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The η_c-meson leading-twist distribution amplitude

Benoît Blossier,[a] Mariane Mangin-Brinet,[b] José Manuel Morgado Chávez[c] and Teseo San José[a,*]

[a] *Laboratoire de Physique des 2 Infinis Irène Joliot-Curie, CNRS/IN2P3, Université Paris-Saclay, 91405 Orsay Cedex, France*

[b] *Laboratoire de Physique Subatomique et de Cosmologie, CNRS/IN2P3, 38026 Grenoble, France*

[c] *Département de Physique Nucléaire, Irfu/CEA-Saclay, 91191 Gif-sur-Yvette Cedex, France*

E-mail: san-jose-perez@ijclab.in2p3.fr, blossier@ijclab.in2p3.fr, mariane@lpsc.in2p3.fr, jose-manuel.morgadochavez@cea.fr

In this project, we employ the short-distance factorization to compute the distribution amplitude of the η_c-meson from Lattice QCD at leading twist. We employ a set of CLS $N_f = 2$ ensembles at three lattice spacings and various quark masses to extrapolate the pseudo distribution to the physical point in the isospin limit. We solve the inverse problem modeling the distribution amplitude, and we match our results to the light-cone in the $\overline{\text{MS}}$-scheme. We include a complete error budget, and we compare to two alternative approaches: non-relativistic QCD and Dyson-Schwinger equations, finding good agreement with the latter but not with the former.

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*Speaker

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Introduction

Ever since the discovery of hadron structure at SLAC, several inclusive and exclusive experimental processes have been discovered that help us with the study of the internal structure of baryons and mesons. From the theoretical side, factorization theorems [1] are a fundamental tool to understand processes with a large momentum transfer: They divide the cross section in a convolution of hard and soft pieces, where the first can be computed in perturbation theory and the second requires a non-perturbative approach. In this study we focus on the calculation of distribution amplitudes (DAs), which appear in meson photoproduction $\gamma^*\gamma^* \to M$ and deeply virtual meson production (DVMP) $\gamma^* p \to Mp$, and describe the momentum distribution among the quarks of the meson along the longitudinal direction. The more traditional methods to estimate these quantities include non-relativistic QCD (NRQCD), Dyson-Schwinger (DS) equations, light-front dynamics, or light-cone sum rules. The fact that this quantity is defined along the light-cone, see eq. (1), prevents its direct calculation using lattice QCD (LQCD). In this study we employ the method of pseudo-distributions [2, 3], which generalizes the definition of DAs and other functions to space-like separations and provides an algorithm to recover the light-cone physics in a certain limit.

Methodology

The DA of charmonium in light-cone coordinates is usually given in terms of the momentum fraction x carried by the quark [4]

$$\phi(x) = \int \frac{\mathrm{d}z^-}{2\pi} e^{-\mathrm{i}(x-1/2)p^+z^-} M^\alpha(p,z), \tag{1}$$

where the matrix element itself is the Ioffe-time distribution amplitude (ITD),

$$M^\alpha(p,z) = \left. \langle \eta_c(p)|\bar{c}(-z/2)\gamma^+\gamma_5 W(-z/2,z/2)c(z/2)|0\rangle \right|_{z^+=z^{\mathrm{T}}=0}, \tag{2}$$

and $\langle\eta_c|$ is the pseudoscalar meson in the final state, $|0\rangle$ is the QCD vacuum, W is a Wilson line assuring gauge invariance, p is the hadron momentum, and c and $\bar{c}$ are the quark fields, which are a distance z^- apart. Unfortunately, the separation along the light cone, $z^2 = 0$, prevents a direct evaluation in Euclidean space. Instead, we use a generalization of the ITD in Euclidean metric to space-like separations $z^2 > 0$ [2, 3]. To connect this quantity, the pseudo-DA (pDA), to the DA on the light-cone at leading twist we need to take several steps: First, we can separate several higher-twist contributions via a Lorentz decomposition,

$$M^\alpha(p,z) = 2p^\alpha \mathcal{M}(\nu,z^2) + z^\alpha \mathcal{M}'(\nu,z^2). \tag{3}$$

Both terms $\mathcal{M}$ and $\mathcal{M}'$ are Lorentz scalars depending on the Ioffe time $\nu \equiv pz$ and the invariant interval z^2. The leading-twist contribution appears in $\mathcal{M}$, and we select it choosing $p = (0,0,p_3,E)$, $z = (0,0,z_3,0)$ and $\alpha = 4$. Second, the matrix element M^α renormalizes multiplicatively [5], and the renormalization factor depends solely on z. We take advantage of the fact that $\nu = 0$ is a fixed point of the ITD to cancel the renormalization factor and avoid its computation entirely. The quantity [6–8]

$$\tilde{\phi}(\nu,z^2) \equiv \frac{\mathcal{M}(p,z)\mathcal{M}(0,0)}{\mathcal{M}(0,z)\mathcal{M}(p,0)} \tag{4}$$

is known as the reduced Ioffe-time pseudo-distribution amplitude (rpITD), it is renormalization group invariant (RGI) and $\tilde{\phi}(\nu = 0, z) = 1$ even at finite lattice spacings. Third, we take the limit $z^2 \to 0$ and match to the light-cone DA in the $\overline{\mathrm{MS}}$-scheme via the relation [9, 10]

$$\tilde{\phi}_{\mathrm{lt}}(\nu, z^2) = \int_0^1 \mathrm{d}w\, C(w, \nu, z\mu) \int_0^1 \mathrm{d}x \cos(wx\nu - w\nu/2)\phi_{\mathrm{lt}}(x, \mu) \tag{5}$$

where the DA now depends on the renormalization scale $\mu = 3\,\mathrm{GeV}$, the kernel $C(w, \nu, z\mu)$ takes care of the divergences appearing when $z^2 \to 0$, and the integral in x corresponds to the Fourier transform between x and ν spaces. In our study we only require the moments of $C(w, \nu, z\mu)$, which are plotted in fig. 1. For their expression and more details of the calculation, see [11]. After introducing the lattice regulator we follow these same steps, and upon arriving to eq. (5) we encounter an inverse problem, with a finite dataset on the left-hand side (lhs) to reconstruct a function on the right-hand side (rhs). To solve this, we introduce extra information by parameterizing the DA on the light-cone as

$$\phi_{\mathrm{lt}}(x, \mu) = (1 - x)^{\lambda - 1/2} x^{\lambda - 1/2} \sum_{n=0}^{\infty} d_{2n}^{(\lambda)} \tilde{G}_{2n}^{(\lambda)}(x), \quad d_0^{(\lambda)} = \frac{4^\lambda}{B(1/2, \lambda + 1/2)} \tag{6}$$

where B is a beta function, $\tilde{G}_{2n}^{(\lambda)}(x)$ are shifted Gegenbauer polynomials defined in the interval $x \in (0, 1)$, and the lattice data constraints the coefficients λ and d_{2n}. A similar approach has been applied to parton distribution functions (PDFs) in [12]. Setting $\lambda = 1.5$, it is possible to recover the conformal expansion of the DA from eq. (6), and one also obtains the asymptotic result $6x(1 - x)$ when $\mu \to \infty$. Indeed, we assume that this expansion, which is true at leading twist and $O(\alpha_s)$, can also describe non-perturbative data leaving free the coefficient λ. After all, the series of polynomials form a basis that should be able to describe any smooth function, and even singularities at the endpoints $x = 0$ and 1. Of course, since in our analysis we truncate the series after the first term, this forces the endpoints to be zero for $\lambda > 1/2$. Replacing eq. (6) in eq. (5) simplifies the relation between the fit coefficients and the data,

$$\tilde{\phi}_{\mathrm{lt}}(\nu, z^2) = \sum_{n=0}^{\infty} \tilde{d}_{2n}^{(\lambda)} \sigma_{2n}^{(\lambda)}(\nu, z^2\mu^2), \quad \tilde{d}_n^{(\lambda)} = \frac{d_n^{(\lambda)}}{4^\lambda}, \tag{7}$$

where we use a new set of functions σ_{2n}, plotted in fig. 1. Their main feature is that they peak in a certain range of Ioffe times and then vanish. This means that, depending on the domain in Ioffe time of our data, we will be sensitive to more or less of these coefficients (see [11] for more details).

The lattice calculation

We employ the set of $N_f = 2$ CLS ensembles gathered in table 1, which employ the Wilson gauge action and Wilson quarks with non-perturbative $O(a)$ improvement. The charm-quark mass was tuned so that $m_{D_s} = m_{D_s,\mathrm{phys}} = 1968\,\mathrm{MeV}$ [13], while the light-quark masses yield pions in the range $190\,\mathrm{MeV} < m_\pi < 440\,\mathrm{MeV}$. We use quark propagators with wall sources diluted in spin, deflated SAP-GCR to solve the Dirac equation, and a custom version of the DD-HMC algorithm

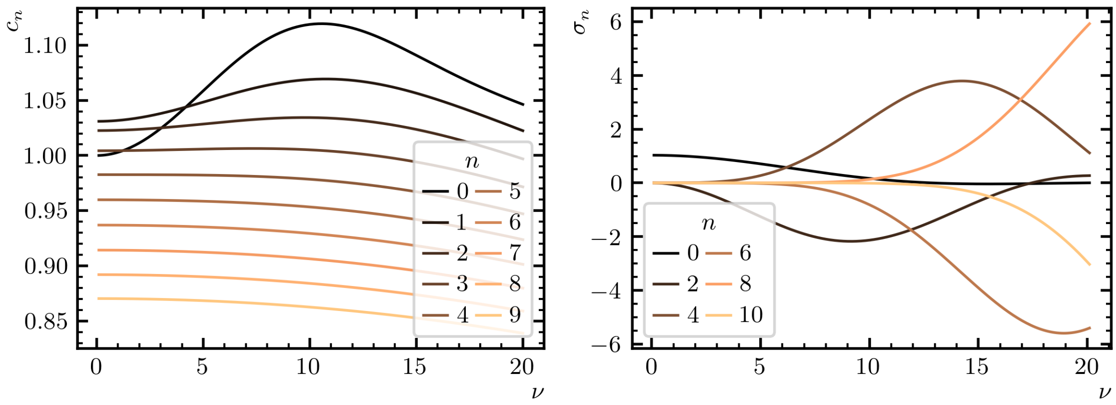

Figure 1: LEFT: Moments $c_n(\nu, z\mu)$ of the matching kernel. RIGHT: Functions $\sigma_n(\nu, z\mu)$. We choose $\lambda = 2.7$ and $z_3 = 5 \times 0.0658\,\mathrm{fm}$.

id	β	a [fm]	L/a	am_π	m_π [MeV]	$m_\pi L$	κ_c	Measurements
A5	5.2	0.0755(9)(7)	32	0.1265(8)	331	4.0	0.12531	1980
B6			48	0.1073(7)	281	5.2	0.12529	1180
D5*	5.3	0.0658(7)(7)	24	0.1499(1)	449	3.6	0.12724	1500
E5			32	0.1458(3)	437	4.7	0.12724	2000
F6			48	0.1036(3)	311	5.0	0.12713	1200
F7			48	0.0885(3)	265	4.3	0.12713	2000
G8			64	0.0617(3)	185	4.1	0.12710	1790
N6	5.5	0.0486(4)(5)	48	0.0838(2)	340	4.0	0.13026	1900
O7			64	0.0660(1)	268	4.2	0.13022	1640

Table 1: CLS ensembles included in our analysis. From left to right, we find the labels, the bare couplings and corresponding lattice spacings, the spatial extension ($T = 2L$), the pion mass in lattice and physical units, the value of $m_\pi L$, the bare charm-quark mass and the statistics for each ensemble. An asterisk indicates the ensemble was used only to check for finite-size effects (FSEs) and was not used in the extrapolation to the continuum.

for the contractions. We form the η_c interpolator solving a 4×4 Generalized Eigenvalue Problem (GEVP) with different Gaussian smearing levels and the bilinear $\bar{c}\gamma_5 c$, while its momentum is set via partially twisted boundary conditions (PTBCs) applied to the charm quark. We only compute the quark-connected Wick contraction of eq. (2). We expect a strong Okubo-Zweig-Iizuka (OZI) suppression of the disconnected piece with a factor $\alpha_s^2(\mu) \sim 0.05$. Computing the rpITD for all ensembles of table 1 yields fig. 2, where $\tilde{\phi}(\nu, z^2)$ appears as a function of Ioffe time and the color code indicates the extension of the Wilson line. We observe the data fall close to a universal line, with precise data up to $\nu \sim 5$. These points are still affected from a variety of artifacts: We need to take the continuum limit, extrapolate to the physical quark masses, and remove the remaining higher-twist contamination which includes the target-mass corrections. The latter are proportional

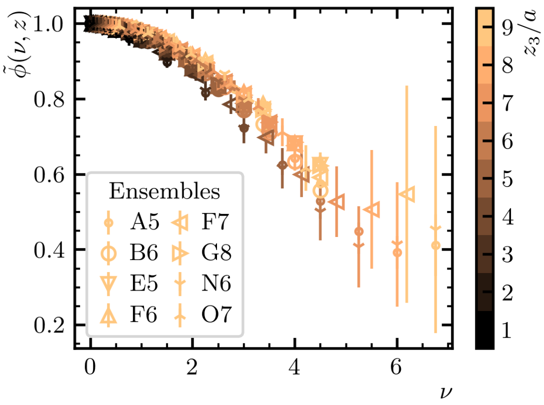

Figure 2: The rpITD for the η_c-meson on all ensembles considered in this analysis.

λ	$2.73 \pm 0.12 \pm 0.12 \pm 0.06$	$2.75(12)$	$2.61(15)$	$2.62(10)$
$a_{1,2}$	$-7.58 \pm 0.05 \pm 0.59 \pm 0.55$	$-8.12(5)$	$-8.68(13)$	$-6.76(4)$
$b_{1,2}$	$0.88 \pm 0.07 \pm 0.08 \pm 0.06$	$0.89(7)$	$0.77(10)$	$0.81(7)$
$c_{1,2}$	$-0.042 \pm 0.002 \pm 0.005 \pm 0.001$	$-0.0428(22)$	$-0.0440(28)$	$-0.0407(24)$
$d_{1,2}$	$-2.221 \pm 0.015 \pm 0.063 \pm 0.150$	$-2.368(15)$	$-2.52(4)$	$-2.000(11)$
$e_{1,2}$	$-0.0897 \pm 0.0010 \pm 0.1590 \pm 0.1160$	$-0.1700(16)$	$-0.321(5)$	$-0.068\,48(12)$

Table 2: Fit parameters in eq. (8). The first column indicates the parameter; the second column the expected value, the statistical and the various systematic uncertainties; the third is the result removing the heaviest pion mass; the fourth keeps only $m_\pi < 300$ MeV and the fifth checks that the results are stable when we only keep $z_3 < 0.5$ fm. The coefficients $a_{1,2}$, $b_{1,2}$, etc., correspond to the auxiliary functions A_1, B_1, etc.

to $z^2 m^2_{\eta_c,\mathrm{phy}}$, which can be sizeable for the η_c-meson. We fit the lattice data $\tilde{\phi}_e(\nu, z^2)$ to the following model to separate all these effects from the leading-twist pDA that can be matched to the light-cone using eq. (7),

$$\begin{aligned} \tilde{\phi}_e(\nu, z^2) = \tilde{\phi}_{\mathrm{lt}}(\nu, z^2) &+ \frac{a}{|z|} A_1(\nu) + a\Lambda B_1(\nu) + z^2\Lambda^2 C_1(\nu) \\ &+ \frac{a}{|z|}\Big(\Lambda^{-1}[m_{\eta_c} - m_{\eta_c,\mathrm{phy}}] D_1(\nu) + \Lambda^{-2}[m^2_\pi - m^2_{\pi,\mathrm{phy}}] E_1(\nu)\Big). \end{aligned} \tag{8}$$

The leading lattice artifacts are $O(a)$ because we do not improve the matrix element eq. (3), only the action. The auxiliary functions A_1, B_1, etc. have a similar form to eq. (7) and have their own fit parameters (their explicit form appears in [11]). The value of λ is shared, as it only serves to specify a basis of polynomials. We render all terms dimensionless using $\Lambda \equiv \Lambda^{(2)}_{\mathrm{QCD}} = 330$ MeV [14]. Fitting the data shown in fig. 2 to eq. (8) yields the results displayed in table 2. We are only sensitive to the first coefficient $d_0^{(\lambda)}$, such that the DA given in eq. (6) is reduced to

$$\phi_{\mathrm{lt}}(x, \mu) = \frac{4^\lambda (1-x)^{\lambda-1/2} x^{\lambda-1/2}}{B(1/2, 1/2+\lambda)} \tag{9}$$

with $\lambda = 2.73(18)$ adding in quadrature all uncertainties of table 2. Equation (9) appears in fig. 3 both in x space and Ioffe-time space. The latter is especially useful to compare to other theoretical

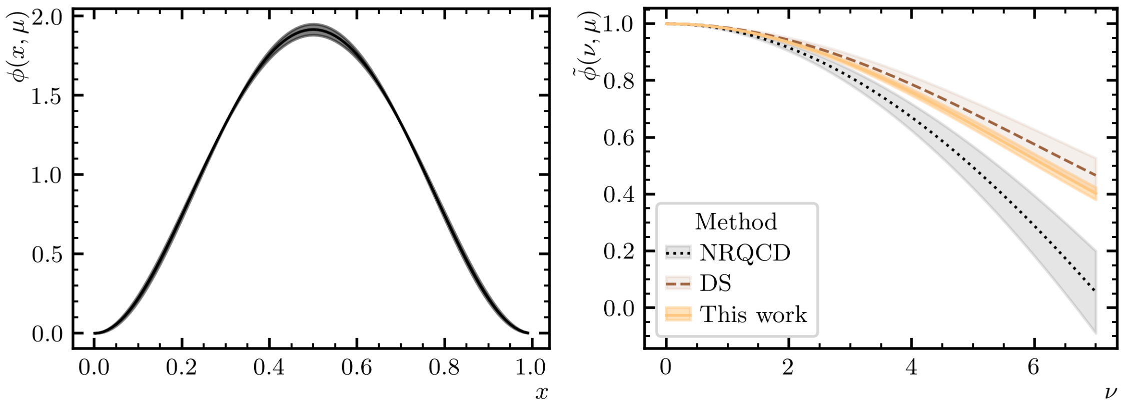

Figure 3: LEFT: The DA in x space given by eq. (9) using $\lambda = 2.73(18)$ at $\mu = 3\,\text{GeV}$. RIGHT: The DA in Ioffe-time space compared to NRQCD [15] and DS equations [16].

determinations because the DA has to be analytic. In particular, we compare to two alternative approaches, one using the NRQCD framework [15] and another employing DS equations [16]. We observe good agreement with the latter, which is also a non-perturbative approach, while NRQCD differs significantly at larger Ioffe times. Since NRQCD relies in a series expansion and the entire ν dependence is given by the first quantum and relativistic corrections, which become very sizeable for large Ioffe times, we think it is important to know the corrections at next order.

Conclusions and outlook

We compute the leading-twist contribution to the DA of the η_c-meson with a set of $N_f = 2$ CLS ensembles using the method of short-distance factorization. Thanks to the various lattice spacings and quark masses we can extrapolate to the physical point in the isospin limit. The DA on the light-cone is parameterized in eq. (9). We explore several sources of systematic uncertainty and conduct various crosschecks that show that our results (given in table 2) are stable. The method developed in this work can now be applied to other states, like J/ψ, with a more complicated Lorentz structure and bigger impact in the upcoming EIC experiment.

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