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B dependence of the QED chiral condensate induced by an external magnetic field.

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We study the non-perturbative behaviour of QED in a strong external magnetic field using Rational Hybrid Monte Carlo (RHMC) simulations of lattice QED. The RHMC method was developed for simulating lattice QCD. Previously we showed that the magnetic field breaks chiral symmetry in the limit of zero bare mass as had been predicted by Schwinger-Dyson methods. We are now simulating at weaker magnetic fields to test that the chiral condensate scales as $(eB)^{3/2}$ as predicted, where B is the constant homogeneous external magnetic field and e is the electric charge. We first chose a much weaker magnetic field. While we saw evidence that chiral symmetry was still broken, it became clear that to measure the zero bare mass dependence of the chiral condensate on eB would require use of much larger lattices than we can manage. We are therefore now performing simulations at a magnetic field with an intermediate strength.

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1. Introduction

Much of the physics of QED can be obtained using perturbation theory. However, there are situations where QED exhibits non-perturbative behaviour. The normal approach to such non-perturbative behaviour is to sum infinite sets of Feynman diagrams. One such method of doing this is the Schwinger-Dyson approach [1–3] which uses identities that such diagrams obey. However, to do this one needs to truncate to a finite number of such identities, which requires assumptions that are not necessarily valid. Instead, we are using Lattice Gauge Theory simulation techniques which were developed for QCD. These use the functional integral method of quantizing this field theory and rotate to imaginary time where the integrand for the theory of interest is real and bounded below and the integral can be evaluated by importance sampling. This requires defining the integrand at a countable set of space-time points, which we select to lie on a lattice, and in a finite space-time volume to make the integrals finite dimensional. The particular simulation method we use is the RHMC method developed for Lattice QCD [4].

The problem we are studying at present is QED in a (strong) external magnetic field. For simplicity we choose a magnetic field which is constant in space and time and oriented in the $+z$ direction. This theory does have a real euclidean action, which is bounded below, as required. Schwinger-Dyson analysis of this theory predicts that the presence of the magnetic field breaks the chiral symmetry at bare mass $m = 0$ giving a dynamical mass to the electron proportional to $\sqrt{eB}$ [5–15] and a chiral condensate $\langle\bar{\psi}\psi\rangle \propto (eB)^{3/2}$ [16, 17]. We have shown previously that for a relatively strong eB, lattice QED does have a non-zero chiral condensate as $m \to 0$ [18]. What we are now trying to show by repeating our simulations at a weaker eB, is that the condensate at this lower eB is consistent with $\langle\bar{\psi}\psi\rangle \propto (eB)^{3/2}$.

We simulate lattice QED on an $N_1 \times N_2 \times N_3 \times N_4$ euclidean lattice with periodic boundary conditions using the RHMC method [4] with a non-compact gauge field action and a compact electron action [18]. The non-compact gauge action is obtained by replacing the derivatives in the continuum gauge action by finite differences. It is therefore quadratic in the gauge field and hence is a free field theory, as is the continuum gauge action. Thus it lacks the non-trivial phase structure of the compact gauge action. It is manifestly gauge invariant. Note that it only involves the dynamic gauge field and not the external gauge field. The fermion action is obtained from the non-interacting staggered electron action by multiplying the fermion fields by factors of $\exp(A^\mu + A_{ext}{}^\mu)$ where A is the dynamic gauge field and A_{ext} is the external gauge field with $e\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}_{ext}$. Since one staggered fermion field represents four continuum fields, we need to replace $\det[\mathbf{D} + m]$ with $\det([\mathbf{D} + m][\mathbf{D} + m]^\dagger)^{1/8}$ where the fractional powers of the positive definite Dirac matrices are replaced with rational approximations. The external gauge field is in the symmetric gauge so that the magnetic field through all but one (12) plaquette in each (12) plane is B, while that through the remaining plaquette is $(1 - N_1 N_2)B$, so that the total flux through each 12 plane is zero. However, if $N_1 N_2 eB = 2\pi n$, where n is an integer with $0 \leq n \leq N_1 N_2/2$, for the compact fermion action $(1 - N_1 N_2)B$ is equivalent to B. So, we need to restrict eB to $eB = 2\pi n/(N_1 N_2)$. For more details, see [18]

For our original simulations with bare fine structure constant $\alpha = 1/5$, we chose $eB = 2\pi \times 100/36^2 = 2\pi \times 25/18^2 = 0.4848\ldots$. Classically, electrons/positrons in such an external magnetic field traverse helical orbits around magnetic field lines. Quantum mechanically, while

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the motion along these field lines is free, that in the plane perpendicular to the magnetic field is quantized, and the energy levels – the Landau levels – have energies:

$$E_n(p_z) = \pm\sqrt{m^2 + 2eBn + p_z^2} \tag{1}$$

Hence for such strong magnetic fields one expects the partition function to be dominated by the lowest Landau level (LLL), $n = 0$, which has radius $\sim 1/\sqrt{eB} \approx 1$, choosing $N_1 = N_2 = 36$ or $N_1 = N_2 = 18$ should be adequate. For our weaker external magnetic field we chose $eB = 2\pi \times 24/36^2 = 2\pi \times 6/18^2 = 0.1163...$, where the radius of the LLL is ≈ 2. As we shall see, while our simulations show evidence that the chiral condensate at small m is dominated by low momentum modes, which suggests that chiral symmetry is broken, we do not have enough data to estimate the chiral condensate at $m = 0$, especially if the condensate is what $(eB)^{3/2}$ scaling predicts. At present, with the data we have, it would be difficult to rule out a zero chiral condensate in the $m = 0$ limit. To extract the $m = 0$ chiral condensate, assuming it is non zero, would require much larger lattices than those we have been using, which would require considerably more computer resources than we can expect to have in the near future. That we need larger lattices is also indicated by the fact that the increase in the chiral condensate between that measured on $18^2 \times 128^2$ lattices and that measured on $18^2 \times 160^2$ lattices is sizable, at both $m = 0.0005$ and $m = 0.001$.

Because testing $(eB)^{3/2}$ scaling at this much weaker external magnetic field is beyond our capabilities, we are now performing simulations with an intermediate strength magnetic field $eB = 2\pi \times 64/36^2 = 2\pi \times 16/18^2 = 0.3102...$, also at $\alpha = 1/5$. Very preliminary results appear promising.

2. Simulations at $\alpha = 1/5$, $eB = 0.1163...$

Our earlier simulations at $\alpha = 1/5$, $eB = 0.4848...$, with electron bare masses as low as $m = 0.001$ showed evidence of chiral symmetry breaking with a chiral condensate extrapolated to $m = 0$ of $\langle\bar{\psi}\psi\rangle = 3.5(5) \times 10^{-3}$. We have since been performing simulations at a much weaker external magnetic field, $eB = 0.1163....$ Since, at this magnetic field, and in the limit $m = 0$, the radius of the LLL is approximately 2 we believe that we can fix $N_1 = N_2 = 36$ or $N_1 = N_2 = 18$ when performing a finite size scaling analysis. Hence we only vary $N_3 = N_4$ in performing such an analysis. In particular, we increase $N_3 = N_4$ until further increases no longer produce an increase in the chiral condensate. Figure 1 shows the present status of such a finite size scaling analysis at $eB = 0.1163....$ What we notice is that at bare masses as low as $m = 0.0005$ and $m = 0.001$ there is still a significant increase in the chiral condensate in going from an $18^2 \times 128^2$ lattice to an $18^3 \times 160^2$ lattice. While this does indicate the presence of low momentum modes which could produce chiral symmetry breaking, we notice that the values of the chiral condensates on $18^3 \times 160^2$ lattices do not appear to be consistent with the value of the $m = 0$ chiral condensate predicted from that at $eB = 0.4848...$ assuming that it should scale as $(eB)^{3/2}$, which we have included in figure 1. This means that we probably need lattices with much larger $N_3 = N_4$, which is not possible in the near future. Note that, at the very least, the increases in the condensates from $N_3 = N_4 = 128$ to $N_3 = N4 = 160$ would require simulations with $N_3 = N_4 > 160$.

We are therefore now simulating at an intermediate strength external magnetic field. We have chosen $eB = 2\pi \times 64/36^2 = 2\pi \times 16/18^2 = 0.3102...$ where the radius of the LLL is ≈ 1.25 and

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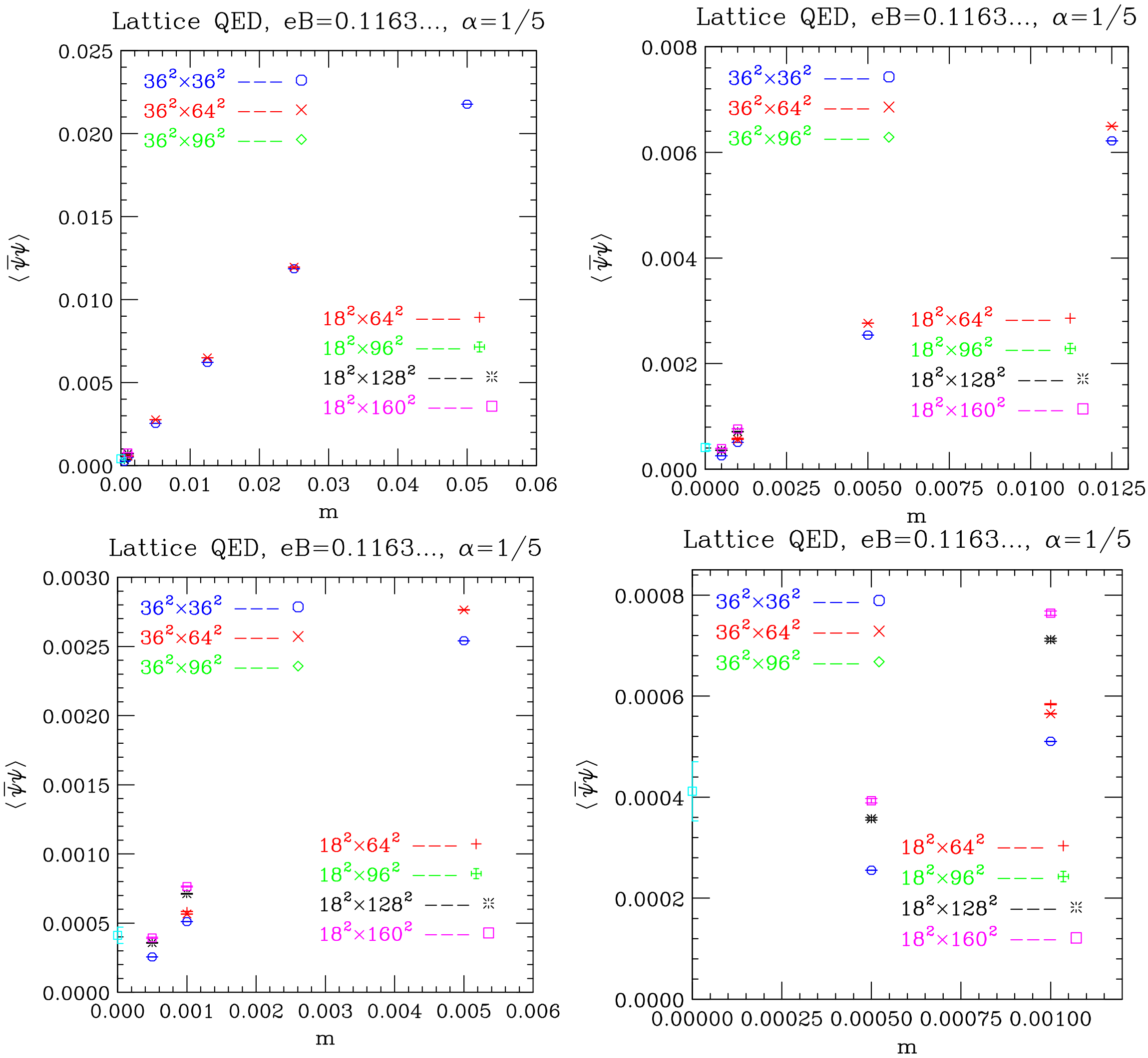

Figure 1: The chiral condensate $\langle\bar{\psi}\psi\rangle$ as a function of m and lattice sizes for $eB = 0.1163....$ The 4 parts are the same data for different m ranges. The cyan strip at $m = 0$ centered just above 0.0004 is the prediction from our measurements and extrapolation at $eB = 0.4848...$ assuming $|eB|^{3/2}$ scaling.

the $m = 0$ chiral condensate is expected to be around half that at the stronger magnetic field. Initial simulation results at this external magnetic field look promising.

3. Discussion and Conclusions

We are continuing our simulations of Lattice QED in strong external magnetic fields using the RHMC algorithm developed for lattice QCD. Our goal is to study the non-perturbative effects produced by these magnetic fields. In particular, we are studying the chiral symmetry breaking produced by these fields. These effects were named magnetic catalysis by those who predicted them from Schwinger-Dyson analyses.

In our earlier work, we had established that such an effect exists from lattice simulations for one external field value. We are now attempting to measure the zero-mass chiral condensate for a

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weaker magnetic field. At our first attempt at a much weaker external field, we determined that we would need much larger lattices to obtain a result, despite the fact that we could see signs that chiral symmetry was broken at $m = 0$. We are now performing simulations at an intermediate strength external magnetic field where we can already see signs that the $m = 0$ chiral condensate is non-zero. So, we are cautiously optimistic that we will be able to test the predicted $(eB)^{3/2}$ scaling of this condensate.

So far, the chiral condensate we are measuring is the bare chiral condensate, whereas the Schwinger-Dyson results are for a renormalized chiral condensate, so a direct comparison is not yet possible. We plan to investigate the renormalization of lattice QED to make a first estimate of the renormalized chiral condensate, so that we can make a direct comparison with the Schwinger-Dyson estimate.

We plan to use our stored configurations to measure other effects that are produced by external magnetic fields such as partial screening and distortion of the coulomb field of a point charge placed in a strong external magnetic field [19–22].

Because some of the more interesting effects of QED in external electromagnetic fields such as the Sauter-Schwinger effect [23, 24], (electron-positron pair production in a (strong) electric field) occur with external electric fields, we need to understand whether there is any way to approach problems with complex actions. Here there are some indications that the complex langevin approach or other modifications of one of the importance sampling methods could work, unlike the case of QCD at finite quark-number chemical potential. One piece of evidence that lattice QED, at least with our choice of lattice actions might be different from lattice QCD in this regard is that even in the absence of fermions, pure gauge QCD is an unstable fixed point of the complex langevin, whereas, with the non-compact pure gauge action, lattice QED without fermions is quadratic in the gauge fields and just a sum of harmonic oscillators and therefore a stable fixed point of the complex langevin.

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