PROCEEDINGS OF SCIENCE

Chiral and deconfinement properties of the QCD crossover have a different volume and baryochemical potential dependence

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The crossover from hadronic to quark matter is understood to be both a deconfinement as well as a chiral symmetry restoring transition. Here, we study observables related to both aspects using lattice simulations: the Polyakov loop and its derivatives and the chiral condensate and its derivatives. At zero baryochemical potential, and infinite volume, the chiral and deconfinement crossover temperatures almost agree. However, chiral and deconfinement related observables have a qualitatively different chemical potential and volume dependence. In general, deconfinement related observables have a milder volume dependence. Furthermore, while the deconfinement transition appears to get broader with increasing μ_B , the width as well as the strength of the chiral transition is approximately constant. Our results are based on simulations at zero and imaginary chemical potentials using 4stout-improved staggered fermions with $N_\tau = 12$ time-slices and physical quark masses.

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1. Introduction

An active field of research is the search for the Critical End Point (CEP) in the (T, μ_B) phase diagram of QCD: that would be the point where the crossover transition between the hadron gas and the quark gluon plasma phase turns into a first order transition [1–5]. A symmetry that is related to this transition is the chiral symmetry, whose order parameter in the limit of vanishing quark mass is the chiral condensate. For this reason a possible way to identify the transition is to check the volume dependence of the chiral susceptibility. The height and inverse width of the chiral susceptibility is expected to scale with the volume in the case of a first order transition.

Another symmetry that distinguishes the two phases is the Z_3 symmetry in the limit of infinite quark masses. In that case the order parameter is the Polyakov loop P, related to the static quark free energy F_Q to remove a quark from a system.

Our goal is then to study the volume dependencies of two groups of observables, related respectively to the chiral and the Z_3 symmetry, both at $\mu_B = 0$ and at finite μ_B. The observables are introduced in section 2 in detail. The results at $\mu_B = 0$ are shown in 3, while those at finite μ_B are in section 4. This contribution is a shortened version of the full work in Ref. [6].

The results have been obtained with 2+1+1 4stout-improved staggered fermions at physical values of the quark masses. The time extension of the lattice is always $N_t = 12$. Concerning the spatial direction, we have $N_s = 20, 24, 28, 32, 40, 48, 64$ for $\mu_B = 0$; for $N_s = 32, 40, 48$ we simulated also at the imaginary chemical potentials $\mathrm{Im}\frac{\mu_\mathrm{B}}{\mathrm{T}}\frac{\pi}{8} = 3, 4, 5, 6, 6.5, 7$. We are also in a strangeness neutrality setup. More information on the simulations can be found in [7].

2. Definitions

In this section we introduce the observables of interest, both the ones related to the chiral and those to the Z_3 symmetry. We plot them for different volumes, at $\mu_B = 0$.

In the limit of vanishing quark mass the order parameter of the chiral symmetry is the chiral condensate:

$$< \bar{\psi}\psi >= \frac{T}{V}\frac{\partial \log Z}{\partial m_{ud}}. \tag{1}$$

The full and the disconnected chiral susceptibilities are defined as follows:

$$\chi = \frac{T}{V}\frac{\partial^2 \log Z}{\partial m_{ud}^2} \qquad \chi_\mathrm{disc} = \frac{T}{V}\left(\frac{\partial^2 \log Z}{\partial m_u \partial m_d}\right)_{m_u = m_d} \tag{2}$$

All these quantities need both a multiplicative and additive renormalization:

$$< \bar{\psi}\psi >^R = -\frac{m_{ud}}{f_\pi^4}[< \bar{\psi}\psi >_T - < \bar{\psi}\psi >_{T=0}] \tag{3}$$

$$\chi = \frac{m_{ud}^2}{f_\pi^4}[\chi_T - \chi_{T=0}] \qquad \chi_{disc} = \frac{m_{ud}^2}{f_\pi^4}[\chi_{disc,T} - \chi_{disc,T=0}] \tag{4}$$

A first evaluation of the volume dependencies of (1, 2) can be done by looking at the plots in Fig. 1 We see that the volume effects are smaller for higher temperatures. Looking at the peaks of

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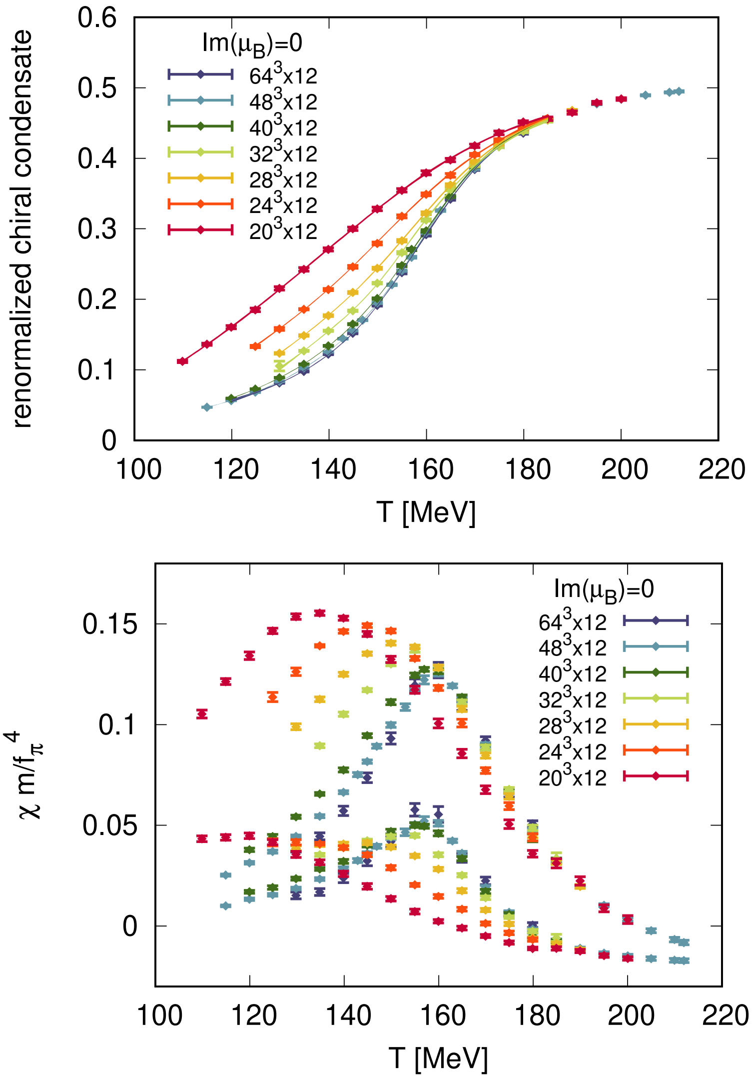


Figure 1: Plots of the chiral condensate (1) and of the chiral susceptibilities (2) at $\mu_B = 0$ for different volumes.

the chiral susceptibilities, we can already expect that the crossover temperature T_c increases with the volume.

The order parameter that is more directly related to the deconfinement transition is the Polyakov loop, that can be computed on the lattice as follows:

$$P(\vec{x}) = \frac{1}{3} \prod_{x_4=0}^{N_t-1} U_4(\vec{x}, x_4). \tag{5}$$

Since $P \sim e^{-F_Q}$, in the infinite quark mass limit we have that the Polyakov loop is 0 for the confined phase and finite otherwise. The static quark free energy can be then computed and additively renormalized:

$$F_Q = -T \log \left(\frac{1}{V} \sum_{\vec{x}} | < P(\vec{x}) >_T | \right) + T_0 \log \left(\frac{1}{V} \sum_{\vec{x}} | < P(\vec{x}) >_{T_0} | \right), \tag{6}$$

where the temperature T_0 for the renormalization is $T_0 = 160$ MeV.

Finding T_c from the inflection point of P would be numerically not easy. More than that, it would be scheme-dependent [8]. For those reason we prefer to use the static quark entropy S_Q defined in [8]. The idea is to interpolate F_Q with a (2,2) rational function fit, then the fitted function is derived in the temperature:

$$S_Q = -\frac{\partial F_Q}{\partial T}. \tag{7}$$

The peak position of S_Q correspond to T_c .

From the plots of F_Q and S_Q (Fig. 2) we immediately see that the volume effects are much milder than for the chiral observables. We can also see that here T_c decreases with the volume.

3. $\mu_B = 0$ results

We compare how the crossover temperatures extracted from the different observables scale with the aspect ratio $LT = N_s/N_t$ in Fig. 3.

We extract T_c from the peak position of the chiral susceptibilities χ, χ_{disc} (2) or from the peak position of the static quark entropy S_Q (7). As an additional check, we see how $T_c(LT)$ changes along curves of fixed values for the chiral condensate $< \bar{\psi}\psi >$ or the static quark free energy F_Q. The fixed values correspond to the infinite volume value at the crossover temperature defined via the full chiral susceptibility.

We observe that T_c from the chiral and the deconfinement observables do not tend for the same limit for larger volumes. Actually, they also have opposite volume dependencies: as could be already be seen from Fig. 1, 2, T_c^{chiral} $(T_c^{(\chi_{\text{disc}})}, T_c^{(\chi)}, T_c^{(<\bar{\psi}\psi>)})$ increases with the volume, while $T_c^{\text{deconfinement}}$ $(T_c^{(S_Q)}, T_c^{(F_Q)})$ decreases. In the latter case the volume effects are milder. For example, the difference between $T_c^{(S_Q)}$ at the largest and at the smallest volume is $O(10\,\text{MeV})$, while for $T_c^{(\chi_{\text{disc}})}$ the same difference is $O(35\,\text{MeV})$.

4. Finite μ_B results

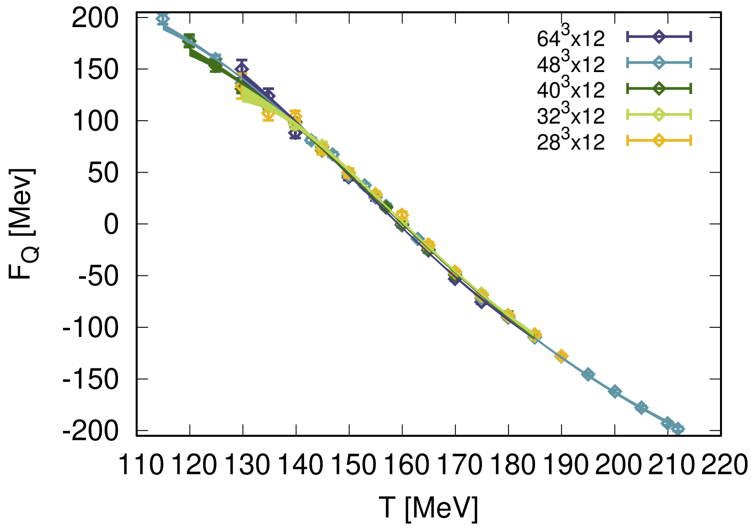
We now investigate the entity of the volume effects for our different definitions of T_c at finite μ_B. To circumvent the sign problem, we perform the simulations at imaginary μ_B. The results are then extrapolated to the real μ_B.

In Fig. 4 we show how $T(\mu_B)$ changes along curves of fixed values for $< \bar{\psi}\psi >$ (curves in the tones of blue) and F_Q (curves in the tone of orange), for the 2 volumes $32^3 \times 12$, $48^3 \times 12$.

That is, for a fixed T^*, we solve

$$< \bar{\psi}\psi > (T(\mu_B), \mu_B) = [< \bar{\psi}\psi > (T^*, 0)], \tag{8}$$

and analogous for F_Q. We see that, while the F_Q-curves overlap for the two different volumes at any T^*, the same holds for $< \bar{\psi}\psi >$ only at the highest temperatures.



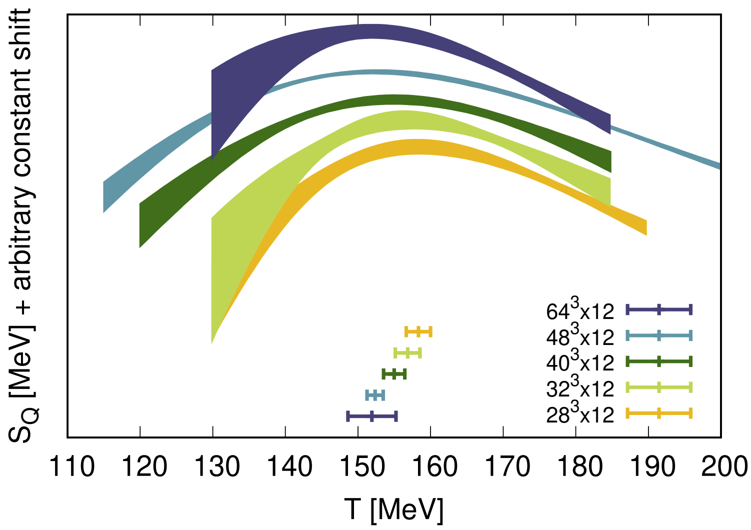


Figure 2: Plots of the static quark free energy (6) and of the static quark entropy (7) at $\mu_B = 0$ for different volumes. The S_Q-curves have been shifted in the vertical direction to avoid their overlapping and making them more visible.

Finally in Fig. 5 we plot the heights of the peaks of $\chi, \chi_{\text{disc}}, S_Q$ for the 3 volumes $32^3 \times 12$, , $40^3 \times 12, 48^3 \times 12$.

In case of a transition, we would expect these quantities to show a rise for larger μ_B^2/T^2 as the volume increases. The only observable that shows such a behaviour is χ_{disc}, while the height of the peak of χ is constant in μ_B^2/T^2 and that of S_Q even decreases. We can then not draw any conclusion on the possibility of a critical end point. However, we can safely observe that the volume effects related to S_Q are much milder than those of χ, χ_{disc}.

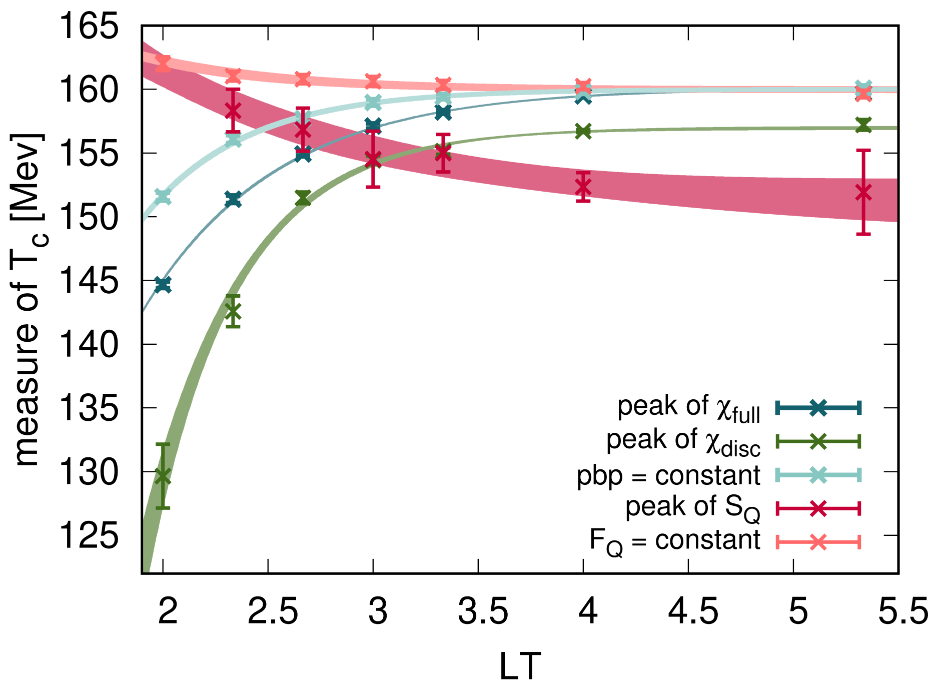


Figure 3: Volume dependence of the crossover temperature T_c at $\mu_B = 0$ from the different definitions introduced in section 2.

5. Conclusion

We extrapolate the crossover temperature T_c at $\mu_B = 0$ and we check its volume dependence for different definitions, related respectively to the chiral and the deconfinement aspect of the Hadron Gas-QGP transition. We find that the latter have milder volume effects.

We check the volume dependence of the same definitions at finite μ_B, relying on analytical continuation to circumvent the sign problem. We find again that deconfinement observables have milder volume effects. We also plot the height of the peaks of $\chi, \chi_{\rm disc}, S_Q$ versus μ_B^2/T^2 for 3 different volumes. Although the behaviour of the peak of $\chi_{\rm disc}$ is interesting, it is alone not enough to draw any conclusion on a possible critical end point.

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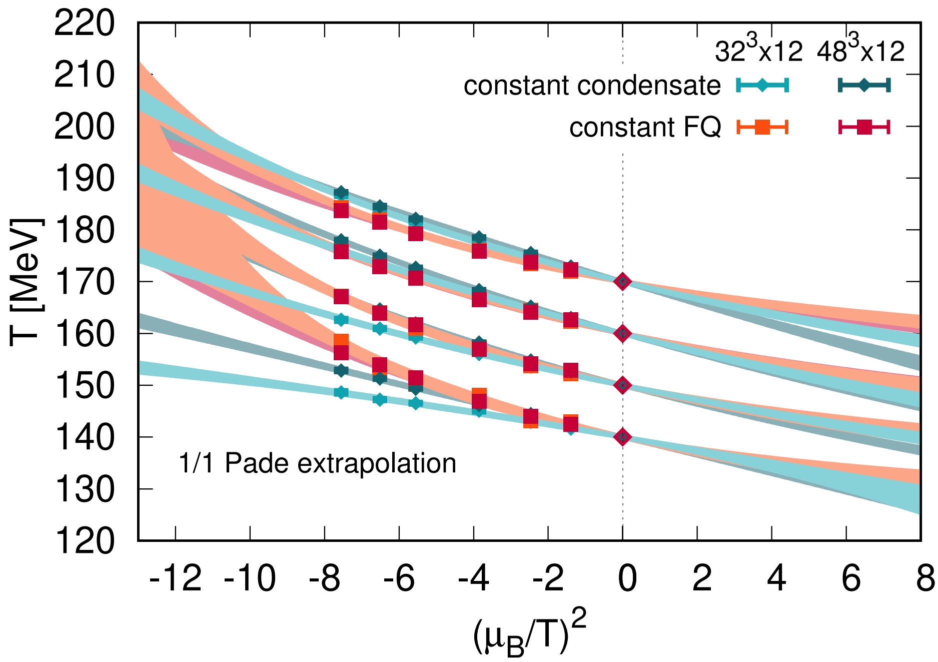

Figure 4: $T(\mu_B)$ curves along fixed values of $<\bar{\psi}\psi>$ and F_Q versus μ_B^2/T^2.

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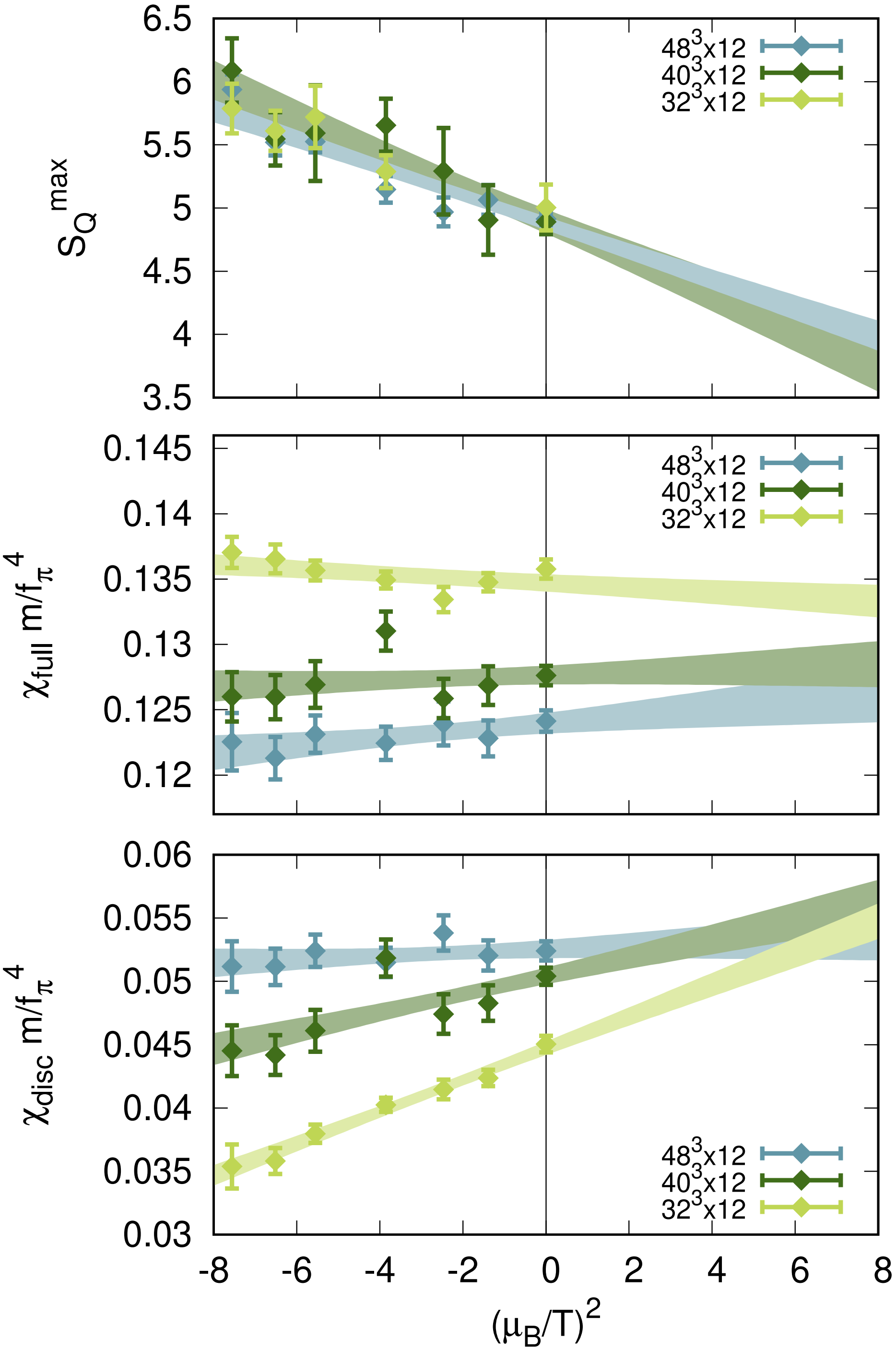

Figure 5: Heights of the peaks of $\chi, \chi_{\text{disc}}, S_Q$ for 3 different volumes.

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