Study of symmetries in finite temperature $N_f = 2$ QCD with Möbius Domain Wall Fermions

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We report on the ongoing study of symmetry of $N_f = 2$ QCD around the critical temperature. Our simulations of $N_f = 2$ QCD employ the Möbius domain-wall fermion action with residual mass ~ 1 MeV or less, maintaining a good chiral symmetry. Using the screening masses from the two point spatial correlators we compare the mass difference between channels connected through various symmetry transformations. Our analysis focuses on restoration of the $SU(2)_L \times SU(2)_R$ as well as anomalously broken axial $U(1)_A$. We also present additional study of a potential $SU(2)_{CS}$ symmetry which may emerge at sufficiently high temperatures.

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1. Introduction

Chiral symmetry is key to determining the phases of quantum chromodynamics(QCD). At low temperatures, spontaneous breaking of chiral symmetry is the origin of hadronic masses. Whereas at high temperatures, such as those in the early universe, restoration of chiral symmetry impacts the fundamental properties of the quark gluon plasma. It is, therefore, important to understand the chiral phase transition not only in particle physics, but also in both nuclear physics and cosmology.

The QCD Lagrangian is invariant under the $SU(N_f)_L \times SU(N_f)_R$ and $U(1)_A$ chiral symmetries in the massless limit. Because the axial $U(1)_A$ symmetry is broken by quantum anomaly at all energies, as a result it is natural to assume to play little role in the phase transition. Therefore, the $SU(N_f)_L \times SU(N_f)_R$ symmetry sector is considered the most relevant to describe the chiral phase transition.

It has been argued that the chiral condensate is not simply an order parameter for chiral symmetry transitions, but may in fact have a relationship to the axial anomaly $U(1)_A$ [1–3]. If the $U(1)_A$ symmetry is effectively restored at the critical temperature, then the order and universality of the phase transition may change [4].

Preservation of chiral symmetry by operators in the lattice quark action is essential to the study of spontaneous restoration/breaking of chiral symmetry at high temperatures. In previous simulations, however, it has been difficult to preserve chiral symmetry and so non-chiral fermions have often been employed. Wilson fermions break the $SU(2)_L \times SU(2)_R$ to a remnant vector-like $SU(2)$; staggered fermions, while successful in many studies, break $SU(2)_L \times SU(2)_R$ to a remnant $U(1)_V \times U(1)_{A'}$ subgroup symmetry. Therefore, study of chiral symmetry using methods such as overlap or domain wall is attractive to produce more theoretically clean evidence of the chiral transition behaviors as the $SU(2)_L \times SU(2)_R$ is preserved.

In this work, we simulate $N_f = 2$ QCD using Möbius Domain Wall Fermions, with a residual mass ~ 0.1 MeV. While some previous results at higher temperatures have already been published [5–8], this most recent work covers previous temperatures and adds two new ensembles at $T = 147\,\text{MeV}(0.9\text{T}_c)$ and $165\,\text{MeV}(\text{T}_c)$ allowing us to study symmetries around the critical temperature. To examine the symmetries of the light mesons we use the screening mass extracted from the spatial two-point meson correlation function in the isospin triplet channel.

In addition to the well known symmetries of QCD, we are interested in studying emergent symmetry structures which are not seen in the original QCD Lagrangian[9–12]. In recent literature [13–15] emergent higher symmetries of QCD e.g. $SU(N_f)_{CS}$, $SU(2N_f)$ have been increasingly discussed, which we have addressed in previous numerical simulations [7, 16–20] and will explore in this work as well.

2. Spatial Mesonic Correlators

Let us consider the quark isospin triplet bilinear operators

$$O_\Gamma(x) = \overline{q}(x)(\Gamma \otimes \frac{\vec{\tau}}{2})q(x), \tag{1}$$

where τ^a is an element of the generators of $SU(2)$, Γ denotes the combination of Dirac γ listed in Table. 1. Using the correlation functions of the $O_\Gamma(x)$ operators we examine symmetries at

high temperature using the screening mass spectrum. Specifically we make use of the two point correlator along the z-axis,

$$C_\Gamma(n_z) = \sum_{n_t, n_x, n_y} \langle O_\Gamma(n_x, n_y, n_z, n_t) O_\Gamma^\dagger(0, 0, 0, 0) \rangle \tag{2}$$

where n_t is the Euclidean time direction and we specify the correlator channel with Γ. Assuming confinement in the spatial direction of QCD, the correlation function at long distances is expected to exponentially decay following $C_\Gamma(z) \sim e^{-M_\Gamma z}$, where M_Γ is the screening mass of the lowest energy state in the Γ channel.

Γ	Reference Name	Abbr.	Symmetry Correspondences		
$\mathbb{I}$	Scalar	S	$\left.\right\} U(1)_A$		
γ_5	Psuedo Scalar	PS			
γ_1, γ_2	Vector	**V**	$\left.\right\} SU(2)_L \times SU(2)_R$		$\left.\right\} SU(2)_{CS}?$
$\gamma_1\gamma_5, \gamma_2\gamma_5$	Axial Vector	**A**			
$\gamma_4\gamma_3$	Tensor	$\mathbf{T}_t$	$\left.\right\} U(1)_A$		
$\gamma_4\gamma_3\gamma_5$	Axial Tensor	$\mathbf{X}_t$			

Table 1: The six channels of interest in this study and connecting symmetry transformations.

3. Emergence of $SU(2)_{CS}$

For $T \to \infty$ the free quark propagator with mass m appears to exhibit an additional set of symmetries. Let us consider a quark propagator which has momenta (p_x, p_y) perpendicular to the z-axis:

$$\langle \overline{q}(z) q(0) \rangle (p_1, p_2) = \sum_{p_0} \int_{-\infty}^{\infty} \frac{dp_3}{2\pi} \frac{m - (i\gamma_0 p_0 + i\gamma_i p_i)}{p_0^2 + p_i^2 + m^2} e^{ip_3 z}, \tag{3}$$

where we have decomposed the Euclidean time direction into the Matsubara frequencies $p_0 = 2\pi T(n + \frac{1}{2})$ and labelled x, y and z momenta with numerical indicies. The p_3 integral has a pole at $-ip_3 = E = \sqrt{p_0^2 + m^2 + p_1^2 + p_2^2}$ resulting in,
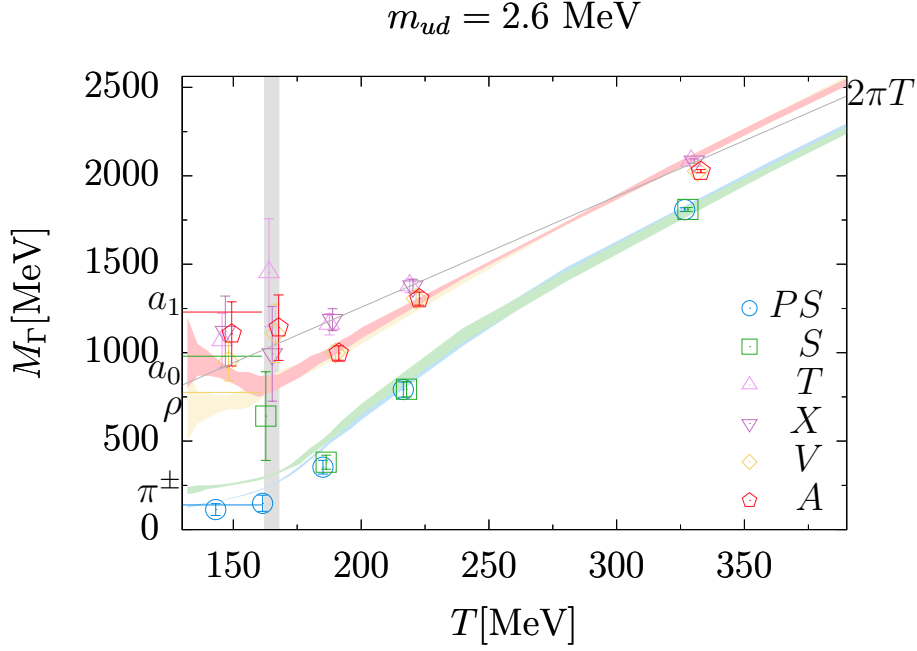
$$\langle \overline{q}(z) q(0) \rangle (p_1, p_2) = \sum_{p_0} \frac{m + \gamma_3 E - (i\gamma_0 p_0 + i\gamma_1 p_1 + i\gamma_2 p_2)}{2E} e^{-Ez}. \tag{4}$$

In the $T \to \infty$ limit we can take $T \gg m^2 + p_1^2 + p_2^2$ and expand the correlator in terms of $1/T$:

$$\langle \overline{q}(z) q(0) \rangle = \sum_{p_0 = \pm \pi T} \gamma_3 \frac{1 + i\text{sgn}(p_0)\gamma_0\gamma_3}{2} e^{-\pi T z} + O(1/T). \tag{5}$$

Here we have approximated the summation over p_0 to be the lowest Matsubara modes with $M = \pm \pi T$ only. This form of the correlator is invariant under an additional set of transformations:

$$q(x) \to \exp\left(\frac{i}{2}\Sigma_a \theta_a\right) q(x)$$

$$\overline{q}(x) \to \overline{q}(x)\gamma_0 \exp\left(-\frac{i}{2}\Sigma_a \theta_a\right)\gamma_0, \tag{6}$$

$$m_{ud} = 2.6 \text{ MeV}$$

Figure 1: For the lightest mass quark ensembles the $N_f = 2$ screening mass data plotted with respect to temperature, shown as points. The shaded bands are $N_f = 2 + 1$ data from HotQCD in [22].

where

$$\Sigma_k = \begin{bmatrix} \gamma_k \\ i\gamma_k\gamma_5 \\ \gamma_5 \end{bmatrix},$$

which is the so-called $SU(2)_{CS}$ chiral-spin symmetry, explored in [7, 9, 12, 13, 18].

For the free quark case shown above, the chiral-spin symmetry is approximate and emerges in the high temperature limit. However, when gluonic interactions are introduced the quark propagator has a nontrivial form which cannot be analyzed akin to the free quark form, yet it may possibly transform under the chiral-spin symmetry. Other work [7, 10, 21] has shown evidence for an emergent $SU(2)_{CS}$ symmetry at temperatures $\sim 1.1 T_c$ and above. While it has also been argued this symmetry is over taken by interactions with the gluonic medium[19, 20] at very high temperatures.

Although there are estimates presented for this symmetry, it remains unclear if such a symmetry is emergent near T_c and whether this symmetry is preserved into a higher temperature regime. For the range of temperatures in our study we also consider analysis of the channels which are related by $SU(2)_{CS}$ symmetry transformation as a further examination of the $SU(2)_{CS}$ around T_c.

4. Numerical Results

Our simulations of $N_f = 2$ flavor QCD employ the tree-level improved Symanzik gauge action, as well as Möbius domain wall fermion action with three steps of stout smearing. For all ensembles $\beta = 4.30$ with a lattice cutoff set to $a^{-1} = 2.463$ GeV. The residual mass of the domain-wall quark is reduced to < 0.1 MeV. For the highest temperature ensembles the aspect ratio is 4:1 and

we do not expect finite size effects on the screening masses. For temperatures $T \sim 147$ MeV and $T \sim 165$ MeV lattices with spatial extent $L = 36$ and $L = 32$ respectively may not be enough to control the long range correlation effects for the spatial correlation functions and so we also consider additional lattices with spatial extents of $L = 48$ and $L = 40$ respectively. For all new ensemble data we considered four values of $am = 0.0010, 0.0025, 0.00375, 0.0050$ for up and down quark masses with the lightest value $am = 0.0010$ corresponding to a mass of 2.6 MeV below the physical point for the lightest quarks. The value for the screening mass is extracted from fitting lattice data to the standard form

$$C(z) = A \cosh(m[z - L/2]). \tag{7}$$

For channels with noisy fits such as the S-channel, and in particular the lowest two temperatures in our study we omit the fit values as both the correlator and the effective mass curves are too unstable to produce a reliable fit value.
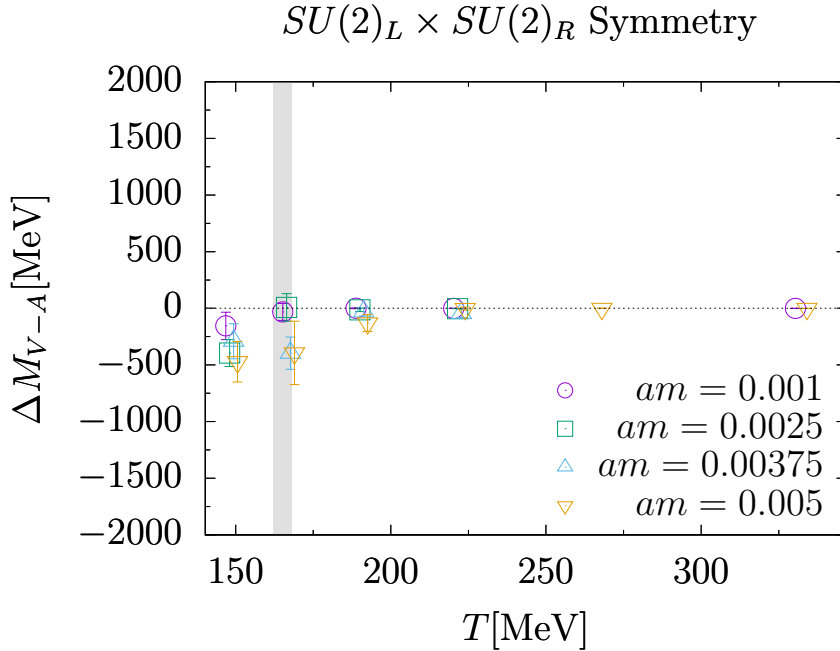
Figure 2: The screening mass difference plot for $SU(2)_L \times SU(2)_R$ with respect to temperature for the range of masses in this study.

5. Summary of Preliminary Results

Figure 1 shows, for the lowest temperature $\sim 0.9T_c$ in the study, a good overlap between screening masses in various channels and the experimental data at $T = 0$; in particular, the pseudoscalar overlaps with the $T = 0$ $\pi^\pm$ mass indicating that the chiral condensate rapidly converges to the zero temperature value below T_c.

The most significant difference between our work and the results by HotQCD, plotted as shaded bands in fig 1, is the behavior of the S channel at low temperatures. In [22], the staggered fermion

action employed by HotQCD breaks isospin symmetry such that a two pion decay channel from the scalar triplet is opened; when the isospin symmetry is preserved the triplet scalar cannot decay as this decay channel is forbidden. Because our simulations are done with Möbius Domain Wall fermions we do not observe the same $2\pi^{\pm}$ lattice artifact, and instead, we observe a large increase in the screening mass in the region of broken chiral symmetry.

The $SU(2)_L \times SU(2)_R$ symmetry restoration, seen through the mass difference in the V and A partner channels, shows a clear signal for the all masses but is then most evident at $am = 0.0010$. From figure 2 at $T = 147$ MeV the $SU(2)_L \times SU(2)_R$ chiral symmetry is broken, and it appears to be restored at the transition point $T_c \sim 165$ MeV and remains zero for all higher temperatures. $U(1)_A$ symmetry is also effectively ''restored'' for $T \sim 165$ MeV from the plot of $X_t - T_t$ fig. 3. While it is still unclear if the transition is exactly at the pseudocritical point at $T = 189$ MeV corresponding to $T \sim 1.1T_c$ we see restoration of $U(1)_A$ which is below the $1.2 - 1.3T_c$ threshold for restoration reported by [22–27] and is observed in the $PS - S$ screening mass difference as well.

In contrast to both of these symmetries we also plot a temperature spectrum for the $SU(2)_{CS}$ predicted in [9, 10, 12], unlike the clear signals of symmetry restoration seen in figs 2 and 3, the plot of the symmetry pair $X - A$ figure 4 does not converge to zero for the temperatures in our study. While there is a significant reduction in noise, we do not see a convergence to, or sustained, zero value indicating emergence of $SU(2)_{CS}$.

Acknowledgments
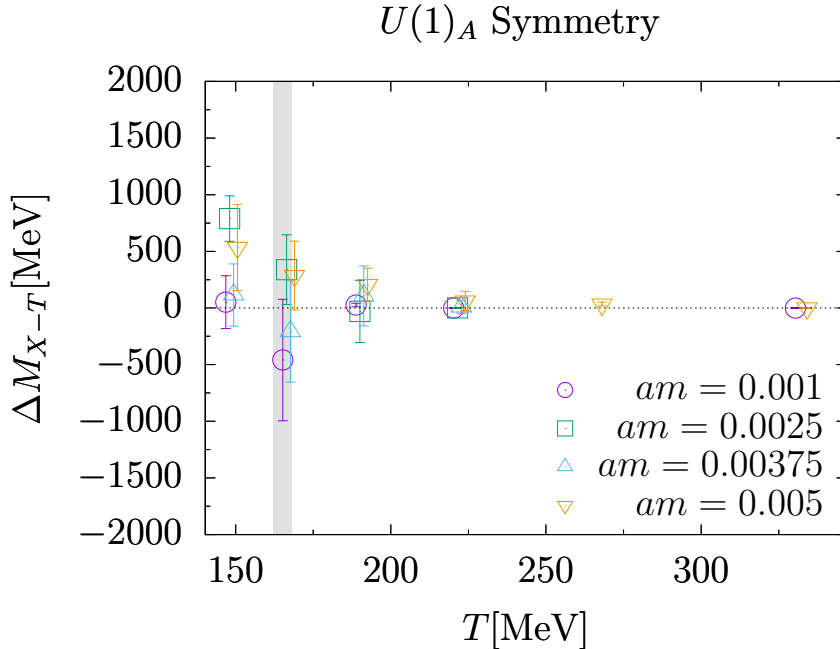
Figure 3: The screening mass difference plot for $U(1)_A$ with respect to temperature, the axial anomaly appears to be suppressed at $T = 189$ MeV.
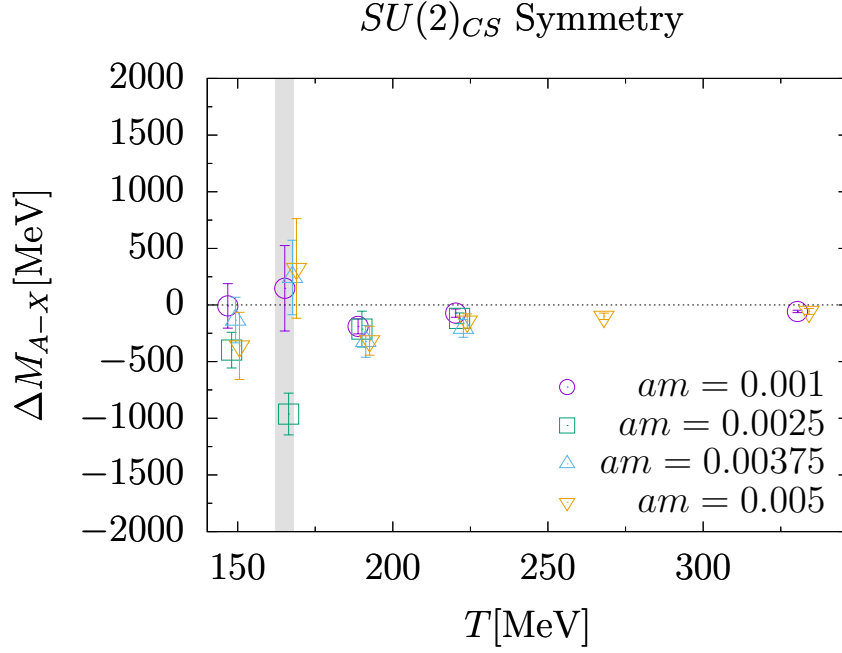
$SU(2)_{CS}$ Symmetry

Figure 4: The screening mass difference plot for $SU(2)_{CS}$ for various temperatures and masses, while it looks close to being restored at $T = 330$ MeV it is ~ 40 MeV and most likely approximate if the symmetry is emergent at this temperature.

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