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Scaling results for isospin charged operators near the QCD conformal window

Alessandra D'Alise,[a,b,*] **Jahmall Bersini,**[c] **Clelia Gambardella**[b,d] **and Francesco Sannino**[a,b,d,e]

[a] *Dipartimento di Fisica "E. Pancini", Università di Napoli Federico II, I-80125 Napoli, Italy*
[b] *INFN, Sezione di Napoli, Complesso Universitario di Monte S. Angelo, Via Cintia Edificio 6, 80126 Napoli, Italy*
[c] *Kavli IPMU (WPI), UTIAS, The University of Tokyo, Kashiwa, Chiba 277-8583, Japan*
[d] *Scuola Superiore Meridionale, Largo S. Marcellino, 10, 80138 Napoli NA, Italy*
[e] *Quantum Theory Center (ħQTC) at IMADA and D-IAS, Southern Denmark Univ., Campusvej 55, 5230 Odense M, Denmark*

E-mail: alessandra.dalise@unina.it, jahmall.bersini@ipmu.jp, c.gambardella@ssmeridionale.it, sannino@qtc.sdu.dk

This work, drawing upon [1], uncovers the leading near-conformal corrections on a cylinder to the scaling dimension of operators with isospin charge Q defined at the lower boundary of the QCD conformal window. The method involves determining the classical ground state energy of the theory on the cylinder using a semiclassical large charge expansion. In the conformal limit, this energy maps, by state-operator correspondence, into the scaling dimension of the lowest-energy operator carrying a generalised isospin charge Q. We find that the leading near-conformal corrections to the scaling dimension display distinctive Q-dependent scalings, arising from the anomalous dimension of the quark mass operator and the one related to the operator working as a potential for the dilaton that dynamically shifts QCD away from the conformal window.

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*Speaker

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1. Introduction

Understanding the complex forces that govern Nature is an ongoing endeavour. The dynamics of strongly coupled theories represents one of the most intriguing challenges in theoretical physics. Although significant progress has been made in recent decades due to a diverse array of analytical and numerical techniques, numerous puzzles persist. At the forefront of these challenges stands Quantum Chromodynamics (QCD), the crown jewel of the Standard Model of particle interactions, which remains fraught with mysteries. The complex physical spectrum, chiral symmetry breaking and confinement, and phase structure concerning light matter fields, temperature, and density pose substantial analytical tasks. In strongly interacting theories, the coupling constant changes across energy scales, a phenomenon known as "running". This variation is due to a balance between opposing effects: an "anti-screening" contribution that strengthens interactions as distances decrease caused by charged gauge bosons, and a "screening" effect from matter fields at the same distances. Under specific conditions, particularly when there is a sufficiently large number of matter fields, the coupling constant can converge to a fixed value at low energies before chiral symmetry breaking occurs. This behaviour signals the emergence of a "conformal phase", in which the theory exhibits scale-invariant characteristics.

Moreover, when the number of distinct matter particles is just below the threshold for entering this conformal phase, the coupling constant shows minimal variation as the energy scale changes, behaving as though it is "walking" rather than "running".

This regime of the theory can be traced back to the pioneering work of Banks and Zaks, who discovered a perturbative infrared fixed point in massless QCD as the number of quark flavours N_f approaches, but stays below, the critical point at which asymptotic freedom is lost. The range of quark flavours and colours that permit conformality is called the *conformal window*. As the number of flavours decreases, the theory approaches the aforementioned quantum phase transition, delineating a boundary below which confinement and chiral symmetry breaking occur. This transition region is characterised by walking dynamics, where the running of the gauge coupling slows across a significant energy range. In the theory's spectrum, near the lower end of the conformal window in the confining and chiral symmetry broken phase, it is argued [2–5] that a dilaton appears as a precursor of the smooth quantum phase transition, alongside the ordinary Goldstones. The initial effective Lagrangian description traces back to Coleman's work [6], and its applicability to walking dynamics is considered in [7–11]. Recent studies of effective approaches across different regimes appeared in [12–24]. One of the key questions regards the dynamics near the conformal window's lower boundary, such as its precise edge and the nature of the quantum phase transition. One possibility is losing conformality via the Berezinskii-Kosterlitz-Thouless (BKT) mechanism [25, 26], suggested for four dimensions in [27–30]. In this context, the underlying theory appears close to achieving scale invariance but has yet to attain it fully. This precise regime of the theory can be seen as a departure from conformality, hence the term "near-conformality". Completing the low-energy framework of QCD, when favouring near-conformal dynamics, is essential. Conformal field theory (CFT) is a powerful tool for analysing QCD's behaviour, particularly near critical points, where understanding confinement and chiral symmetry breaking is important. The interplay between CFT and QCD is crucial, as CFT offers a framework to study the critical phenomena in QCD, especially where the theory approaches conformal symmetry.

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The central quantities in any CFT are the scaling dimensions of local operators. By the state-operator correspondence, these are the energies of the corresponding states on a non-trivial gravitational background. For example, the scaling dimension of the lowest-lying operator of charge Q, denoted with Δ_Q, is mapped into the ground state energy E_Q on the cylinder via the relation

$$\Delta_Q = R\, E_Q \,, \tag{1}$$

with R the radius of the cylinder. Fixed-charge sectors offer novel opportunities to investigate near-conformal dynamics, providing precious information on the sectors responsible for breaking conformality. In the large charge limit one can compute scaling dimensions of fixed charge operators using semiclassical computations [31–43]. Intriguingly, the large charge framework allows us to perform analytical calculations in strongly coupled quantum field theories and its predictions have been tested via numerical Monte Carlo simulations in [32, 44–47]. For the following discussion we extend the relation in eq.(1) to near-conformal field theories [21, 33] by introducing the quantity:

$$\Delta_Q^{ex} \equiv R\, E_Q = \Delta_Q + \text{ near CFT terms} \,. \tag{2}$$

The near CFT terms that emerge in our framework are proportional to the parameters deforming the underlying CFT. For our purposes, we work with two sources of conformal breaking: the first is due to the explicit quark mass term, and the second comes from the presence of an operator of dimension Δ working as a dilaton potential. We show that the near-conformal corrections are proportional to the powers of the dilaton and pion masses. Furthermore, we enrich our theory with the presence of the topological CP-violating term, and we show that the θ-angle dependence explicitly emerges as a redefinition of the pion mass. The precise scalings of the charge of the near-conformal corrections are induced by the quark mass operator anomalous dimension γ and the one characterising the dilaton potential, Δ.

This contribution shows an example of applications of the large charge formalism generalised to conformal theories featuring a dilaton in the spectrum. These investigations are presented by providing a correspondence between the dilaton properties and the large charge operator spectrum in theories featuring an approximate moduli space. In particular, the large charge limit allows model-independent predictions for the Q-scalings of near-conformal corrections to observables in the large charge sector of the theory. The example we share is QCD at finite isospin density whose effective description via a chiral Lagrangian is introduced in section 2 while the enriched effective field theory (EFT) when including a dilaton field is presented in section 3. Finally, we reveal the leading near-conformal corrections to the scaling dimension of the lowest-lying operators carrying isospin charge defined at the lower boundary of the QCD conformal window in section 4. Our conclusions are given in section 5.

2. Chiral Lagrangian at finite isospin and θ-angle

The chiral Lagrangian below describes the low-energy dynamics of QCD at finite generalised isospin density

$$\mathcal{L} = \nu^2 Tr\{\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\} + m_\pi^2 \nu^2 Tr\{M\Sigma + M^\dagger \Sigma^\dagger\} + 2i\mu\nu^2 Tr\{I\partial_0 \Sigma\Sigma^\dagger - I\Sigma^\dagger \partial_0 \Sigma\} + 2\mu^2 \nu^2 Tr\{II - \Sigma^\dagger I \Sigma I\} \,, \tag{3}$$

with v the half the pion decay constant, μ the (generalised) isospin chemical potential, and

$$\Sigma = e^{i\varphi/v}\,, \qquad \varphi = \Pi^a T^a + \frac{S}{\sqrt{N_f}}\,, \tag{4}$$

where T^a are the $SU(N_f)$ generators whose normalisation is $\mathrm{Tr}\{T^a T^b\} = \delta^{ab}$. Moreover, we assume the (pseudo-)Goldstones to have the same mass m_π and, to study the dynamics of the theory at finite charge density, we introduce the charge matrix I to define the charge. Explicitly, the mass and the charge matrices are

$$M = \mathbb{1}_{N_f}\,, \qquad I = \frac{1}{2}\begin{pmatrix} \mathbb{1}_{N_f/2} & 0 \\ 0 & -\mathbb{1}_{N_f/2} \end{pmatrix}. \tag{5}$$

The ground state of our theory generalises the $N_f = 2$ case of [48] and is

$$\Sigma_c = \mathbb{1}_{N_f}\cos\varphi + i\Sigma_I \sin\varphi\,, \tag{6}$$

with

$$\Sigma_I = \begin{pmatrix} 0 & \mathbb{1}_{N_f/2} \\ \mathbb{1}_{N_f/2} & 0 \end{pmatrix}\cos\eta \; + i\begin{pmatrix} 0 & -\mathbb{1}_{N_f/2} \\ \mathbb{1}_{N_f/2} & 0 \end{pmatrix}\sin\eta\,. \tag{7}$$

The potential is minimized through the interplay of two terms on the right side of eq.(6), each reducing either the mass or isospin terms. Their contributions are weighted by the angle φ, fixed by the equations of motion (EOM). Finally, the CP-violating topological sector is included through the θ-angle term [49, 50],

$$\Delta\mathcal{L}_\theta = -av^2\left(\theta - \frac{i}{2}Tr\{\log\Sigma - \log\Sigma^\dagger\}\right)^2\,, \tag{8}$$

whose introduction is well-justified only in the large number of colours limit[1].

We then enrich the vacuum ansatz of the theory with the introduction of the Witten variables α_i [49] to take into account the effect of the θ-angle on the vacuum state. The ground state then becomes

$$\Sigma_0 = U(\alpha_i)\Sigma_c\,, \text{ with } U(\alpha_i) = \mathrm{diag}\{e^{-i\alpha_1}, \ldots, e^{-i\alpha_{N_f}}\}\,. \tag{9}$$

Defining the quantities

$$\bar\theta = \theta - \sum_i^{N_f}\alpha_i\,, \quad X = \sum_{i=1}^{N_f}\cos\alpha_i\,, \tag{10}$$

allows the Lagrangian of the theory, when evaluated on the ground state ansatz, to be expressed as:

$$\mathcal{L}[\Sigma_0] = 2m_\pi^2 v^2 X\cos\varphi + N_f\mu^2 v^2\sin^2\varphi - av^2\bar\theta^2\,. \tag{11}$$

[1]However, we work at the leading order in an expansion in m_π^2/a in the normal phase and in $m_\pi^2/(a\mu^2)$ in the superfluid phase, which is equivalent to incorporating the θ-angle directly in the mass term (as done in e.g. [51–53]). The corresponding analysis does not rely on the large N limit.

The EOM determines the angle φ and the Witten variables α_i as

$$\sin\varphi\left(N_f\cos\varphi-\frac{m_\pi^2 X}{\mu^2}\right)=0\,, \tag{12}$$

$$m_\pi^2\sin\alpha_i\cos\varphi=a\bar{\theta}\,,\quad i=1,\ldots,N_f\,. \tag{13}$$

The first EOM has two solutions, $\varphi=0$ and $\cos\varphi=\frac{m_\pi^2 X}{N_f\mu^2}$, with the latter indicating that the theory is in a superfluid phase characterized by pion condensation. We observe that when $a\gg m_\pi^2$ the θ-dependence results in an effective pion mass $m_\pi^2(\theta)=m_\pi^2 X/N_f$. Eq.(13) is solved by a small m_π^2/a expansion. In the superfluid phase, the EOM becomes

$$\frac{m_\pi^4}{N_f\mu^2}X\sin\alpha_i=a\bar{\theta}\,,\quad i=1,\ldots,N_f \tag{14}$$

and has the same leading-order solution in the expansion parameter $m_\pi^4/(a\mu^2)$ as (16). In both cases, the leading-order solutions read

$$\alpha_i=\begin{cases}\pi-\alpha(\theta)\,, & i=1,\ldots,n\\ \alpha(\theta)\,, & i=n+1,\ldots,N_f\,,\end{cases} \tag{15}$$

where

$$\alpha(\theta)=\frac{\theta+(2k-n)\pi}{(N_f-2n)},\quad k=0,\ldots,N_f-2n-1\,,\quad n=0,...,\left[\frac{N_f-1}{2}\right]\,, \tag{16}$$

and the parameters n and k label the different EOM solutions. The value range for k is limited since, at fixed n, solutions are periodic in k with period N_f-2n, and the energy-minimizing solutions are

$$\alpha(\theta)=\frac{\theta}{N_f}\quad\text{for}\quad\theta\in[0,\pi]\quad\text{and}\quad\frac{\theta-2\pi}{N_f}\quad\text{for}\quad\theta\in[\pi,2\pi]\,. \tag{17}$$

3. Dilaton augmented chiral Lagrangian and the large charge expansion

Time is ripe to focus on the dynamics near the lower edge of the conformal window to determine the ground state energy of charged states on a non-trivial background that can be associated with scaling dimensions of QCD operators carrying (generalised) isospin charge. To smoothly approach the conformal phase of the theory, we non-linearly realise scale invariance by *dressing* the Lagrangian via a dilaton field $\sigma(x)$ that partially serves as a conformal compensator [6–8, 10, 11, 54]. Under a scale transformation $x\mapsto e^{\alpha}x$, each operator O_k of dimension k is assumed to transform as follows

$$O_k\mapsto e^{(k-4)\sigma f}\,O_k\,, \tag{18}$$

where f is the order parameter of the spontaneous scale symmetry breaking whose pseudo-Goldstone boson transforms as

$$\sigma\mapsto\sigma-\frac{\alpha}{f}\,. \tag{19}$$

Additionally, conformality can be explicitly broken by perturbing the CFT with a relevant operator O that has a conformal dimension Δ by adding to the CFT Lagrangian the term

$$\delta L_O=\lambda_O O\,, \tag{20}$$

where λ_O is the associated coupling. We consider this relevant operator to be the following dilaton potential

$$V(\sigma) = f^{-4}\, e^{-4\sigma f} \sum_{n=0}^{\infty} c_n\, e^{-n(\Delta-4)f\sigma} \,, \tag{21}$$

with the c_ns theory-dependent. Since we are interested in near-conformal dynamics, we assume small explicit breaking. This takes place when $\lambda_O \ll 1$ and/or O are nearly marginal, expecting $c_n \sim \lambda_O^n$ in the first scenario. Truncating the expansion in eq.(21) to the first two terms we obtain [54] [2][3]

$$V(\sigma) = \frac{m_\sigma^2 e^{-4f\sigma}}{4(4-\Delta)f^2}\left(1 - \frac{4}{\Delta}e^{-(\Delta-4)f\sigma}\right) + O(\lambda_O^2) \,, \tag{22}$$

with m_σ the dilaton mass. The key point here is that Δ will not be fixed to a precise value. Instead, the discussion remains on a general level, and it will later be shown how the large charge expansion allows for a general result, with the charge acting as a new tunable parameter governing the final expression. In particular, the non-conformal corrections do depend on the specific form of the potential deforming the CFT, but they are accompanied by a scaling in the charge that holds more generally.

Using the large charge expansion framework [31, 34, 41], we determine the scaling dimension of the lowest-charge operator carrying charge Q. Approximate Weyl invariance maps this near-conformal theory onto the cylinder $\mathbb{R}\times S^3$. The volume, radius, and Ricci scalar of S^3 are represented as V, R, and $\mathcal{R} = \frac{6}{R^2}$, respectively. Hence, the dilaton-pion effective Lagrangian on $\mathbb{R}\times S^3$ reads

$$\begin{aligned}\mathcal{L}_\sigma &= \nu^2 Tr\{\partial_\mu\Sigma\partial^\mu\Sigma^\dagger\}\, e^{-2\sigma f} + m_\pi^2\nu^2 Tr\{M\Sigma + M^\dagger\Sigma^\dagger\}\, e^{-y\sigma f} + 2\mu^2\nu^2 Tr\{II - \Sigma^\dagger I\Sigma I\}e^{-2\sigma f}\\ &+ 2i\mu\nu^2 Tr\{I\partial_0\Sigma\Sigma^\dagger - I\Sigma^\dagger\partial_0\Sigma\}e^{-2\sigma f} - a\nu^2\left(\theta - \frac{i}{2}Tr\{\log\Sigma - \log\Sigma^\dagger\}\right)^2 e^{-4\sigma f} - \Lambda_0^4\, e^{-4\sigma f}\\ &+ \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - \frac{\mathcal{R}}{6f^2}\right) e^{-2\sigma f} - V(\sigma)\,,\end{aligned} \tag{23}$$

where Λ_0 is the bare cosmological constant, and the fermion-induced mass term operator has dimension $y = 3 - \gamma$. The anomalous dimension of the fermion condensate is constrained to $0 < \gamma < 2$ by the unitarity bound. However, around $\gamma \simeq 1$, the four fermion operator is nearly marginal so we carry the following analysis only in the interval $0 < \gamma < 1$. We recall that, in a CFT, Δ_Q can be computed via a semiclassical expansion in the double scaling limit [41]

$$\Lambda_0 f \to 0\,, \quad Q \to \infty\,, \quad Q(\Lambda_0 f)^4 = \text{fixed.} \tag{24}$$

Accordingly, the scaling dimension of the lowest-lying operator assumes the following form

$$RE_Q = \Delta_Q = \sum_{j=-1} \frac{1}{Q^j}\Delta_j\left(Q(\Lambda_0 f)^4\right) \,. \tag{25}$$

[2]This potential has been used in recent investigations [19], including comparisons with lattice simulations [23].

[3]We stress that another potential acquired central stage in a series of interesting papers by Golterman and Shamir [15, 17, 55–59].

The leading order $Q\Delta_{-1}$ of the above expression corresponds to the leading order in the large charge expansion given by the classical ground state energy on the cylinder multiplied by its radius [36, 37, 41]. However, the theory that we are investigating is approximately conformal, with slight perturbations from its underlying conformality by operators like $V(\sigma)$. On the cylinder, this leads to terms linked to symmetry-breaking parameters, such as the pion and dilaton masses that in turn will affect the ground state energy. We stress that under the assumption that these parameters are small, the explicit breaking of conformality is small and this leads us to naturally consider the following extended quantity

$$\Delta_Q^{ex} = RE_Q^{ex} \equiv \Delta_Q + \text{corrections to the CFT}, \tag{26}$$

where Δ_Q is the conformal dimension of the lowest-lying charged operator emerging from the conformal contribution to the ground state energy while we interpret its non-conformal contributions as the near-conformal corrections to Δ_Q. In the next section, we will determine the classical ground state energy.

4. Large charge expansion: leading order

The state-operator correspondence is crucial for finding the scaling dimension of the lowest-lying operator with (generalised) isospin charge Q. To this aim, we need to determine the energy of the vacuum structure that induces the superfluid phase transition. Hence, we evaluate the Lagrangian in eq.(23) on the ansatz in eq.(9)

$$\mathcal{L}_\sigma\left[\Sigma_0, \sigma_0\right] = -e^{-4f\sigma_0}\Lambda_0^4 - V(\sigma_0) - \frac{\mathcal{R}\, e^{-2f\sigma_0}}{12f^2} + 2m_\pi^2 \nu^2 X \cos\varphi\, e^{-f\sigma_0 y} + N_f \mu^2 \nu^2 e^{-2f\sigma_0}\sin^2\varphi - a\nu^2 e^{-4f\sigma_0}\bar{\theta}^2, \tag{27}$$

with σ_0 denoting the classical dilaton solution. Then, we compute the classical ground state energy by solving the EOM

$$\frac{\delta\mathcal{L}}{\delta\alpha} = \frac{\delta\mathcal{L}}{\delta\varphi} = \frac{\delta\mathcal{L}}{\delta\sigma_0} = 0, \qquad \frac{\delta\mathcal{L}}{\delta\mu} = \frac{Q}{V}, \tag{28}$$

in the variables φ, α_i, σ_0 and μ and by plugging the solution into eq.(27). Specifically, we solve the EOM perturbatively in powers of m_σ^2 and m_π^2, arriving at the final result

$$\begin{aligned}\Delta_Q^{ex} = {} & \frac{\pi^2}{8f^2}\left(6N_f (f\nu\mu R)^2 + 1\right)\left(\frac{2N_f (f\nu\mu R)^2 - 1}{f^2\Lambda^4}\right) + m_\sigma^2 \frac{\pi^2 2^{1-\Delta} R^{4-\Delta}}{(\Delta-4)\Delta f^2}\left(\frac{2N_f (f\nu\mu R)^2 - 1}{f^2\Lambda^4}\right)^{\Delta/2} \\ & - m_\pi^4 N_f \cos^2(\alpha(\theta))\, 2^{2\gamma-3}\left(\frac{\pi\nu R^{\gamma+1}}{\mu R}\right)^2 \left(\frac{2N_f (f\nu\mu R)^2 - 1}{f^2\Lambda^4}\right)^{2-\gamma} + O\left(m_\sigma^4, m_\pi^8, m_\sigma^2 m_\pi^4\right),\end{aligned} \tag{29}$$

where μ is related to Q as

$$\mu R = \frac{\left(6\pi^4\nu^2 N_f\right)^{1/3} + \left(\sqrt{81 f^6\Lambda^8 Q^2 - 6\pi^4\nu^2 N_f} + 9f^3\Lambda^4 Q\right)^{2/3}}{f\left(6\pi\nu^2 N_f\right)^{2/3}\left(\sqrt{81 f^6\Lambda^8 Q^2 - 6\pi^4\nu^2 N_f} + 9f^3\Lambda^4 Q\right)^{1/3}}. \tag{30}$$

The first term in eq.(29) denotes the scaling dimension in the conformal limit $m_\pi = m_\sigma = 0$; in fact, it only depends on the dimensionless combination μR which is set by the charge Q through eq.(30). The leading pion mass correction is of order m_π^4, linked to geometry through the chiral condensate's anomalous dimension by the universal factor $R^{2(\gamma+1)}$. Additionally, the cosmological constant is redefined by the first term in the dilaton potential in eq.(21) as

$$\Lambda^4 \equiv \Lambda_0^4 + \frac{m_\sigma^2}{4 f^2 (4-\Delta)} \, . \tag{31}$$

The contribution from the second dilaton potential term of eq.(22) is expanded in powers of m_σ, with the leading order term quadratic in the dilaton mass, universally depending on the sphere's radius through the factor $R^{4-\Delta}$. We then expand the results in eq.(29) in the large charge limit $Q(\Lambda_0 f)^4 \gg 1$ to connect with the universal predictions of large charge EFT [31, 34]. Consequently, we arrive at

$$\tilde{\Delta}_Q^{ex} = \tilde{\Delta}_Q + \left(\frac{m_\sigma}{4\pi\nu}\right)^2 Q^{\frac{\Delta}{3}} B_1 + \left(\frac{m_\pi}{4\pi\nu}\right)^4 \cos^2(\alpha(\theta))\, Q^{\frac{2}{3}(1-\gamma)} B_2 + O\left(m_\sigma^4, m_\pi^8, m_\sigma^2 m_\pi^4\right) \tag{32}$$

where

$$\tilde{\Delta}_Q = c_{4/3} Q^{4/3} + c_{2/3} Q^{2/3} + O\left(Q^0\right) \tag{33}$$

is the scaling dimension in the conformal limit at the leading order in the double scaling limit of eq.(24), which depends only on the dimensionless coefficients defined below

$$c_{4/3} = \frac{3}{8}\left(\frac{2\Lambda^2}{\pi N_f \nu^2}\right)^{2/3} \, , \qquad c_{2/3} = \frac{1}{4 f^2}\left(\frac{2\pi^2}{N_f \nu^2 \Lambda^4}\right)^{1/3} \, , \tag{34}$$

and exhibits the general structure predicted by the large charge EFT. The non-conformal corrections present a Q-scaling as shown in eq.(32) that rely on the parameters γ and Δ breaking scale invariance explicitly. Furthermore, the coefficients B_1 and B_2 read

$$B_1 = \frac{c_{2/3} 2^{9-2\Delta} 3^{\frac{\Delta}{2}-1} (\pi \nu R)^{4-\Delta} \left(c_{4/3} N_f\right)^{1-\frac{\Delta}{2}}}{(\Delta-4)\Delta} \left(1 - \frac{\Delta\, c_{2/3}}{4 c_{4/3}} Q^{-2/3} + O\left(Q^{-4/3}\right)\right) \, , \tag{35}$$

$$B_2 = -\, 3^{4-\gamma} 2^{4\gamma-3} \pi^{2\gamma+2} c_{4/3}^{\gamma-4} N_f^{\gamma-1} (\nu R)^{2(\gamma+1)} \left(1 + \frac{(\gamma-4) c_{2/3}}{2 c_{4/3}} Q^{-2/3} + O\left(Q^{-4/3}\right)\right) \, , \tag{36}$$

comprising two components: the first (blue) encodes geometric information, and the second (yellow) provides general large charge expansion data. We emphasize that eq.(32) is model-independent regarding the dilaton potential, with corrections influenced only by the dilaton mass and charge expansion, potentially altering the coefficients B_1 and B_2 by constants.

5. Conclusions

Via the semiclassical large charge expansion we determined the leading near-conformal corrections to the scaling dimensions of the lowest-lying operators with isospin charge defined at the lower bottom of the QCD conformal window. We unveiled precise scalings in the charge governed by the parameters that drive the theory away from conformality showing also that the near-conformal corrections depend on the way the CFT is deformed.

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