative to q_T subtractions at N^{3}LO.

Our implementation of Drell-Yan production with leptonic decays at N^{3}LO in MCFM. Our current public implementation of W- and Z-boson production in CuTe-MCFM[2] [54, 55, 101] allows for fully differential predictions at N^{3}LO in fixed-order QCD, and including the effect of q_T resummation at N^{4}LL. Electroweak effects can be included at NLO [102]. The third-order α_s^3 predictions have residual QCD truncation uncertainties at the level of $1-3\%$ inclusively and at small transverse momenta using q_T resummation. We find that the large negative cross-section corrections of $-2\,\%$ to $-3\,\%$, that have been previously found, are compensated by the use of (approximate) N^{3}LO PDFs. This is demonstrated in fig. 1 for W^+ production, see also fig. 5 in ref. [55]. The impact of PDFs on kinematic distributions is in general substantial, and now constitutes one of the major uncertainties. In fig. 2 we show the impact of recent NNLO PDF fits and their (approximate) N^{3}LO versions on the differential q_T distribution of Z-bosons in the region $q_T < 30\,\text{GeV}$ where the measurement is most precise at the level of 1%. The right panel shows the use of PDF fits from the NNPDF collaboration, while the left panel shows fits by MSHT and the comparison to the NNPDF N^{3}LO fit. At small q_T the overall spread of all fits is larger than 10%, a multiple of the residual QCD truncation uncertainty as well as other effects. The difference between both N^{3}LO fits is smaller, but still substantial. Note though that uncertainty bands are at the 1σ level.

The use of q_T subtractions in our calculation comes with significant computing requirements, although still moderate compared to other state-of-the-art calculations [103]. To obtain cross-sections and distributions as shown in refs. [54, 55] with a numerical error of better than 0.5%, about 800-1500 node hours (64–128 nodes running for 12 hours) are necessary on Perlmutter at NERSC, equivalent to about 150k CPU core hours on similar modern systems. We rely on a 1-jettiness calculation for the NNLO V+jet process, so the overall calculation relies on a nested slicing in q_T and 1-jettiness, see fig. 3. To obtain reliable fixed-order N^{3}LO predictions, the inclusion of (linear) fiducial power corrections is crucial, see fig. 4. While we are able to obtain Z-boson results with negligible numerical and cutoff uncertainties using a q_T-slicing and resummation-matching-correction cutoff

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[2] `https://mcfm.fnal.gov`

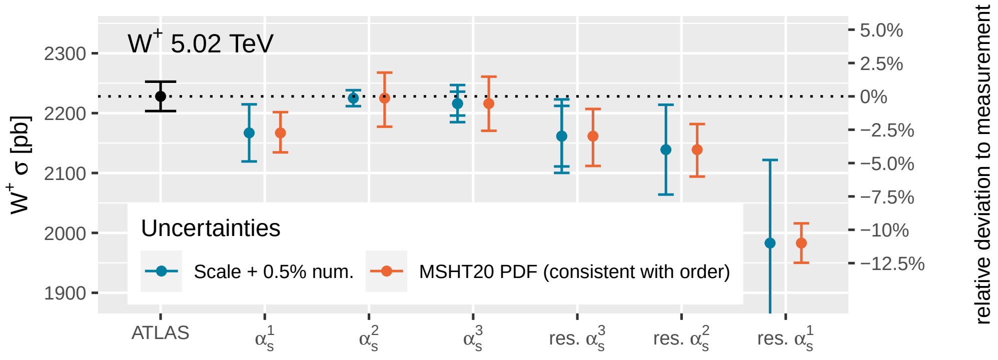

Figure 1: W^+ cross-sections at various perturbative orders in α_s, with and without q_T resummation, in comparison with the 5.02 TeV ATLAS measurement [9]. Error bars show uncertainties from scale variation and from the `MSHT20` PDF sets [56, 104] corresponding to the perturbative order. The α_s^3 results have an additional numerical and slicing cutoff uncertainty of 0.5% that was added linearly to the scale uncertainties for display. This figure is taken from ref. [55]

of 5 GeV, the power corrections for W-boson production are substantially larger and required a cutoff of about 3 GeV, compare in fig. 5.

Challenges and outlook. To summarize, the experimental precision reached for W and Z-boson production demonstrates the LHC's tremendous capabilities and the success of modern data-analysis techniques, as well as advances in theoretical modeling. Is is a showcase for a level of precision that is expected to be reached by a wider range of processes at the HL-LHC. To ensure that data can be interpreted, theoretical predictions will need to match, posing a significant challenge in higher order perturbative methods, but also in non-perturbative inputs like PDFs. It will require and unprecedented community effort in novel developments and open collaboration to overcome these challenges efficiently, in particular in light of dwindling resources for core theory research that directly supports LHC precision.

With refs. [54, 55] we have completed the set of public calculations of W and Z-boson production at α_s in QCD, at fixed order N^{3}LO and with q_T resummation at order N^{3}LO+N^{4}LL.

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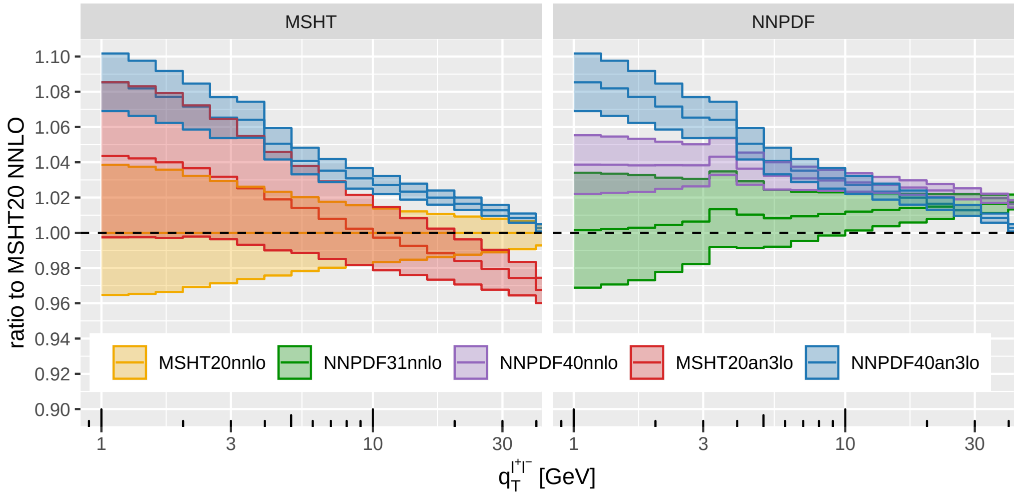

Figure 2: N^{4}LL resummed transverse momentum distribution of the Z-boson (with fiducial cuts as in ref. [54]) using NNLO and (approximate) N^{3}LO PDF sets by the MSHT and NNPDF collaborations [56, 57], normalized to `MSHT20nnlo` [104]. Note that the `MSHT20an3lo` uncertainty band includes missing higher order (MHO) effects and not just the fitting uncertainty.

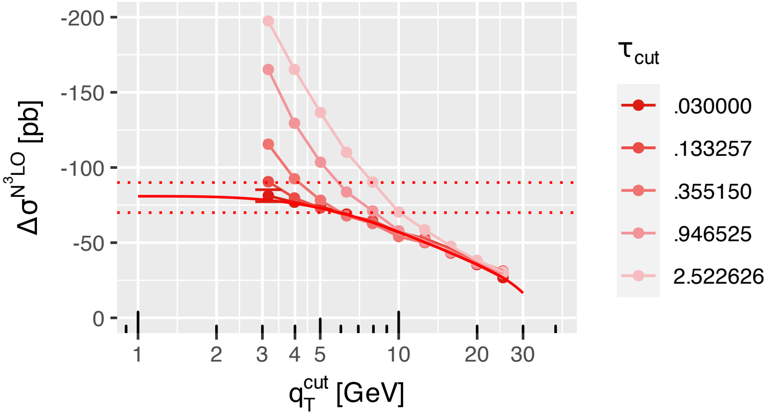

Figure 3: Nested slicing extrapolation in the q_T and 1-jettiness cutoffs for the W^+ N^{3}LO cross-section coefficient, see ref. [55] for details.

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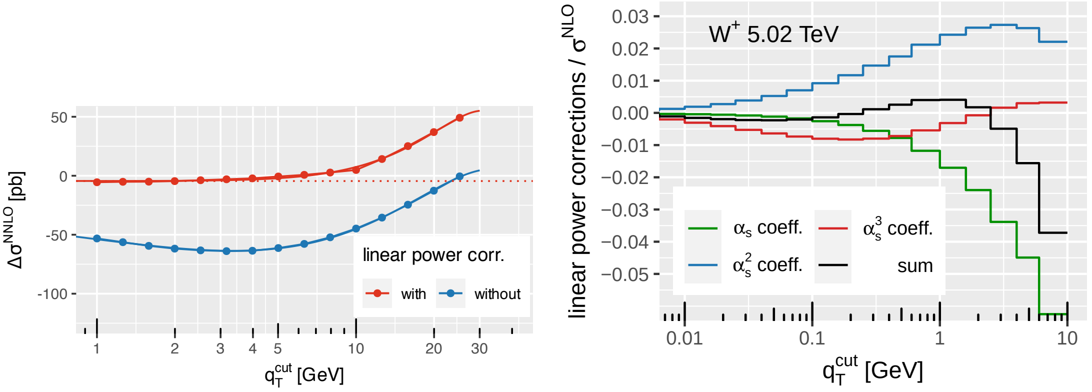

Figure 4: Left: NNLO cross-section coefficient of W^+ production with symmetric lepton cuts in dependence of the q_T slicing cutoff, with and without the inclusion of fiducial power corrections. Right: Fiducial power corrections calculated using a recoil prescription of the W^+ cross-section coefficients in dependence of the q_T slicing cutoff, see ref. [55] for details.

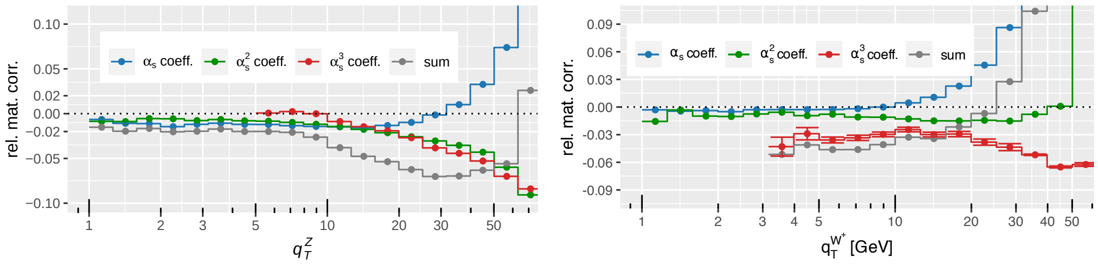

Figure 5: Left: Relative size of the matching corrections for the matched N^{3}LO+N^{4}LL q_T-resummed calculation of Z-boson production [54]. Matching corrections are negligible below 5 GeV. Right: The same for W^+-boson production [55] where a 3 GeV cutoff leads to residual cutoff effects of about 0.5% in inclusive quantities.

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