PROCEEDINGS OF SCIENCE

Towards a discretization of supersymmetric QCD

Enno Carstensen[a,b,*] **and Georg Bergner**[b]

[a] *Institute of Physics, NAWI Graz, University of Graz, Universitätsplatz 5, Graz, Austria*

[b] *TPI, Friedrich-Schiller-Universität Jena, Fröbelstieg 1, Jena, Germany*

E-mail: enno.carstensen@uni-graz.at, georg.bergner@uni-jena.de

We study possible discretizations of the action of supersymmetric QCD. Supersymmetry is broken on the lattice and improved lattice formulations should reduce the amount of fine-tuning required to recover it in the continuum limit. The discrepancy between the conventional scalar field discretization and the Wilson fermion discretization contributes to the breaking of supersymmetry even in the free theory. We present an alternative formulation of the scalar sector, that avoids part of the mismatch. Using this discretization, we explore properties of the scalar sector of $\mathcal{N} = 1$ super QCD. This sector is similar to a 2-Higgs model.

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*Speaker

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1. Introduction

Supersymmetry is broken by any local interacting lattice theory due to the absence of the Leibniz rule [1–3]. To achieve, nevertheless, a supersymmetric continuum limit, one has to tune parameters to compensate the breaking as $a \to 0$. The number of these parameters is especially large for theories with scalar fields. Since there are only rough perturbative estimates of the parameters, the fine-tuning requires intense additional measurements and parameter scans. After successes with lattice simulations of super Yang-Mills [4, 5], our current goal is to generalize the approach towards a supersymmetric continuum limit of $\mathcal{N} = 1$ super QCD in four dimensions. The fine-tuning for this theory is significantly more challenging due the large number of tuning parameters [6–8]. As a first steps, we implement an improved scalar derivative operator which is expected to reduce supersymmetry breaking. In the present work a few initial results of numerical simulations are presented, in particular some indications of phase transitions in the bare parameter space.

2. Supersymmetry improvements of the scalar lattice discretization

A lattice discretization of the fermion action requires either an additional mass term or additional fermion fields (doublers) are introduced in the continuum limit in accordance with the Nielsen-Ninomiya theorem. Only in some specific cases of theories with extended supersymmetry, the additional fields are matched by bosonic counterparts. The Wilson fermion formulation is applied to reduce the fermions in the continuum limit at the cost of an additional mass term, which breaks not only chiral symmetry. In addition, it also leads to a breaking supersymmetry since it requires the same mass for all superpartners. Note that this breaking appears even in the free theory, when supersymmetry breaking is not implied by the absence of the Leibniz rule. An improved lattice formulation should be implemented with the aim to realize at least the tree level symmetry relations.

One possible solution for such an improvement is to add a Wilson term in the same way for fermions and their scalar superpartners. Perturbative studies of the Wess-Zumino model have indicated the advantages of this formulation despite the fact that the symmetry still remains broken at higher loop level [9]. Our aim is to perform first numerical tests of this formulations once the theory is coupled to gauge fields. A direct relation to the fermion action can be found if four complex scalar fields are introduced. The Wilson-Dirac operator $\not{D}_W$ of the fermion part, can in this case be used to represent the scalar theory

$$\begin{aligned}\mathcal{L} &= \phi^\dagger \not{D}_W^\dagger \not{D}_W \phi = \phi^\dagger \left(-\gamma^\mu D_\mu + m_W\right)\left(\gamma^\nu D_\nu + m_W\right)\phi \\ &= \phi^\dagger \left(-D^\mu D_\mu + m_W^2\right)\phi, \end{aligned} \tag{1}$$

where

$$m_W \phi(x) = (m_0 - \frac{r}{2}\Box)\phi(x) \tag{2}$$

is the mass term including the bare mass m_0. D_μ represents the usual symmetric fermion derivative operator. This operator introduces doublers also for the bosonic fields, which are removed by the bosonic Wilson mass.

This explains the similarities between fermion and boson action due to the additional mass term. Note that in the on-shell action of the Wess-Zumino model the modified mass term would also appear in the interaction vertices.

As a first test, we have simulated the free ungauged scalar theory with the improved lattice formulation. We have confirmed that the bosonic two point function shows no sign of doubler fields. From a fit of the correlators, the scalar mass has been determined on a $8^3 \times 48$ lattice.

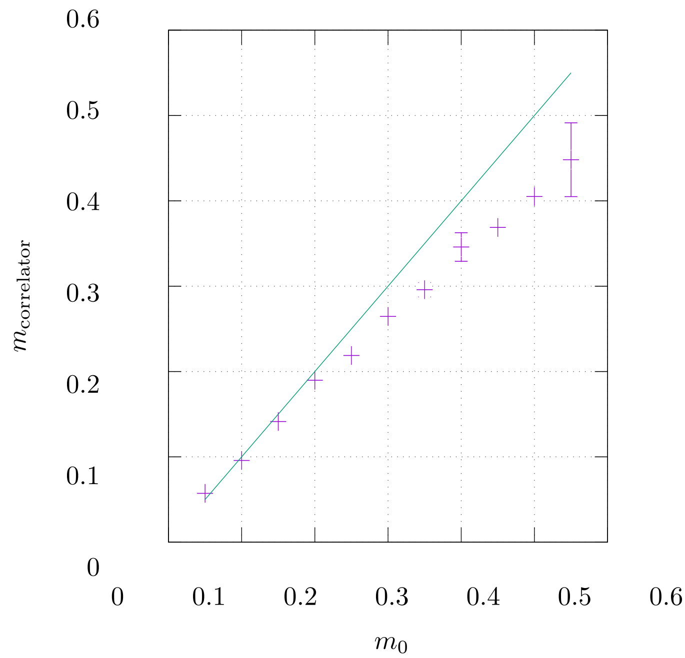

Figure 1: Simulations of the improved scalar action for the free theory. Comparison of bare mass parameter m_0 and the mass determined from a fit of the correlator $m_{\text{correlator}}$.

The relation between the bare mass parameter and the correlator mass is shown in Fig. 1. As expected, the obtained mass agrees nicely with the bare mass parameter. At larger bare mass, the lattice artefacts increase leading to a deviation from the continuum relation.

3. Super QCD

Super Yang-Mills and the matter multiplet of a Wess-Zumino model can be combined to Super QCD, a supersymmetric version of the standard model strong interaction. The super Yang-Mills part describes the gluons and their superpartners the gluinos, which are Majorana fermions, while the now gauged Wess-Zumino part describes the quarks and their superpartners the squarks, which are scalars. There are two scalar superpartner fields to account for the left- and right-handed degrees of freedom of the fermion. Supersymmetry requires additional interaction terms have besides the standard gauge couplings: Yukawa interactions between the quarks, gluinos and squarks, in addition to a scalar ϕ^4 potential. In the continuum, all couplings depend on a single parameter, the gauge coupling and the complete Lagrangean is

$$\mathcal{L} = \mathcal{L}_{\text{SYM}} + \mathcal{L}_{\text{GWZ}} + \mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{\text{ScalPot}} \tag{3}$$

$$\mathcal{L}_{\text{SYM}} = \frac{1}{4} F^{c}_{\mu\nu} F^{\mu\nu\, c} + \frac{1}{2} \bar{\lambda}^c \gamma_\mu \left(D^\mu \lambda^c \right) \tag{4}$$

$$\mathcal{L}_{\text{GWZ}} = \bar{\psi} \not{D} \psi + m \bar{\psi} \psi + D^\mu \phi_1^\dagger D_\mu \phi_1 + D^\mu \phi_2 D_\mu \phi_2^\dagger + m^2 (\phi_1^\dagger \phi_1 + \phi_2 \phi_2^\dagger) \tag{5}$$

$$\mathcal{L}_{\text{Yukawa}} = i\sqrt{2} g \left(\phi_1^\dagger \bar{\lambda}^c T^c P_+ \psi - \overline{\psi} P_- \lambda^c T^c \phi_1 + \phi_2 \bar{\lambda}^c T^c P_- \psi - \overline{\psi} P_+ \lambda^c T^c \phi_2^\dagger \right) \tag{6}$$

$$\mathcal{L}_{\text{ScalPot}} = \frac{g^2}{2} \left(\phi_1^\dagger T^c \phi_1 - \phi_2 T^c \phi_2^\dagger \right)^2 \tag{7}$$

In this case D_μ represents the gauge covariant derivative. $F^c_{\mu\nu}$ is the gluon gauge field strength, the corresponding superpartners, the gluinos λ^c, are Majorana fermions. The quarks are represented by the Dirac spinor field ψ, their superpartners, the squarks, are two scalar fields, ϕ_1 and ϕ_2. T^c are the generators of the gauge group and $P_\pm = \frac{1}{2}(1 \pm \gamma_5)$.

This theory has only two independent parameters: The mass m (lattice counterpart m_0) of the quarks and squarks and the gauge coupling g. The generators T^c are determined by the gauge group.

When supersymmetry is not enforced, the number of parameters increases from 2 to 11 (4 mass terms, 2 Yukawa couplings, 5 scalar self-interactions), which makes fine-tuning a challenging task.

To approach this challenge, we consider only the scalar sector and ignore all fermion contributions. This has several advantages: The computational time gets reduced by a few orders of magnitude, exploring the phase diagram is easier since fewer parameters are present, and the effects of improved scalar actions are easier to observe. Fermions would also lead to a more complex phase diagram as they accentuate flat directions of the scalar potential. Omitting all fermion fields results in the following action:

$$\mathcal{L} = \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + D^\mu \phi^\dagger D_\mu \phi + m^2 \left(\phi_1^\dagger \phi_1 + \phi_2 \phi_2^\dagger \right) + \frac{g^2}{2} \left(\phi_1^\dagger T^c \phi_1 - \phi_2 T^c \phi_2^\dagger \right)^2 . \tag{8}$$

4. Two Higgs model

This Lagrangian can be considered as a special case of a two component Higgs model. They both have the same field content, two massive gauged scalar fields, and the scalar potential of (8)

$$\begin{aligned} \left(\phi_1^\dagger T^c \phi_1 - \phi_2 T^c \phi_2^\dagger \right)^2 &= \left(\phi_1^\dagger T^c \phi_1 \right)^2 - 2 \left(\phi_1^\dagger T^c \phi_1 \right) \left(\phi_2 T^c \phi_2^\dagger \right) + \left(\phi_2 T^c \phi_2^\dagger \right)^2 \\ &= \frac{1}{2} \left(1 - \frac{1}{N_c} \right) \left(\left(\phi_1^\dagger \phi_1 \right)^2 + \left(\phi_2 \phi_2^\dagger \right)^2 \right) \\ &\quad + \frac{1}{N_c} \left(\phi_2 \phi_2^\dagger \right) \left(\phi_1^\dagger \phi_1 \right) - \left(\phi_2 \phi_1 \right) \left(\phi_1^\dagger \phi_2^\dagger \right) \end{aligned} \tag{9}$$

is an extension of a two Higgs model with equal masses and coupling constants for both ϕ_1 and ϕ_2,

$$\mathcal{L}_{\text{2-Higgs}} = \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + D^\mu \phi^\dagger D_\mu \phi + m^2 \left(\phi_1^\dagger \phi_1 + \phi_2 \phi_2^\dagger \right) + \lambda \left(\left(\phi_1^\dagger \phi_1 \right)^2 + \left(\phi_2 \phi_2^\dagger \right)^2 \right) . \tag{10}$$

5. Phase transition from the lattice

We expect both scalar theories to exhibit two phases, a symmetric phase with intact ground-state symmetry and zero vacuum field expectation value, corresponding to large masses, and a phase with broken ground-state symmetry and some non-zero field expectation value, corresponding to small masses. A useful observable to determine phase transitions of this kind is the scalar condensate. It is not a true order parameter, since it is not exactly zero, but just significantly smaller in the symmetric phase. Nevertheless, it is a reasonable observable to determine the phase transition.

The mass parameter of the lattice action can be represented by κ using:

$$m^2 = \frac{m_0^2}{a^2} = \frac{1}{a^2}\left(\frac{1-2\lambda}{\kappa} - 8\right). \tag{11}$$

The simulations were done on a 8^4 lattice at $\lambda = 0.1$. κ was varied between 0.12 and 0.29 for different β between 0.5 and 2.0.

5.1 Two Higgs model

In this theory, there exists a phase transition between $\kappa = 0.16$ and $\kappa = 0.25$ for different β. For larger values of β the critical κ decreases while the strength of the phase transition decreases.

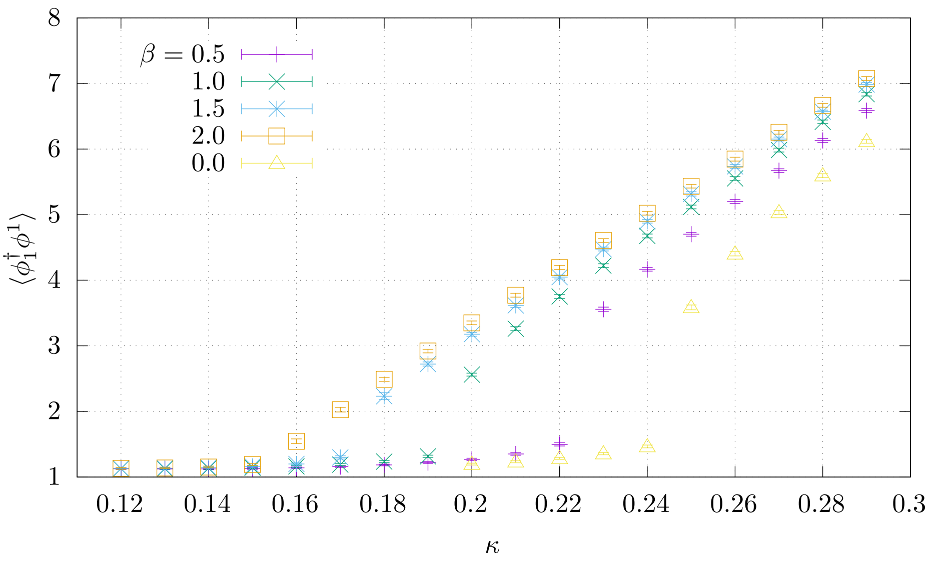

Figure 2: A phase transition in the two Higgs model is clearly visible in the condensate $\langle \phi_1^\dagger \phi_1 \rangle$ for κ between 0.25 and 0.16.

Qualitatively the results are similar to a single Higgs model, while there are some noticeable differences at the quantitative level. In particular the values of the gauge coupling considered here are different from similar studies of this theory. Nevertheless the improved scalar formulation captures the main features of the theory.

5.2 The scalar part of super QCD

After the studies of the two Higgs model, the next step is a numerical study of the scalar part of super QCD. This numerical study provides insights into a subset of the parameter space that needs to be explored for the fine tuning of the full theory. Similar to the two Higgs model, a phase transition is expected. The flat directions of the scalar potential are lifted by scalar loop contributions in contrast to the full supersymmetric case. For simplicity, we enforce the tree level relation of the parameters in the scalar potential. This means that the scalar couplings in (9) are fixed by the single parameter β.

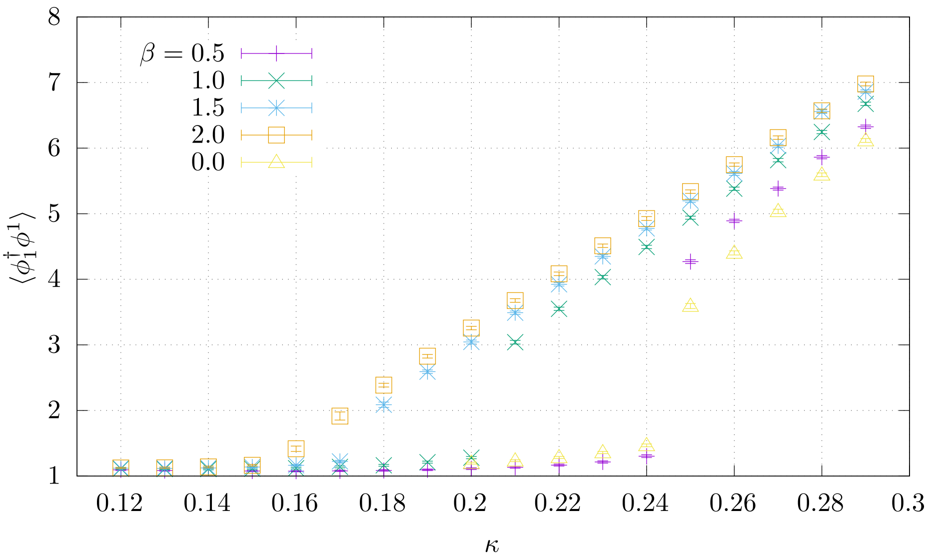

Figure 3: Also for the scalar SuperQCD, a Phase transition in the condensate exists for $0.16 < \kappa < 0.25$.

In comparison of the diagrams in bare parameter space to the two Higgs model, the phase transitions for low β occur later and are stronger. At larger β no significant differences are visible. For very low and high κ both theories look identical, irrespective of β. For both the $\lambda = 0.1$ two Higgs theory and scalar Super QCD, the κ at which phase transition occurs is limited to the interval of $[0.15, 0.25]$.

These results provide a starting point for the fine-tuning of the full SQCD theory. Introducing heavy fermions to the theory and observing changes in the phase diagram when lowering their masses would be a possible approach towards the full theory.

6. Conclusion

In this work, the Super QCD action was reduced down to a gauged scalar action to test improved scalar formulations and sketch a phase diagram as a starting point for fine-tuning the full theory. The improved scalar action implements a Wilson term in the scalar sector to reduce supersymmetry breaking on the lattice. We have confirmed a correct representation of the free scalar theory. As a starting point for the parameter scan of the scalar SQCD action, a comparison to a two Higgs model was done. The difference is modification of the scalar potential. Similar to the two Higgs model, we have also observed a phase transition in the scalar part of Super QCD.

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