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Symmetries of the loop-string-hadron framework: towards quantum simulating non-Abelian gauge theories

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Reformulating the Hamiltonian formulation of non-Abelian lattice gauge theories entirely in terms of gauge invariant loop-string-hadron degrees of freedom provides a set of advantages for simulating the theory on quantum hardware and in turn is expected to address a series of physics quests. The framework is manifestly (non-Abelian) gauge invariant, yet possesses a set of remnant Abelian gauge symmetries along with its global symmetry properties. In this talk, we describe all the symmetries of this framework and discuss the advantages/ challenges of the symmetry structure being present/preserved in a Hamiltonian simulation towards understanding real-time dynamics of non-Abelian gauge theories.

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1. Introduction

Rapid advances in quantum technology, specifically scaling up of quantum computation capability by quantum hardware manufacturers have triggered a steep hike in the interests of the lattice gauge theorists in performing useful quantum computation aiming at a better understanding of strongly correlated quantum systems [1**?** –20]. This research direction has opened up natural collaborations and conversation between the nuclear-particle physics community - traditional lattice gauge theorists the condensed-matter physics community, atomic and molecular physics community as well as Mathematicians and and computer scientists interested in developing quantum algorithms and implementation of the same. The last decade has witnessed quite a few successful quantum simulations implemented on NISQ-era hardware - both noisy digital gate-based devices and quantum simulators built for gauge theory simulations [6, 12, 21–24]. While the progress stands remarkable in defining a new direction of computation, the aim of quantum simulating the complete standard model of particle physics or Quantum chromodynamics (QCD) remains a far-term goal [25, 26]. One of the major step to achieve the goal is to find or develop a convenient framework for quantum computing for non-Abelian gauge theories [9, 27–50]. As the state-of-the-art suggests, the conventional framework - using irrep basis or strong coupling basis appears expensive for SU(2) when the interest is in simulating dynamics only in the gauge invariant physical subspace [**? ?**], and is extremely difficult to generalize to SU(3) [15]. To address these issues, starting from the prepotential framework of lattice gauge theories [51, 52], a novel Loop String Hadron (LSH) framework [28, 29] is being developed for non-Abelian gauge theories. For SU(2) theories, the framework is complete in any arbitrary dimensions for pure gauge fields and gauge fields coupled with one flavour of staggered fermions [28]. While, the generalization to SU(3) is found to be non-trivial [53, 54] but is in progress by completing each step at a time [29, 55]. However, the fully developed LSH frameworks ($d + 1$ dimensional SU(2) [28] and $1 + 1$ dimensional SU(3)) [29] are ready for computation and it is instructive to look into the symmetry sectors of these theories. Understanding the symmetry sectors is crucial for developing efficient algorithms and also for understanding the physics including their thermalization properties. In the context of quantum simulation, the full realization of the symmetries would lead to efficient algorithm development - while protecting the symmetries in the simulated dynamics is a major step in mitigating errors in quantum simulation.

In this article, we highlight the global symmetries of the loop-string-hadron framework. We confine the discussion to a lattice in one spatial dimension subjected to open boundary conditions. The current article is organized as follows: In section 2 we briefly review the loop-string-hadron framework. The local and global symmetries are discussed in section 3 and 4. Finally, we discuss how the underlying symmetry structure leads to novel quantum simulation protocols and unveil unknown physics in section 5.

2. Hamiltonian lattice gauge theory translated to Loop-String-Hadron framework

Hamiltonian framework for SU(N) non-Abelian lattice gauge theories in one spatial dimension is given by the Kogut-Susskind Hamiltonian [56], where the electric field and the link operators act as the canonical conjugate variables of the theory.

$$H^{(\mathrm{KS})} = H_I^{(\mathrm{KS})} + H_E^{(\mathrm{KS})} + H_M^{(\mathrm{KS})}. \tag{1}$$

Here, $H_I^{(\mathrm{KS})}$ denotes interactions among the fermionic and gauge fields as given in

$$H_I^{(\mathrm{KS})} = \frac{1}{2a}\sum_x \left[\psi^\dagger_\alpha(x)\hat{U}^\alpha{}_\beta(x)\psi^\beta(x+1) + \mathrm{h.c.}\right], \tag{2}$$

where, α, β runs from $1, 2, .., N$, for N being the dimension of the fundamental irreps for SU(N). The mass and electric parts of the Hamiltonian are given by,

$$H_E^{(\mathrm{KS})} = \frac{g^2 a}{2}\sum_x \hat{E}(x)^{\mathrm{a}}\hat{E}(x)^{\mathrm{a}} \quad , \quad H_M^{(\mathrm{KS})} = m\sum_x (-1)^x \psi^\dagger_\alpha(x)\psi^\alpha(x). \tag{3}$$

Here, g is a coupling, m is the mass of the two-component staggered fermionic fields considered and a denotes lattice spacing. The index ‘a’ runs from 1 to $N^2 - 1$ for the gauge group SU(N).

The Hamiltonian given in (1) commutes with the generators of gauge transformation at each site (x) given by

$$\hat{G}^{\mathrm{a}}(x) = \hat{E}^{\mathrm{a}}_L(x) + \hat{E}^{\mathrm{a}}_R(x-1) + \rho^{\mathrm{a}}(x) \tag{4}$$

where ρ^{a} denotes staggered matter density for the theory. The physical Hilbert space of the theory is defined to contain states which are gauge invariant, i.e. the physical states must satisfy:

$$\hat{G}^{\mathrm{a}}(x)|\Psi_{\mathrm{phys}}\rangle = 0 \ \ \forall \mathrm{a}, x, \tag{5}$$

while $\hat{G}^{\mathrm{a}}$ satisfy the Lie algebra of the gauge group. Construction of the physical Hilbert space for SU(2) gauge theory is possible, but turns out to be tedious and expensive for classical computation. Simulating the same on a quantum computer is extremely difficult as it requires a large number of qubits/qudits to encode the fermionic and gauge degrees of freedom and a precise many-body control to impose the Gauss law constraint. So far in the literature, a fully gauge fixed and purely fermionic formalism has been popular in the context of quantum computing for gauge theory [57–59] which is restrictive to boundary conditions and not scalable to arbitrary dimension.

We focus on a particular framework, namely the loop-string-hadron (LSH) framework, that has been developed for SU(2) [28] and SU(3) [29] gauge theories. The SU(2) framework has been studied extensively and found to be advantageous over other available frameworks to describe the same physics. The key feature of LSH formalism is that it is constructed to be manifestly gauge invariant. The Hilbert space is characterized by on-site quantum numbers, which correspond to gauge invariant and physical loop-string-hadron excitations at each site and their dynamics are described by a Hamiltonian which contains combinations of occupation number operator and ladder operators for the associated loop-string-hadron excitations.

2.1 LSH basis

The construction of a basis is essential to perform Hamiltonian calculations, while the choice is not unique. In the context of lattice gauge theories, a strong coupling basis or eigenbasis of the electric part of the Hamiltonian is a canonical choice. The gauge theory Hilbert space is characterized by the tensor products of on-link rigid rotors and on-site fermionic state across all the links and sites of the lattice. The strong coupling vacuum $|0\rangle$ is such a state which corresponds to the lowest eigenvalue for electric and mass part of the Hamiltonian. Irreducible representation (irrep) of the gauge group provides a choice of strong coupling eigenbasis. The irrep basis is not gauge invariant yet the construction of gauge singlets is well understood for SU(2). Whereas for SU(3), irreps are more complicated and the direct products of SU(3) irreps exhibit multiplicities characterized by the Littlewood Richardson coefficients. Constructing a singlet for SU(3) by combining irreps appears to be a notorious task and is beyond the capability of near-term quantum hardware. The LSH basis is constructed for SU(2) [28] and SU(3) [29] and the key features are highlighted below.

The LSH basis is defined locally at each lattice site, which is characterized by a set of integer-valued quantum numbers, namely the loop and string quantum numbers. Loop quantum numbers are bosonic, implying that the Hilbert space has an infinite dimension, while string quantum numbers are fermionic. However, for practical computation, one needs to impose a finite cut-off on the bosonic quantum number so that the Hilbert space has a finite dimension.

For SU(2) gauge theory, the LSH basis states at a site r in 1 + 1-d is given by

$$|n_l, n_i, n_o\rangle_x \tag{6}$$

, where $n_l \in \{0, \infty\}$ is the loop quantum number and $n_i, n_o \in \{0, 1\}$ are fermionic quantum numbers. Imposing a cut-off Λ_B, one would obtain an onsite Hilbert space of dimension $2^2(\Lambda + 1)$.

For the case of SU(3) gauge theory defined on a one-dimensional spatial lattice, the on-site Hilbert space is characterized by two loop and three string quantum numbers as

$$|n_P, n_Q, \nu_{\underline{1}}, \nu_0, \nu_1\rangle_x, \tag{7}$$

where, $n_P, n_Q \in \{0, \infty\}$ and $\nu_{\underline{1}}, \nu_0, \nu_1 \in \{0, 1\}$. Pictorial representations of the LSH states are given in Fig. 1. For a bosonic cut-off Λ_B, the dimension of the on-site LSH Hilbert space is $2^3(\Lambda_B + 1)^2$.

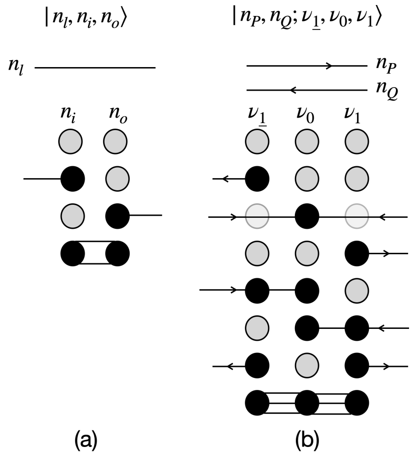

Figure 1: On-site LSH state for (a) SU(2) and (b) SU(3) gauge theories in 1 + 1 dimension. Note that the fermionic excitations are denoted as circles, which can be either empty or filled denoting the corresponding fermionic occupation numbers to be 0 and 1 respectively. Within the LSH framework, the fermionic occupation numbers are associated with incoming and outgoing strings in different directions.

2.2 LSH Operators

In the LSH framework, one can define on-site occupation number operators for the LSH basis both for the SU(2) and SU(3) case, and the onsite contribution of electric and mass term is obtained as a function of occupation numbers of the LSH excitations at that site:

$$\hat{n}_l|\{n_l\},\{n_f\}\rangle = n_l|\{n_l\},\{n_f\}\rangle \quad , \quad \hat{n}_f|\{n_l\},\{n_f\}\rangle = n_f|\{n_l\},\{n_f\}\rangle \tag{8}$$

where $\{n_l\}$ denotes the set of bosonic quantum numbers, i.e. n_l for SU(2) and n_P & n_Q for SU(3).

We also consider the following set of normalized ladder operators, which acting on an on-site LSH state, changes its excitations by ± 1 unit.

$$\hat{\Gamma}^{\dagger}_L|\{n_l\},\{n_f\}\rangle = |\{n_l|n_L+1\},\{n_f\}\rangle \tag{9}$$

$$\hat{\chi}^{\dagger}_F|\{n_l\},\{n_f\}\rangle = (1-\delta_{1,n_F})|\{n_l\},\{n_f|n_F+1\}\rangle \tag{10}$$

$$\hat{\Gamma}_L|\{n_l\},\{n_f\}\rangle = |\{n_l|n_L-1\},\{n_f\}\rangle \tag{11}$$

$$\hat{\chi}_F|\{n_l\},\{n_f\}\rangle = (1-\delta_{0,n_F})|\{n_l\},\{n_f|n_F-1\}\rangle \tag{12}$$

The notation $\{n_l|n_L\pm 1\}$ and $\{n_f|n_F\pm 1\}$ within the ket denotes that only the specific bosonic (L) or fermionic (F) LSH quantum number has changed, while keeping all other in the set fixed.

2.3 Hamiltonian represented in LSH basis

As in the Kogut-Susskind Hamiltonian, the LSH Hamiltonian in $1+1$-dimension contains three terms H_E, H_M, H_I which while represented in the LSH basis are reexpressed in terms of the LSH occupation numbers and ladder operators. Note that, the LSH basis is a strong coupling eigenbasis, hence H_E and H_M are solely expressed in terms of LSH number operators. The dynamics of LSH states are governed by the matter-gauge interaction Hamiltonian H_I, which contains particular combinations of LSH ladder operators. We discuss the symmetries of LSH Hamiltonian in the next section. Below each term of the Hamiltonian for SU(2) and SU(3) gauge theory is presented.

$$H_E^{SU(2)LSH} = \frac{g^2 a}{2}\sum_x \frac{\hat{N}_L}{2}\left(\frac{\hat{N}_L}{2}+1\right) \tag{13}$$

$$H_E^{SU(3)LSH} = \frac{g^2 a}{2}\sum_r \frac{1}{3}\left(\hat{P}_1^2+\hat{Q}_1^2+\hat{P}_1\hat{Q}_1\right)+\hat{P}_1+\hat{Q}_1, \tag{14}$$

where the on-site operators are:

$$\text{for SU(2):}\quad \hat{N}_L = \hat{n}_l+\hat{n}_o(1-\hat{n}_i) \tag{15}$$

$$\text{for SU(3):}\quad \hat{P}_1 = \hat{n}_P+\hat{\nu}_1\,(1-\hat{\nu}_0)\,,\ \ \&\ \hat{Q}_1 = \hat{n}_Q+\hat{\nu}_0(r)\left(1-\hat{\nu}_{\underline{1}}\right) \tag{16}$$

The mass terms of the Hamiltonians are also given by LSH (only fermionic) occupation numbers as:

$$H_M^{SU(2)LSH} = \sum_x(-1)^x\,(\hat{n}_i+\hat{n}_o) \tag{17}$$

$$H_M^{SU(3)LSH} = \sum_x(-1)^x\left(\hat{\nu}_{\underline{1}}+\hat{\nu}_0+\hat{\nu}_1\right) \tag{18}$$

For a $1+1$ dimensional theory, the only term responsible for dynamics is the hopping or 'matter-gauge interaction' term H_I. The construction of this interaction Hamiltonian H_I for SU(3) is nontrivial, compared to its SU(2) counterpart as evident in the detailed construction given in [29]. This is because matter content at an SU(3) site contains a much more detailed inner structure than just being present as string end operators for the SU(2) case. This can be qualitatively appreciated from FIG. 1. The interaction Hamiltonian contains on-link contributions:

$$H_I = \frac{1}{2a} \sum_x h_I(x, x+1) + \text{h.c.} \tag{19}$$

For SU(2) theory, the $H_I(x, x+1)$ is expressed in terms of the string-end operators at neighbouring sites given by:

$$h_I(x, x+1)^{SU(2)LSH} = \frac{1}{\hat{N}_L(x)+1} \left[\sum_{\sigma=\pm} \mathcal{S}^{+,\sigma}_{\text{out}}(x) \mathcal{S}^{\sigma,-}_{\text{in}}(x+1) \right] \frac{1}{\hat{N}_L(x)+1} \tag{20}$$

where the string end operators are given by

$$\mathcal{S}^{++}_{\text{in}} = \hat{\chi}^\dagger_i \, (\hat{\Gamma}_l{}^\dagger)^{\hat{n}_o} \sqrt{\hat{n}_l + 2 - \hat{n}_o} \quad , \quad \mathcal{S}^{--}_{\text{in}} = \hat{\chi}_i \, (\hat{\Gamma}_l)^{\hat{n}_o} \sqrt{\hat{n}_l + 2(1 - \hat{n}_o)} \tag{21}$$

$$\mathcal{S}^{++}_{\text{out}} = \hat{\chi}^\dagger_o \, (\hat{\Gamma}_l{}^\dagger)^{\hat{n}_i} \sqrt{\hat{n}_l + 2 - \hat{n}_i} \quad , \quad \mathcal{S}^{--}_{\text{out}} = \hat{\chi}_o \, (\hat{\Gamma}_l)^{\hat{n}_i} \sqrt{\hat{n}_l + 2(1 - \hat{n}_i)} \tag{22}$$

$$\mathcal{S}^{-+}_{\text{in}} = \hat{\chi}^\dagger_o \, (\hat{\Gamma}_l)^{1-\hat{n}_i} \sqrt{\hat{n}_l + 2\hat{n}_i} \quad , \quad \mathcal{S}^{+-}_{\text{in}} = \hat{\chi}_o \, (\hat{\Gamma}_l{}^\dagger)^{1-\hat{n}_i} \sqrt{\hat{n}_l + 1 + \hat{n}_i} \tag{23}$$

$$\mathcal{S}^{+-}_{\text{out}} = \hat{\chi}^\dagger_i \, (\hat{\Gamma}_l)^{1-\hat{n}_o} \sqrt{\hat{n}_l + 2\hat{n}_o} \quad , \quad \mathcal{S}^{-+}_{\text{out}} = \hat{\chi}_i \, (\hat{\Gamma}_l{}^\dagger)^{1-\hat{n}_o} \sqrt{\hat{n}_l + 1 + \hat{n}_o} \tag{24}$$

For SU(3) gauge theories, a representation of H_I in LSH basis is obtained via a series of analyses presented in detail in ref. [29]. The inner structure of gauge invariant dynamics for this theory is captured by the following structure:

$$\begin{aligned} h_I(x, x+1)^{SU(3)LSH} \equiv & \left[\hat{\chi}^\dagger_1 (\hat{\Gamma}^\dagger_P)^{\hat{\nu}_0} \sqrt{1 - \hat{\nu}_0/(\hat{n}_P + 2)} \sqrt{1 - \hat{\nu}_{\underline{1}}/(\hat{n}_P + \hat{n}_Q + 3)} \right]_r \\ & \otimes \left[\sqrt{1 + \hat{\nu}_0/(\hat{n}_P + 1)} \sqrt{1 + \hat{\nu}_{\underline{1}}/(\hat{n}_P + \hat{n}_Q + 2)} \, \hat{\chi}_1 (\hat{\Gamma}^\dagger_P)^{1-\hat{\nu}_0} \right]_{r+1} \\ & + \left[\hat{\chi}^\dagger_{\underline{1}} (\hat{\Gamma}_Q)^{1-\hat{\nu}_0} \sqrt{1 + \hat{\nu}_0/(\hat{n}_Q + 1)} \sqrt{1 + \hat{\nu}_1/(\hat{n}_P + \hat{n}_Q + 2)} \right]_r \\ & \otimes \left[\sqrt{1 - \hat{\nu}_0/(\hat{n}_Q + 2)} \sqrt{1 - \hat{\nu}_1/(\hat{n}_P + \hat{n}_Q + 3)} \, \hat{\chi}_{\underline{1}} (\hat{\Gamma}_Q)^{\hat{\nu}_0} \right]_{r+1} \\ & + \left[\hat{\chi}^\dagger_0 (\hat{\Gamma}_P)^{1-\hat{\nu}_1} (\hat{\Gamma}^\dagger_Q)^{\hat{\nu}_{\underline{1}}} \sqrt{1 + \hat{\nu}_1/(\hat{n}_P + 1)} \sqrt{1 - \hat{\nu}_{\underline{1}}/(\hat{n}_Q + 2)} \right]_r \\ & \otimes \left[\sqrt{1 - \hat{\nu}_1/(\hat{n}_P + 2)} \sqrt{1 + \hat{\nu}_{\underline{1}}/(\hat{n}_Q + 1)} \, \hat{\chi}_0 (\hat{\Gamma}_P)^{\hat{\nu}_1} (\hat{\Gamma}^\dagger_Q)^{1-\hat{\nu}_{\underline{1}}} \right]_{r+1} + \text{h.c.}, \end{aligned} \tag{25}$$

The next section spells out the symmetries, using the complete structure of the Hamiltonian on an LSH basis.

3. Remnant Abelian gauge invariance

The LSH basis states, defined locally at neighboring lattice sites are required to satisfy constraints to be mapped to on-site segments of the non-local Wilson loop or string states of the theory.

This is depicted in Fig. 1, where each on-site configuration is associated with open flux lines at both ends. A global LSH state is a tensor product of on-site LSH states:

$$|\Psi_{LSH}\rangle = \prod_x |\{n_l\}, \{n_F\}\rangle_x$$

However, a valid physical state for SU(2) gauge theory must satisfy

$$\left[\left(\hat{n}_l(x) + \hat{n}_o(x)(1-\hat{n}_i(x))\right) - \left(\hat{n}_l(x+1) + \hat{n}_i(x+1)(1-\hat{n}_o(x+1))\right)\right]|\Psi_{SU(2)LSH}\rangle = 0 \;\; \forall x, \tag{26}$$

in order to guarantee the continuity of flux lines across neighbouring lattice sites. This is denoted as the Abelian Gauss law constraint for SU(2) LSH framework. It is worth noting that this constraint was also present in the original Kogut-Susskind Hamiltonian yielding rigid rotors state to describe the gauge theory Hilbert space.

As opposed to undirected flux lines for SU(2), the flux lines are directed in the SU(3) case as depicted in Fig. 1. This results in the following set of two Abelian Gauss law constraints to be satisfied by a global physical state of SU(3) gauge theory:

$$\left[\hat{P}_1(x) - \hat{P}_{\underline{1}}(x+1)\right]|\Psi_{SU(3)LSH}\rangle = 0 \;\; \& \;\; \left[\hat{Q}_1(x) - \hat{Q}_{\underline{1}}(x+1)\right]|\Psi_{SU(3)LSH}\rangle = 0 \;\; \forall x, \tag{27}$$

where, $\hat{P}_{\underline{1}} = \hat{n}_P + \hat{\nu}_0\,(1-\hat{\nu}_1)\,,\;\; \hat{Q}_{\underline{1}} = \hat{n}_Q + \hat{\nu}_{\underline{1}}\,(1-\hat{\nu}_0)$. The LSH Hamiltonian presented in the last section commutes with all of the Abelian Gauss law constraints. Dynamics within the physical Hilbert space are also manifested in the explicit presence of only $\hat{N}_L$ in (13,20) and $\hat{P}_1, \hat{Q}_1$ in (14, 25).

4. Conserved Global Charges

The electric and mass Hamiltonian, being diagonal in the LSH basis hence presents the integrable part of the theory. However, the matter-gauge interaction Hamiltonian H_I presented for SU(2) and SU(3) gauge theories in one spatial dimension restricts the dynamics even in smaller subspaces of the physical state space. The immediate consequence is a block diagonal structure of the Hamiltonian where each block is of much smaller size than the entire physical Hilbert space dimension and the dynamics in each block can be computed parallely. Below we highlight the global conserved charges of the system.

SU(2) case: The H_I apparently conserves the bare fermion number - while accommodating particle creation and annihilation dynamics within the framework of staggered fermions. A careful inspection of (20) yields that the summation of sigma indices follows the Abelian Gauss laws and hence results in a particular combination of the LSH creation operators $\hat{\chi}_{i/o}$ and hence yield $\sum_x n_i(x)$ & $\sum_x n_o(x)$ as the conserved charges of the theory. One can also define global charges

$$Q = \sum_x \left(n_i(x) + n_o(x)\right) \;\; \& \;\; q = \sum_x \left(n_i(x) - n_o(x)\right) \tag{28}$$

For a N staggered site lattice with open boundary condition, $0 \le Q \le 2N$ and $-N \le q \le N$. Hence each block of the Hamiltonian is characterized by a set of global quantum numbers (q, Q). It is also important to note that the blocks are identical for $q \to -q$, yielding a huge saving in computational cost in studying the system via Hamiltonian simulation. For periodic boundary conditions, for all values of Q, it is always $q = 0$. Note that, q precisely give the imbalance of incoming and outgoing flux lines for the lattice, which has to be zero for a valid loop/ physical state on periodic lattice. With periodic boundary conditions, the emergent discrete symmetries due to translation invariance yield a block diagonal structure of the Hamiltonian.

SU(3) case: Albeit the LSH construction is involved for SU(3), the final interaction Hamiltonian given in (25) is of identical structure to that of SU(2) and conserves the global charges

$$q_{\underline{1}} = \sum_x \nu_{\underline{1}}(x) \ , \ q_0 = \sum_x \nu_0(x) \ \& \ q_1 = \sum_x \nu_1(x).$$

A linear combination of these charges denote the imbalance between incoming and outgoing flux lines in each direction, which are given by $\mathcal{P} = q_1 - q_0$ & $\mathcal{Q} = q_0 - q_{\underline{1}}$, together with $\mathcal{F} = q_0 + q_1 + q_{\underline{1}}$. Thus the blocks of the Hamiltonian matrix are characterized by $(q_0, q_1, q_{\underline{1}})$ or $(\mathcal{F}, \mathcal{P}, \mathcal{Q})$. Unlike SU(2) case, the connection of these global symmetries to invariance under local Abelian Gauss laws is not evident in the Hamiltonian construction but is reflected in numerical simulations presented in [60].

5. Consequences in quantum simulation strategies and calculating dynamics

Unveiling the global symmetry structures is of many-fold use for quantum simulation. Primarily, using the LSH framework allows one not to worry about imposing a non-Abelian symmetry in a quantum simulation algorithm or analog simulation protocol - an advantage towards simulating QCD. An efficient quantum simulation would require an efficient symmetry protection protocol, which for any gauge theory is difficult. LSH framework in 1 + 1-d allows connecting all of its local symmetries to global conserved charges while preserving locality for both SU(2) [61] and SU(3) [60]. Resolving all the global symmetries of the theory is crucial to finding a true answer to the quest of whether the theory thermalizes or not [62, 63] and also to explore if there is any quantum scar [64] present for QCD-like theories. The same questions remain valid in higher dimensions. Fully understanding the global symmetries of higher dimensional SU(2) gauge theories is in progress and will be reported soon. Understanding discrete symmetries of 1 + 1-d SU(3) LSH framework with a periodic boundary condition is also aimed at a future study while the SU(3) higher dimensional framework develops. Works are in progress to deliver a useful framework and algorithms towards quantum computing for the full QCD in the foreseeable future.

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