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Soft Theorems and Dilaton Effective Theory

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We discuss model independent single- and double-soft dilaton theorems, taking into account the spacetime dependence of the dilation commutator $[iQ_D, \mathcal{O}(x)] = (\Delta_{\mathcal{O}} + x\cdot\partial)\mathcal{O}(x)$. The procedure restores positivity in the (pseudo)-Goldstone masses and sets the constraint $\Delta_{\mathcal{O}} = d-2$ for an operator $\mathcal{O}$ generating a dilaton mass. We then apply these findings to QCD-like gauge theories where it has been speculated that a dilaton phase might emerge in the chiral limit. It is found that the quark bilinear is of scaling dimension $\Delta_{\bar{q}q} = d-2$, therefore satisfying the soft theorem. We show that some findings are realised in $\mathcal{N}=1$ supersymmetric gauge theories and argue that the extension below the conformal window makes sense in that case. We briefly discuss the infrared implementation of conformal symmetry by the example of gravitational form factors.

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PoS(LATTICE2024)151

1. Introduction

The dilaton in our context is the Goldstone boson due to spontaneous scale symmetry breaking. Originally introduced and developed to describe the strong interactions e.g. [1, 2] it has found numerous application in particle physics and cosmology. Recent interest has been spurred by a light $J^{PC} = 0^{++}$ state emerging in lattice simulations at finite quark mass. We will first review model-independent soft dilaton theorems, verify them in dilaton effective theory before discussing the possibility that gauge theories in the chirally broken phase, including QCD, are described by an infrared fixed point (IRFP) with a dilaton. For the latter, the most prominent features are the extraction of scaling dimensions and impact on gravitational form factors.

2. Model-independent Soft Dilaton Theorem

Soft theorems were part of the current algebra arsenal and assume that a (pseudo) Goldstone is much lighter than the remaining particles (or hadrons) $m_{pG} \ll m_{\text{had}}$. Practically, it amounts to replacing the soft Goldstone by a symmetry transformation.[1] Formulated for the spontaneously broken scale invariance it reads

$$\lim_{q\to 0} \langle D(q)\beta|O(0)|\alpha\rangle = -\frac{1}{F_D}\langle\beta|i[Q_D, O(0)]|\alpha\rangle + \lim_{q\to 0} iq\cdot R + O(m_D^2/m_{\text{had}}^2) , \qquad (1)$$

where R_μ is the remainder which vanish unless the $|\alpha, \beta\rangle$ are degenerate with intermediate states

$$R_\mu = -\frac{i}{F_D}\int d^d z e^{iq\cdot z}\langle\beta|TJ_\mu^D(z)O(0)|\alpha\rangle , \qquad (2)$$

$Q_D = \int d^{d-1}zJ_0^D(z)$ the dilatation charge. The quantity F_D is the dilaton decay constant and order parameter defined from

$$\langle 0|T_{\mu\nu}|D(q)\rangle = \frac{F_D}{d-1}(m_D^2\eta_{\mu\nu} - q_\mu q_\nu) , \qquad (3)$$

with $\eta_{\mu\nu}$ the mostly minus Minkowski metric and $T_{\mu\nu}$ the energy momentum tensor, related by $J_D^\mu = x_\nu T^{\mu\nu}$ to the Goldstone current.[2] The novel element lies in the the application of the dilatation commutator to a primary operator

$$i[Q_D, O(x)] = \frac{1}{F_D}(\Delta_O + x\cdot\partial)O(x) , \qquad (4)$$

for which the $x\cdot\partial$-piece expresses the scale transformation of the argument. In (4) $\Delta_O = d_O + \gamma_O$ is the scaling dimension which differs from the engineering dimension by the anomalous part. In order to appreciate this aspect we first proceed to repeat the textbook derivation of (1) which starts by applying the LSZ procedure

$$\langle D(q)\alpha|O(0)|\beta\rangle = \frac{i}{Z_D}\lim_{q^2\to m_D^2}(m_D^2 - q^2)\int d^d z e^{iq\cdot z}\langle\alpha|T\partial\cdot J^D(z)O(0)|\beta\rangle , \qquad (5)$$

[1] In the language of correlation function this corresponds to the reduction of a n- to a n-1-point function. For example, a scalar-vertex with zero momentum insertion can be replaced by mass-differentiation.

[2] The version for pions of (1) is obtained by replacing $F_D \to F_\pi$, $\langle D| \to \langle\pi^a|$ and $J^D \to J_5^a$.

where $\langle D|\partial \cdot J^D|0\rangle = Z_D$ serves as an interpolating operator for the dilaton. Next one considers the standard Ward identity

$$\partial^\mu \langle \alpha|TJ^D_\mu(z)O|\beta\rangle = \delta(z_0)\langle\alpha|[J^D_0(z), O]|\beta\rangle + \langle\alpha|T\partial \cdot J^D(z)O|\beta\rangle , \tag{6}$$

where the first term on the right corresponds to the symmetry transformation of the operator O and the second one parameterises possible explicit or anomalous symmetry breaking. In the case of the pion the latter corresponds to the partially conserved axial current proportional to the pion mass (explicit breaking). Up to this point everything is exact. The soft theorem assumption is that the matrix element in (5) changes smoothly

$$\langle D(q)\alpha|O(0)|\beta\rangle = \lim_{q\to 0}\langle D(q)\beta|O(0)|\alpha\rangle|_{(1)} + O(m_D^2/m_{\text{had}}^2) , \tag{7}$$

as the Goldstone is made soft $q \to 0$ with which one immediately recovers the soft theorem (1). Note that Eq. (7) gives meaning to the statement of an "off-shell matrix element". We now proceed to apply the soft theorem using (4) for the two special cases $\langle D|O|0\rangle$ and $\langle D|O|D\rangle$.

Single-soft theorem: evaluating $\langle D|O|0\rangle$ is straightforward as the x-dependence in (4) drops one, one gets, with $\langle O\rangle \equiv \langle 0|O|0\rangle$,

$$\langle D|O|0\rangle = -\frac{1}{F_D}\Delta_O\langle O\rangle . \tag{8}$$

Double-soft theorem: the evaluation of $\langle D(q)|O|D(q')\rangle$ proceeds in steps. Similarly, we get

$$\langle D(q)|O(x)|D(0)\rangle = -\frac{1}{F_D}(\Delta_O + x\cdot\partial)\langle D(q)|O(x)|0\rangle , \tag{9}$$

but this time the derivative cannot be neglected since $\langle D(q)|O(x)|0\rangle \propto e^{iq\cdot x}$ is x-dependent. To understand what to do one has to remember that operators have to be smeared $O_h = \int d^dx O(x)h(x)$, by test function $h \in \mathcal{S}(\mathbb{R}^d)$ in Schwartz space (sufficiently fast fall off at infinity, e.g. $h(x) = r^n e^{-|c|r^2}$). Taking this into account one finds $\langle D|O_h|D\rangle = 1/F_D^2(\Delta_O - d)\Delta_O\langle O_h\rangle$ by integration by parts, and removing the test function this leads to

$$\langle D|O|D\rangle = \frac{1}{F_D^2}(\Delta_O - d)\Delta_O\langle O\rangle . \tag{10}$$

Whereas the single-soft theorem is standard, the double-soft theorem (10) is a new result [3] to the best of our knowledge. The derivation given here is more detailed than in that reference.

In order to further refine results we consider the following two fundamental formulae

$$F_D m_D^2 = \langle D(q)|T^\rho{}_\rho|0\rangle , \quad 2m_D^2 = \langle D(q)|T^\rho{}_\rho|D(q)\rangle , \tag{11}$$

where the first one follows from (3) and the second one follows from $P_\mu = \int d^{d-1}xT_{\mu 0}$. We assume that a single operator $O \subset T^\rho{}_\rho$, , which is not relevant i.e. $\Delta_O \leq d$, is responsible for generating the mass or the matrix elements in (11). We note that vacuum expectation value must be lower than the perturbative vacuum and thus negative $\langle O\rangle < 0$.

a) First we notice that the d-term assures *positivity* of the dilaton mass since $\Delta_O \leq d$.

b) Applying the single- and double-soft theorem to (11) one gets

$$m_D^2 = -\frac{1}{F_D^2}\Delta_O\langle O\rangle \;, \quad m_D^2 = \frac{1}{2F_D^2}(\Delta_O - d)\Delta_O\langle O\rangle \;, \tag{12}$$

respectively, which hold simultaneously if and only if

$$\Delta_O = d-2 \;. \tag{13}$$

c) Note that for $\Delta_O \neq d-2$ the only solution is $\langle O\rangle = 0$.

In summary the d-term resolves a positivity problem and the operator scaling dimension must be $d-2$. At last two cautionary remarks. If $T^\rho_{\ \rho} \supset \sum_i O_i$ with $\Delta_{O_i} \neq \Delta_{O_j}$ are contributing to (11), then no such strong statement can be made. This is for example the case in the Gross-Neveu-Yukawa theory [6] in $d=3$ where the ϕ^3 and the Yukawa-operator take on this role. In the case there are other Goldstone such as the pion with $m_D > 2m_\pi$, then the dilaton becomes unstable and one must involve a more elaborate formalism. However, we may expect that the qualitative results still hold.

3. Dilaton Effective Theory

Here, we consider the dilaton on its own without further Goldstone bosons such as the pions. The latter are readily added and in particular the dilaton harmonises with them as they solve the Goldstone improvement problem [5]. The leading order dilaton Lagrangian reads

$$\mathcal{L}_{\rm LO} = \frac{1}{2}\hat{\chi}^{d-4}(\partial\chi)^2 + \frac{\kappa_d}{4}R\,\chi^{d-2} + V(\hat{\chi}) \;, \quad \kappa_d \equiv \frac{2}{(d-1)(d-2)} \;, \tag{14}$$

where $\chi \equiv F_D e^{-D/F_D}$ and hatted quantities are divided by appropriate powers of F_D. The first two terms are invariant under *local* Weyl transformation

$$g_{\mu\nu} \to e^{-2\alpha}g_{\mu\nu} \;, \quad D \to D - \alpha F_D \;, \quad \Rightarrow \quad \chi \to e^{\alpha}\chi \;, \tag{15}$$

such that its exponential representation is said to realise the symmetry non-linearly. Choosing the convention $F_D > 0$, then implies $\chi > 0$. The term proportional to the Ricci scalar is known as *the improvement term*; it realises the fundamental matrix element (3) in the effective theory and affects gravitational form factors decisively [5].

3.1 A quick note on a dilaton potential

Turning to the potential $V(\hat{\chi})$, we may make a surprising connection with the double-soft theorem. Firstly, for a massless dilaton the potential vanishes $V(\hat{\chi}) = 0$, and when a dilaton mass is generated by a single operator O, as in the previous section, then it assumes the form

$$V_\Delta(\hat{\chi}) \;=\; \frac{m_D^2 F_D^2}{\Delta - d}\left(\frac{1}{\Delta}\hat{\chi}^\Delta - \frac{1}{d}\hat{\chi}^d\right) = c_V + \frac{1}{2}m_D^2 D^2 + O(D^3) \;, \tag{16}$$

with c_V a constant and $\Delta = \Delta_O$. A long time ago, it was observed by Zumino [2] that $V_{\rm Zumino} \propto \hat{\chi}^d$ is the only term allowed by scale invariance and that for minimisation puposes a second term is needed $V = V_{\rm Zumino} + c\hat{\chi}^\Delta$. The new observation is that the role of the Zumino-term, in the context of the double-soft theorem, is taken on by $x\cdot\partial$ leading to the factor d in the second equation in (12), since $\langle D|\hat{\chi}^\Delta - \frac{\Delta}{d}\hat{\chi}^d|D\rangle = \frac{1}{2}\Delta(\Delta - d)$.

3.2 Explicit commutators in the effective theory

It may be instructive to realise the fundamental commutator (4) in the effective theory for the operator $O = \hat{\chi}^{\Delta_O}$ of scaling dimension Δ_O. Thus the question how to compute a commutator. One may either use the Bjorken-Johnson-Low formula or simply integrate the Ward identity (6) to obtain

$$\langle i[Q_D, O(x)]\rangle = -i \int d^d z \langle T\partial \cdot J^D(z) O(x)\rangle \ , \tag{17}$$

assuming the vanishing of the correlator at infinity. Formally,

$$\partial \cdot J^D = T^\rho{}_\rho + z_\nu \partial_\mu T^{\mu\nu} \ , \tag{18}$$

follows from $J^\mu_D(z) = z_\nu T^{\mu\nu}(z)$ and one easily anticipates that the first and the second term are responsible for the scaling dimension and the $x \cdot \partial$ term respectively. To progress concretely we need the energy momentum tensor

$$\begin{aligned} T_{\mu\nu} = & -2\frac{\delta}{\delta g^{\mu\nu}} \int d^d x \sqrt{-g}\mathcal{L}_{\mathrm{LO}}\big|_{g_{\mu\nu}=\eta_{\mu\nu}} \\ & = \chi^{d-4}\partial_\mu\chi\partial_\nu\chi - g_{\mu\nu}(\frac{1}{2}\hat{\chi}^{d-4}(\partial\chi)^2 - V(\hat{\chi})) + \frac{\kappa_d}{2}\big((g_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu)\big)\chi^{d-2} \ . \end{aligned} \tag{19}$$

The individual terms in (18) evaluate to

$$T^\rho{}_\rho = \chi\partial^2\chi + dV = -\partial^2\hat{D} + \dots \ , \quad \partial_\mu T^{\mu\nu} = \partial^\nu\chi(\partial^2\chi + V'(\hat{\chi})) = (\partial_\nu\hat{D})\partial^2\hat{D} + \dots \ ,$$

where we have set the potential to zero and dots denote higher order in D. Note that we are not to use the classical equation of motion but

$$\partial^2\langle T D(z) f(D(x)) \dots\rangle = -i\delta^{(d)}(z-x)\langle f'(D(x)) \dots\rangle + \dots \ , \tag{20}$$

which follows from $\partial^2\langle TD(z)D(x)\rangle = -i\delta^{(d)}(z-x)$ and the dots stand for other operator insertion leading to further contact terms. We may now evaluate

$$\begin{aligned} -i\int d^d z\langle T T^\rho{}_\rho(z) f(\hat{D}(x))\rangle &= -\langle f'(\hat{D}(x))\dots\rangle + \dots \ , \\ -i\int d^d z\langle T z_\nu \partial_\mu T^{\mu\nu}(z) f(\hat{D}(x))\rangle &= -\langle x\cdot\partial\hat{D}(x) f'(\hat{D}(x))\dots\rangle + \dots \ , \end{aligned} \tag{21}$$

where the dots have the same meaning as above. Assembling bits and pieces we get

$$\langle i[Q_D, f(\hat{D}(x))]\dots\rangle = \langle(1+x\cdot\partial\hat{D})(-f'(\hat{D}(x))\dots\rangle + \dots = \langle(\Delta_O + x\cdot\partial) f(\hat{D}(x))\dots\rangle + \dots \ , \tag{22}$$

for $f(\hat{D}) = e^{-\Delta_O \hat{D}}$. We have therefore explicitly computed the commutator (4) in the effective theory. Note that, for non-integer Δ_O the derivation goes beyond the free field theory.[3]

[3]For a non-primary operator (e.g. a descendant) the fundamental commutator (4) is altered. For a free free theory where $E = (\partial^2 + m_\varphi^2)\varphi^2$ is an equation of motion operator, $\langle[Q_D, E(x)]\rangle = 0$ must hold which we have verified explicitly.

4. Gauge Theories with an IRFP and a Dilaton

Let us return to the question whether gauge theories in the chirally broken phase are compatible with an IRFP interpretation and a dilaton. There is consensus in the community that if such a phase were to occur in QCD, then the $f_0(500)$, known as the σ-meson, would take on the role of the dilaton. The σ-meson is the lowest lying resonance in QCD which continuous to inspire particle physicists [7], it decays quickly into two pions, defies the Regge trajectory and large N_c counting. Its pole location $\sqrt{s_\sigma} = m_\sigma - \frac{i}{2}\Gamma_\sigma = (400-550) - i(200-350)$ [8] explains why it is difficult to access. What happens to this pole when the quark masses are sent to zero is unknown. One may consider the following three logical possibilities

$$(a)\ \ \bar{m}_N = O(1)\bar{m}_\sigma\ , \quad (b)\ \ \bar{m}_\sigma \ll \bar{m}_N\ , \quad (c)\ \ \bar{m}_\sigma = 0\ , \tag{23}$$

with the bar denoting the chiral limit. If (a) was the case then the FP interpretation and the soft theorem are not useful for QCD but might still be valuable for a gauge theory closer to the conformal window, say $N_f = 8$ for $N_c = 3$. If (b) was the case then it is interesting and the dilaton of a new gauge theory could still be a Higgs candidate provided that $F_\sigma/F_\pi \approx 1$ [3]. If (c) was true then there is the additional benefit that soft theorems become exact, along with further predictions cf. section 4.3.

4.1 The pion sector - disregarding the dilaton

We may simply consider the pions on their own in the chiral limit, disregarding the dilaton altogether. In the deep IR the massless pions correspond to a free field theory which is particular fixed point.[4] Can anything be learned in that case? Yes, we can extract scaling dimensions by matching the IRFP-assumption with chiral perturbation theory (χPT)

$$\langle S^a(x)S^a(0)\rangle_{\mathrm{IRFP}} = \langle S^a(x)S^a(0)\rangle_{\chi\mathrm{PT}}\ , \quad \text{for } x^2 \to \infty\ , \tag{24}$$

in the deep IR [4]. Here, we have chosen $S^a = \bar{q}T^a q$ as an operator for which the scaling dimension is given by $\Delta_{S^a} = (d-1) - \gamma_*$ with $\gamma_* = \gamma_m|_{\mu=0}$ being the quark anomalous dimension. Using that for a conformal theory one has $\langle O(x)O^\dagger(0)\rangle \propto (x^2)^{-\Delta_O}$ the left hand side is known and using χPT source theory one readily obtains $S^a|_{\mathrm{LO}} \propto d^{abc}\pi^b\pi^c + O(1/F_\pi^2)$ such that the χPT-evaluation reads $\langle S^a(x)S^a(0)\rangle_{\chi\mathrm{PT}} \propto 1/x^4$. Matching as in (24), gives

$$\langle S^a(x)S^a(0)\rangle_{\mathrm{IRFP}} \propto \frac{1}{(x^2)^{3-\gamma_*}} \propto \frac{1}{x^4} \propto \langle S^a(x)S^a(0)\rangle_{\chi\mathrm{PT}} \quad \Leftrightarrow \quad \gamma_* = 1\ . \tag{25}$$

This result can be deduced in other ways: from hyperscaling arguments, matching the Feynman-Hellmann theorem and the trace anomaly for the pion [4], the decoupling of the dilaton for gravitational form factors [5] and the other correlators [3]. Amongst those the Feynman-Hellmannt-type argument is, perhaps, the strongest. Most notably the result (25) is consistent with lattice simulations, perturbation theory and all model computations. Again, the result connects nicely to the soft theorem in that we may consider the perturbation $T^\rho{}_\rho = (1+\gamma_*)N_f m_q\bar{q}q$ in (12) and observe that

[4]This is for instance assumed when applying the a-theorem to QCD. Albeit there are subtleties with regards to conformality without the massless dilaton e.g. [5].

$\Delta_{\bar{q}q} = d - 2$ does indeed hold since $\Delta_{\bar{q}q} = \Delta_{S^a}$, and is therefore a consistent perturbation [3]; or conversely can be seen as another determination of $\gamma_* = 1$. Such relations are known as dilaton Gell-Mann-Oakes-Renner relations in this context. Another scaling dimension is the one of the field strength tensor G^2 which is $\Delta_{G^2} = d + \beta'_*$ and it is found that $\beta'_* = 0$ from renormalisation group arguments and matching correlators [3]. There are many advantages to it, including that the trace anomaly is a next-leading order in the effective theory. Now, $\Delta_{G^2} = d$ means that one must be cautious in applying partially conserved dilatation current formulae since equation (11) are contradictory unless $\langle \beta/gG^2 \rangle = 0$ [3] (for $m_q \to 0$).

4.2 Consistency with $\mathcal{N} = 1$ supersymmetric gauge theories

The new element of $\mathcal{N} = 1$ gauge theories are the Seiberg dualities, stating that an electric $SU(N_c)$ gauge theory is infrared dual to a magnetic $SU(N_f - N_c)$ gauge theory with an additional neutral “meson field", as for instance reviewed in [9]. A conformal window extends from $3N_c \geq N_f \geq \frac{3}{2}N_c$. From the all order β-function it follows that $\gamma_*^{\text{el}} + \gamma_*^{\text{mag}} = 1$ which implies that at the lower end of the conformal window ($N_f = \frac{3}{2}N_c$) one has $\gamma_*^{\text{el}} = 1$ since $\gamma_*^{\text{mag}} = 0$ is the weak coupling limit. By matching correlation functions of the trace of the energy momentum tensor

$$\frac{1}{(x^2)^{4+\beta_*^{'\text{el}}}} \propto \langle T^\rho{}_\rho(x) T^\rho{}_\rho(0) \rangle_{\text{el}} \propto \langle T^\rho{}_\rho(x) T^\rho{}_\rho(0) \rangle_{\text{mag}} \propto \frac{1}{(x^2)^{4+\beta_*^{'\text{mag}}}} \quad \Leftrightarrow \quad \beta_*^{'\text{el}} = \beta_*^{'\text{mag}} , \tag{26}$$

using $\Delta_{T^\rho{}_\rho} = \Delta_{G^2} = 4 + \beta'_*$. Similarly as above one has $\beta_*^{'\text{mag}} = 0$ at the end of the lower end of the conformal window and thus $\beta_*^{'\text{el}} = 0$ [10].

Hence, both $\gamma_* = 1$ and $\beta'_* = 0$ seem to hold for $\mathcal{N} = 1$ at the end of the conformal window. One might wonder whether the extension of $\gamma_* = 1$ into the chirally broken phase makes any sense below the boundary of the conformal window. Seiberg duality seems to suggests that this is the case since the squark-antisquark bilinear $\tilde{Q}Q$ in the electric theory is matched to the free meson M in the magnetic dual [3]. Concretely, we have ($\Delta_{\tilde{Q}Q} = 2 - \gamma_*$)

$$\frac{1}{(x^2)^{\Delta_M}} \propto \langle M(x) M(0) \rangle \propto \langle \tilde{Q}Q(x) \tilde{Q}Q(0) \rangle \propto \frac{1}{(x^2)^{2-\gamma_*}} \quad \Leftrightarrow \quad \gamma_* = 2 - \Delta_M , \tag{27}$$

that $\gamma_* = 1$ must hold in the IR-free regime $N_c + 1 \leq N_f \leq \frac{3}{2}N_c$ since the meson's scaling dimension must be $\Delta_M = 1$. Hence, the $\mathcal{N} = 1$ theory indicates that the IRFP scenario continuous to make sense below the conformal window.

4.3 Massless dilaton

One might wonder whether a renormalisation group flow and a massless dilaton are compatible. Colloquially speaking: “does the Goldstone candidate remember that the spontaneously broken scale symmetry is only emerging?" The general answer to this question is not known but at least the Gross-Neveu-Yukawa theory in $d = 3$ provides an affirmative answer [6]. In that case the conformal symmetry must be implemented at low energy. As an interesting example we consider the gravitational form factor of scalar particle φ, defined by

$$\Gamma_{\mu\nu}(q^2) \equiv \langle \varphi(p') | T_{\mu\nu}(0) | \varphi(p) \rangle = 2\mathcal{P}_\mu \mathcal{P}_\nu A(q^2) + \frac{1}{2}(q_\mu q_\nu - q^2 \eta_{\mu\nu}) \mathcal{D}(q^2) , \tag{28}$$

with $q \equiv p' - p$ and $\mathcal{P} \equiv \frac{1}{2}(p + p')$ such that $q^\mu \Gamma_{\mu\nu} = 0$ translational invariance holds. Further, one has $A(0) = 1$ since $P_\mu = \int d^{d-1}x T_{\mu 0}$. If $\mathcal{D}(q^2)$ is regular for $q^2 \to 0$ one has the well-known textbook formula $\Gamma^\rho{}_\rho(0) = 2m_\varphi^2$ (cf. (11)). However, when a massless dilaton is present one finds that [5, 11]

$$\mathcal{D}(q^2) = \frac{4}{d-1}\frac{m_\varphi^2}{q^2} + O(1) \quad \Rightarrow \quad \Gamma^\rho{}_\rho(0) = 0 \,, \tag{29}$$

the dilaton implements the conformal Ward identity in analogous manner to the pion for $SU(N_f)$ flavour symmetry. In fact what leads to this cancellation is the Goldberger-Treiman mechanism for the dilaton where the analogy can be seen from the trilinear coupling relation

$$g_{D\phi\phi} = \frac{2m_\varphi^2}{F_D} \,, \quad g_{\pi\phi\phi} = \frac{2m_\varphi^2}{F_\pi} \,. \tag{30}$$

In our view many more things could be said and certainly many more aspects remain to be explored. It is clear that lattice QCD can play an important role in finding out whether or not gauge theories have more to do with IRFPs than is generally assumed.

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PoS(LATTICE2024)151