The hadronic decay of vector charmonium

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The extraction of decay parameters using lattice techniques is a computationally expensive task, requiring several volumes and group irreps to relate the spectrum on a lattice simulation to the infinite volume scattering. In this project we employ an alternative method based on a narrow-width approximation to extract the hadronic mixing $\langle \bar{D}D | \psi(3770) \rangle$, which is needed to compute the decay $\Gamma(\psi(3770) \to \bar{D}D)$ between the second excited state of vector charmonium and a pair of D-mesons in a p-wave. We carry out our lattice simulations on two CLS ensembles at $m_\pi \sim 440$ MeV and $a \sim 0.066$ fm and obtain results compatible with experiment. Furthermore, we interpret our results analytically using the $^3P_0$ quark model.

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1. Introduction

In these proceedings, we study the hadronic decay $\psi(3770) \rightarrow \bar{D}D$ of the second excited state of vector charmonium $\psi(3770)$ into a pair of $\bar{D}D$ mesons in a p-wave configuration. This process is the main decay channel of $\psi(3770)$ and takes place close to threshold [1]. The standard method to predict a decay width on the lattice relates the spectrum in several finite volumes to the infinite-volume scattering phase-shifts [2–5]. For example, the authors of [6, 7] have employed this procedure to predict the decay width of several charmonium states, including $\psi(3770)$. With our exploratory study carried out in [8] and summarized here, we follow an alternative method developed by Michael and Pennanen [9] that employs ratios of correlators to estimate the decay width. The method, explained in section 2, relies implicitly on a narrow-width approximation, which is satisfied for our decay of interest. Alternative approaches to study hadronic decays include the $\mathcal{K}$-matrix approach [10], which assumes unitarity and analyticity of the underlying scattering amplitudes, but it also requires $e^+e^- \rightarrow$ hadrons data from experiment in the energy region around the $\psi(3770)$ pole to fix its free parameters. Several works compute the decay width of $\psi(3770)$ using this method, see [11] for instance. In our project, we compare our results with the $^3P_0$ quark model [12, 13]. This is a quark rearrangement model which assumes the initial charm quarks are spectators, preserving their quantum numbers and momentum, and a pair of light quarks is created from the vacuum. Then, the four quarks rearrange themselves to obtain the correct quantum numbers of the final state.

Our work predicts the hadronic mixing, $x_{31} \equiv \langle \bar{D}D | \psi(3770) \rangle$, employing the method described in [9]. We establish the connection to the decay width and the energy shift of the states employing non-relativistic quantum mechanics. In particular, we consider the QCD Hamiltonian $H_0 = |\psi(3770)\rangle \langle \psi(3770)| + |\bar{D}D\rangle \langle \bar{D}D|$ and the perturbation $H_1 = |\bar{D}D\rangle \langle \psi(3770)| + |\psi(3770)\rangle \langle \bar{D}D|$. On the lattice, we have a two-level system in the centre-of-mass (CM) frame of the charmonium state and the energy shift of the states is obtained solving the eigenvalue problem

$$\begin{pmatrix} -\Delta & x_{31} \\ x_{31}^* & \Delta \end{pmatrix}, \tag{1}$$

where $\Delta = (m_\psi - E_{\bar{D}D})/2$. Including the first-order perturbation, and assuming the charmonium state is heavier, the energies are shifted to

$$m_\psi \rightarrow m_\psi + \sqrt{|x_{31}|^2 + \Delta^2}, \qquad E_{\bar{D}D} \rightarrow E_{\bar{D}D} - \sqrt{|x_{31}|^2 + \Delta^2}. \tag{2}$$

Note that in the case of degenerate states with H_0, the energy shift is given by the hadronic mixing alone. The situation in experiment is quite different. Indeed, the decay channel is closed in lattice simulations, as there cannot be a decay in a finite box, and even kinematics may not allow it depending on the quark masses. This means that we can study the effect of the mixing explicitly. This allows us to test the effects of different theoretical methods. Meanwhile, the situation in experiment corresponds to the decay of a discrete state to a continuum of final states, giving a Breit-Wigner distribution in momentum space and an energy shift in the same direction to both initial and final states. In particular, we will directly use the hadronic mixing extracted on the lattice to obtain the decay width using Fermi's golden rule. Theoretically, the situation in experiment for non-relativistic states can be understood using [14].

2. Methodology

To extract the hadronic mixing $x_{31} \equiv \langle \bar{D}D | \psi(3770) \rangle$ in finite volume, we consider several vector charmonium interpolators O_i^ψ with $i = 1, 2, \ldots$ Solving a generalized eigenvalue problem (GEVP) yields the real right-eigenvectors $\bar{v}_\alpha$ for the spectrum level α,

$$P^{\bar{D}D} = \left\langle O^{\bar{D}D}(t) O^{\bar{D}D}(0) \right\rangle, \quad \bar{P}_{\alpha\alpha}^\psi = \bar{v}_{\alpha j} \left\langle O_j^\psi(t) O_i^\psi(0) \right\rangle \bar{v}_{\alpha i}, \quad \bar{T}_\alpha = \bar{v}_{\alpha i} \left\langle O_i^\psi(t) O^{\bar{D}D}(0) \right\rangle. \quad (3)$$

We choose $\alpha = 3$ to isolate $\psi(3770)$, and we assume that the excited vector charmonium and the $\bar{D}D$ pair may not be necessarily degenerate, $m_\psi \neq E_{\bar{D}D}$. Then, we can study the spectral decomposition of the correlations in equation (3) to isolate x_{31} in terms of the energy difference Δ [8, 15],

$$\mathcal{R}(t) = \frac{|\bar{T}_3(t)|}{\sqrt{\bar{P}_{33}^\psi(t) P^{DD}(t)}} \underset{t \gg 1}{=} \frac{|x_{31}|}{\Delta} \sinh(t\Delta) + A e^{-t\Delta}. \quad (4)$$

If the tuning of the masses is perfect such that $\Delta \to 0$, $\mathcal{R}(t) = x_{31} t + A$, where A is a constant. Equation (4) is the main relation to extract the mixing, but other relations exist if we transit between ground states with compatible quantum numbers. We can also form the ratio [8, 15]

$$x_{\mathrm{T}}(t) \equiv \frac{\bar{T}_3(t)}{\sqrt{\bar{P}_{33}^\psi(t) P^{DD}(t)}} \frac{\lambda^{t/2}}{1 + \lambda + \cdots + \lambda^t} \underset{t \gg 1}{\to} x_{31}, \quad (5)$$

where $\lambda = \exp(2\Delta)$, and which should tend to a constant if the D-meson ground state is well isolated without a GEVP. This is not required for equation (4), and our data do not form clear plateaux to fit equation (5). That is why we prefer to use equation (4) for our final results, but also show equation (5) in figure 4. Knowing the hadronic mixing, we can compute the decay width assuming that the final $\bar{D}D$ are non-interacting, such that each of them can be expressed as a plane wave with periodic boundary conditions (PBCs) in a box of size L [8, 15],

$$\Gamma = \frac{L^3}{24\pi} p_i m_\psi |x_{31}(E = m_\psi)|^2 \quad \text{with} \quad p_i^2 = \frac{m_\psi^2}{4} - m_D^2. \quad (6)$$

In order for equation (6) to tend to the physical result when $L \to \infty$, the hadronic mixing should compensate the factor L^3. Note that at no point we establish an explicit connection to the infinite volume decay, and it is clear the method relies on a narrow-width approximation to work. In order to compare our results for x_{31} with the $^3P_0$ quark model [13, 16], which uses a different normalization for the hadronic mixing, we will compare directly the decay widths. And since both lattice and quark model predict the mixing for any value of the D-meson momentum, including off-shell kinematics, we lift the condition for energy conservation from the definition of the decay width. In this way we can compare the two theoretical predictions in the entire kinematics, see figure 5.

3. Setup

We compute the necessary correlators for equation (4) in two CLS ensembles [17, 18] that employ the Wilson-plaquette gauge action and $N_f = 2$ mass-degenerate flavors of non-perturbatively

id	β	a [fm]	L/a	κ_ℓ	m_π [MeV]	$m_\pi L$
D5	5.3	0.0658(7)(7)	24	0.13625	449	3.6
E5	5.3	0.0658(7)(7)	32	0.13625	437	4.7

Table 1: The two CLS ensembles used in this work. From left to right, we indicate the ensemble label, the bare coupling constant, the lattice spacing, the number of nodes in every spatial direction ($T = 2L$), the light-quark hopping parameter, the corresponding pion mass, and the value of $m_\pi L$.
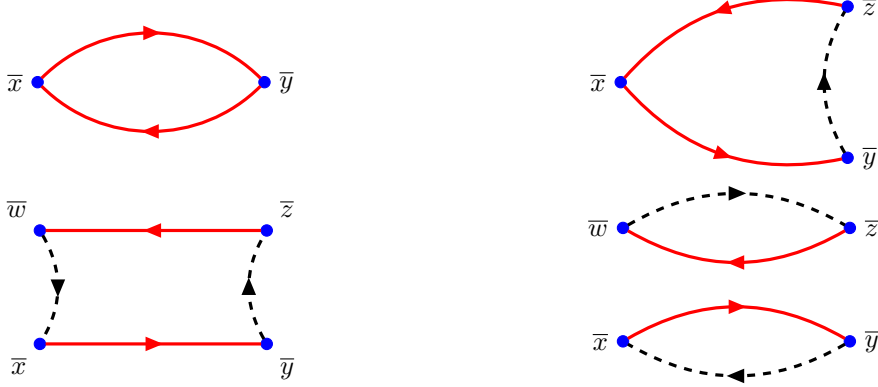
Figure 1: The Wick contractions considered in this project. The charm quark appears as a solid line and the light-quarks as a dashed line. We do not consider diagrams with charm-quark annihilation.

$O(a)$-improved Wilson quarks. Table 1 gathers the label of the ensembles, their bare-coupling and lattice spacing, the lattice extension, the light-quark hopping parameter, the pion mass, and the value of $m_\pi L$. A value of $m_\pi L > 4$ is usually required to reduce finite-volume effects (FVEs) to a manageable level. Figure 1 shows the Wick contractions considered in this work. The red lines indicate a charm-quark propagator, while the dashed lines indicate a light quark. We consider the two charm quarks to belong to two different flavors such that Wick contractions with charm-quark annihilation vanish. To set the momentum of the final-state D-mesons, we use partially twisted boundary conditions (PTBCs) [19] on the charm quarks, such that $c(n) \to \exp(-\mathrm{i}a\boldsymbol{n}\cdot\boldsymbol{\theta}/L)\, c(n)$ and $\bar{c}(n) \to \exp(\mathrm{i}a\boldsymbol{n}\cdot\boldsymbol{\theta}/L)\bar{c}(n)$. This setup leaves the charmonium state at rest while it boosts the D-mesons in opposite directions. Furthermore, leaving the light-quarks with anti-periodic boundary conditions (anti-PBCs) preserves isospin symmetry and the final states $\bar{D}^0 D^0$ and $D^- D^+$ remain mass-degenerate in our simulations. We apply the same twist angle θ in every spatial direction with Gaussian smearing on the quark fields to better interpolate the physical states. We employ $s = 20$ and 50 smearing iterations for the charm bilinears $\bar{c}\gamma_i c$, $\bar{c}\gamma_4\gamma_i c$, $\bar{c}\overleftrightarrow{\nabla}_i\gamma_i\overleftrightarrow{\nabla}_i c$, and $\bar{c}\overleftrightarrow{\nabla}_i\gamma_4\gamma_i\overleftrightarrow{\nabla}_i c$. All are simulated at zero momentum and located at the sink. We used $s = 50$ smearing iterations for the D meson interpolator $\bar{c}\gamma_5 u$. The $\bar{D}D$ system at the source is in a p-wave,

$$2O^{\bar{D}D} = \bar{D}^0(\boldsymbol{p},t)D^0(-\boldsymbol{p},t) - \bar{D}^0(-\boldsymbol{p},t)D^0(\boldsymbol{p},t) + D^-(\boldsymbol{p},t)D^+(-\boldsymbol{p},t) - D^-(-\boldsymbol{p},t)D^+(\boldsymbol{p},t). \quad (7)$$

However, not all Wick contractions can be computed in this way. The two lower diagrams in figure 1, which we name box and direct contributions to the $\bar{D}D$ propagator, include terms where a momentum change is needed, which is incompatible with PTBCs. In this case, we subdivide
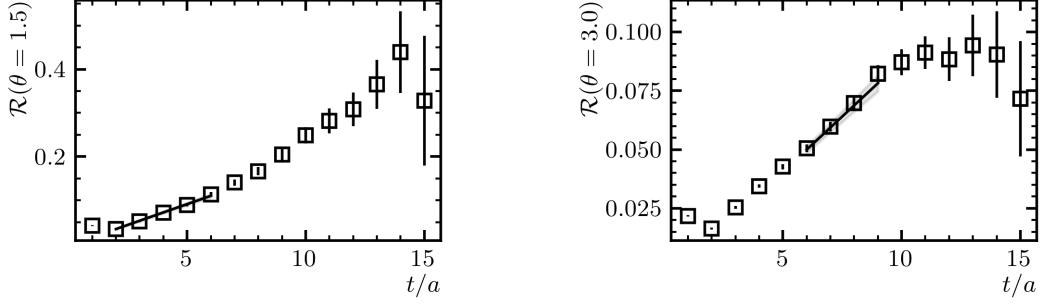
Figure 2: Ratio $\mathcal{R}(t)$ of equation (4) as a function of time, together with fit to the right-hand side (RHS) of equation (4). Ensemble D5 at twist angle $\theta = 1.5$ rad (LEFT), and ensemble E5 at $\theta = 3$ rad (RIGHT). These twist angles lie close to the on-shell condition on their respective datasets, see table 4.

the correlator into two pieces, one that can be computed using PTBC, and another containing the momentum exchange that is computed with Fourier modes and interpolated linearly to the exact momentum. At $\boldsymbol{p} = \boldsymbol{0}$ both components cancel exactly, while at large momentum the second piece becomes negligible.

4. Analysis

In this section, we extract the mixing x_{31} and compute both the decay width of $\psi(3770)$ and the energy shift. First, we need to determine the mass of $\psi(3770)$ in our simulations, $m_\psi(3770)$. To do that, we solve an 8×8 GEVP with the interpolators and smearings of section 3. The spectrum is extracted computing the effective mass of the eigenvalues and fitting them to a constant. The fit intervals, quality, and the extracted masses appear in table 2. Second, we require the D-meson dispersion relation. For each meson momentum, we fit the correlator to a single exponential. In figure 3, we show the results compared to the free dispersion relation expected in the continuum for both ensembles. These plots indicate the system can be safely treated as non-relativistic. We compute the twist angle corresponding to the on-shell decay assuming the two D-mesons are noninteracting, so that $m_\psi(3770) = 2E_D$ at

$$\sqrt{3}a\theta_0/L = \sqrt{(am_\psi/2)^2 - (am_D)^2}. \tag{8}$$

The value in radians for each ensemble appears in table 4. Finally, we can extract the hadronic mixing x_{31} using equation (4) or equation (5). Figure 2 shows examples of the fits to equation (4) on ensembles D5 and E5. All results appear in figure 4 as a function of the final D-meson momentum modulus, $p \equiv |\boldsymbol{p}|$. As mentioned in section 2, equation (4) is much better fulfilled in our setup and we employ its output for the final results. Indeed, we observe these points can be fitted to a parabola that vanishes at $p = 0$,

$$|x_{31}|(p) = c_1 - c_2(p - p_0)^2, \tag{9}$$

where $p = \sqrt{3}\theta/L$, and $p_0 = \sqrt{c_1/c_2}$. The results from this fit appear in table 3. The results employing equation (5) agree qualitatively, with a better determination on the larger volume. Plugging equation (9) in equation (6), we obtain the decay width for the entire kinematical range,

Level	Fit interval	χ^2/dof	$am_{c\bar{c}}$	$m_{c\bar{c}}$ [GeV]
1	[5, 20]	25.96/15	1.02423(63)	3074.1(2.4)
2	[3, 12]	5.93/9	1.286(17)	3859(52)
3	[5, 9]	0.34/4	1.3148(43)	3946(13)
4	[4, 8]	3.88/4	1.379(22)	4139(67)

Table 2: Charmonium spectrum obtained from an 8×8 GEVP on ensemble D5. The effective mass of each eigenvalue is fitted in the interval specified, obtaining the corresponding fit quality. The energy corresponds to the state at rest, given in lattice units and MeV. We employ the same numbers on ensemble E5.
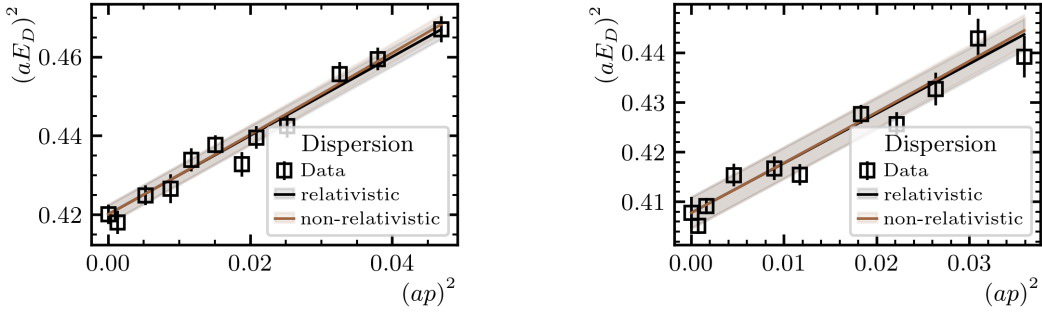
Figure 3: Dispersion relation for the D-meson on ensembles D5 (LEFT) and E5 (RIGHT) as a function of the D-meson momentum modulus, $p \equiv |\boldsymbol{p}|$. The lattice data is fully compatible with a non-relativistic particle in the continuum.

Ensemble	χ^2/dof	ac_1	c_2/a
D5	34.39/39	0.01869(32)	1.169(44)
E5	42.51/35	0.01276(56)	1.183(52)

Table 3: Fit of equation (9) to the ratio in equation (4).

see figure 5. The on-shell condition is indicated by the vertical dotted line. The main results for an on-shell transition appear in table 4. In particular, we indicate the decay width in lattice and physical units, and the energy shift computed using equation (2). Since ensemble E5 has a larger volume, we choose its values as the main results of this project. In particular, the decay width is compatible with the experiment. Finally, we plot the prediction of the $^3P_0$ quark model [16] as a grey band in figure 5. We used the results of our spectroscopy calculation to set the parameters of the $\psi(3770)$ and D wave functions. The remaining free parameter β, which gives the quark-pair creation strength, is fixed fitting the model to the lattice data, see [8] for more details. We observe that the quark model describes our data on a qualitative level, and the agreement is especially good on ensemble E5. This suggests that the physical picture given by the quark model is qualitatively correct.
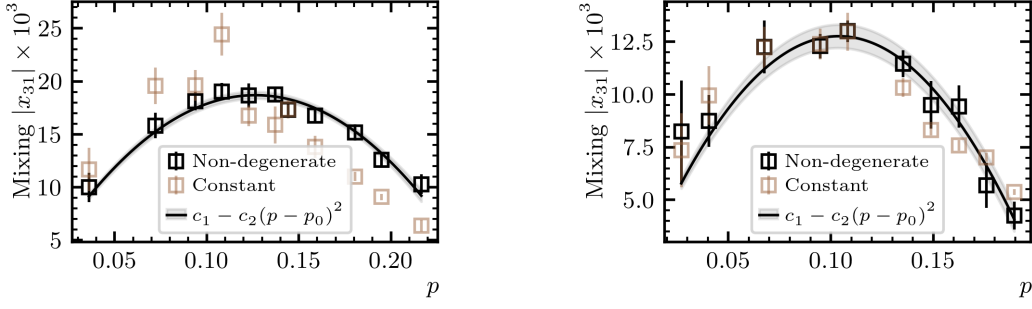
Figure 4: The hadronic mixing between $\psi(3770)$ and $\bar{D}D$ on ensembles D5 (LEFT) and E5 (RIGHT) as a function of the 3-momentum modulus of each D-meson. The darker (brighter) points are obtained fitting $\mathcal{R}(t)$ ($x_T(t)$) to equation (4) (equation (5)). We parametrize the darker points with the parabola in equation (9).
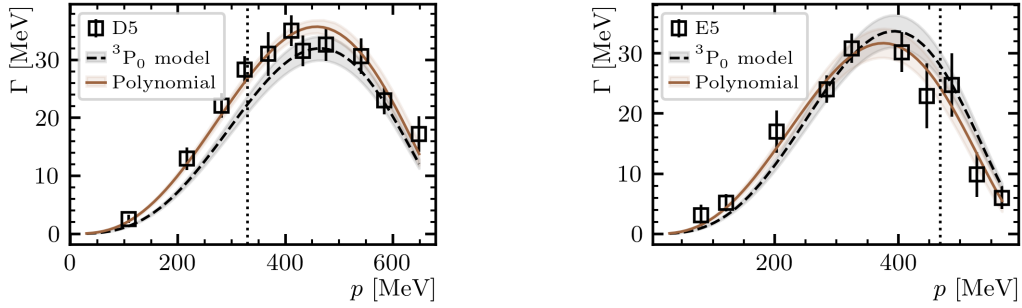

Figure 5: The decay width $\Gamma(\psi(3770) \to \bar{D}D)$ as a function of the 3-momentum modulus of each D-meson. The slashed line shows the $^3P_0$ quark-model prediction, and the full line the prediction given by our lattice determination. The decay may only occur on-shell, marked by the vertical line. We generalize the expression of the decay width for off-shell transitions to compare our results to the quark-model prediction, which employs a different normalization for the hadronic mixing x_{31}.

Ensemble	θ_0 [rad]	p [MeV]	$a\|x_{31}\|$	ϵ [MeV]	Γ/a	Γ [MeV]
D5	1.52(24)	329(52)	0.01837(73)	55.1(2.2)	0.0089(21)	26.8(6.2)
E5	2.89(25)	468(40)	0.0095(16)	28.5(4.9)	0.0081(21)	24.2(6.4)

Table 4: The on-shell twist angle, the corresponding momentum modulus of each D-meson, the hadronic mixing and mass shift, and the decay width in lattice and physical units.

5. Conclusions and Outlook

In these proceedings (which summarize our work presented in [8]), we employ the ratio method described in section 2 to compute the decay width $\Gamma(\psi(3770) \to \bar{D}D)$ as well as the energy shift in the spectrum. Our main results are gathered in table 4, and in particular we extract a decay width that is compatible with the experimental determination. This project shows that it is possible to extract the decay parameters of an excited state when the narrow-width approximation holds. Our lattice implementation relies on the extraction of the charmonium spectrum via a GEVP and

the use of PTBC to tune the $\bar{D}D$ system in a p-wave. Our determination of the hadronic mixing $\langle \bar{D}D | \psi(3770) \rangle$ is not limited to on-shell kinematics, and we compare our results to the prediction by the $^3P_0$ quark model, which shows remarkable agreement, all while being an analytical method to deduce the decay parameters. This suggests that the physical picture presented by the quark-rearrangement model is qualitatively correct. Although we limit ourselves to two ensembles with different volumes, the method presented here can be extended to a more complete dataset. In particular, an extrapolation to the continuum limit in the near future is reasonable. In a similar fashion, lowering the light-quark masses to the physical point should have only a limited effect.

Acknowledgments

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