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Progress on Holographic Vacuum Misalignment

Ali Fatemiabhari,[a,*] **Daniel Elander,** **and Maurizio Piai**[a]

[a] *Department of Physics, Faculty of Science and Engineering, Swansea University, Singleton Park, Swansea, United Kingdom*

E-mail: 2127756@swansea.ac.uk, m.piai@swansea.ac.uk, daniel.elander@gmail.com

We summarise highlights from an ongoing research programme that aims, in the long run, at the ambitious goal of building a realistic, complete holographic composite-Higgs model. This contribution focuses on vacuum misalignment, by showing how to unify its description, as a phenomenon arising from weak coupling considerations, in the holographic description of a strongly coupled field theory in terms of a dual gravity theory. This is achieved by a non-trivial treatment of boundary-localised terms in the gravity action. The gravity backgrounds considered are completely regular and smooth. We provide numerical examples showing that the mass spectrum of particles in the four-dimensional theory is free of pathologies, and that a small hierarchy arises naturally, between the light states that, in this simplified set up, are analogous to the standard-model particles, and all the other, new composite states emerging in the strongly coupled theory.

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*Speaker

PoS(LATTICE2024)142

1. Introduction

Composite Higgs Models (CHMs), in which the fields in the Higgs doublet of the Standard Model (SM) emerge as a set of Pseudo-Nambu-Goldstone-Bosons (PNGBs) [1–3], in a more fundamental, strongly coupled, confining field theory, offer a promising framework within which to address some of the big open questions of modern particle physics [4–6] (see also Refs. [7–9]), such as the electroweak (big or small) hierarchy problem. The new physics sector is endowed with an approximate global symmetry, described by a Lie group, G, broken to a subgroup, H, both by explicit symmetry breaking terms, as well as the emergence of composite condensates. At low energies the theory can be replaced by an Effective Field Theory (EFT), in which the PNGBs are described by fields taking values in the coset, G/H, along the same lines as the chiral Lagrangian. The distinctive feature of CHMs is that while the electroweak gauge group is embedded as a subgroup of G, the presence of a perturbative instability, itself originating from the coupling to SM fields, induces misalignment with the vacuum [10], and electroweak symmetry breaking.

As the phenomenology, in particular the mass spectrum of the composite states, is determined by the underlying strongly coupled dynamics, it is natural to study it on the lattice.[1] Unfortunately, for the minimal CHM, based upon the $SO(5)/SO(4)$ coset, the low-energy spectrum of which consists only of the known SM fields, a simple formulation, amenable to numerical lattice studies is not known (see Ref. [64]). Moreover, the aforementioned vacuum misalignment phenomenon is perturbative in nature, and its lattice treatment unwieldy. Gauge-gravity dualities [65–68] offer a promising alternative way to address calculability. Well known realisations of CHMs based on the $SO(5)/SO(4)$ coset [69–76] are formulated as simple bottom-up holographic models, in which confinement is modelled by a hard cut-off in the theory.

In this proceedings contribution, we summarize highlights from an ambitious research programme [77–80] (see also Refs. [81–83]), which ultimately aims at building a complete holographic CHM model, in which confinement is captured dynamically in the gravity theory. We show how to combine the spontaneous breaking of an approximate $SO(5)$ symmetry, arising in the background geometry, with weak interactions, localised at the boundary, to induce vacuum misalignment. We present a simplified, bottom-up holographic model, describing a four-dimensional gauge theory in which a gauged $SO(4)$ subgroup of the $SO(5)$ approximate global symmetry is Higgsed to its $SO(3)$ subgroup, due to misalignment with the vacuum structure of the underlying strongly coupled dynamics. We provide examples of the resulting spectrum, computed using the gauge-invariant formalism developed in Refs. [84–88]—see also Refs. [82, 89–96]. Our results demonstrate the opening up of a (small) hierarchy in the spectrum. We dispense with the many, non-trivial, technical details necessary in the construction, which can be found in the extensive, accompanying publication in Ref. [77]. We comment on the next programmatic model-building steps that would lead to a fully realistic model of holographic CHM with minimal $SO(5)/SO(4)$ coset.

[1] Pertinent numerical lattice calculations exist in theories with gauge group $SU(2)$ [11–20], $Sp(4)$ [21–42] and $SU(4)$ [43–50]. Results for the $SU(3)$ theory with $N_f = 8$ Dirac fermions [51–58] have been reinterpreted in terms of new CHMs, embedded in the dilaton EFT framework [59, 60]—see also Refs. [61–63]

PoS(LATTICE2024)142

Table 1: Table I of Ref. [77]. Field content, organised in terms of irreducible representations of the symmetries in $D = 6$ dimensions ($SO(5)$ multiplets), $D = 5$ dimensions ($SO(4)$ multiplets, with $\langle \mathcal{X} \rangle \neq 0$), and $D = 4$ dimensions ($SO(3)$ multiplets, with $\langle \vec{\pi} \rangle \neq 0$). In $D = 4$ dimensions, we refer to gauge-invariant combinations, massive representations of the Poincaré group.

$D=6$, $SO(5)$, massless irreps.			$D=5$, $SO(4)$, massless irreps.			$D=4$, $SO(3)$, massive irreps.		
Field	$SO(5)$	N_{dof}	Field	$SO(4)$	N_{dof}	Field	$SO(3)$	N_{dof}
$\hat{g}_{\hat{M}\hat{N}}$	1	9	g_{MN}	1	5	$g_{\mu\nu}$	1	5
						$g_{\mu 5}$	1	–
						g_{55}	1	–
			χ_M	1	3	χ_μ	1	3
						χ_5	1	–
			χ	1	1	χ	1	1
$\mathcal{X}_\alpha$	5	5	ϕ	1	1	ϕ	1	1
			$\pi^{\hat{A}}$	4	4	$\pi^{\hat{\mathcal{A}}}$	3	3
						π^4	1	1
$\mathcal{A}_{\hat{M}\,\alpha}{}^{\beta}$	10	40	$\mathcal{A}_M{}^{\hat{A}}$	4	12	$\mathcal{A}_\mu{}^{\hat{\mathcal{A}}}$	3	9
						$\mathcal{A}_\mu{}^4$	1	3
						$\mathcal{A}_5{}^{\hat{\mathcal{A}}}$	3	–
						$\mathcal{A}_5{}^4$	1	–
			$\mathcal{A}_6{}^{\hat{A}}$	4	4	$\mathcal{A}_6{}^{\hat{\mathcal{A}}}$	3	3
						$\mathcal{A}_6{}^4$	1	1
			$\mathcal{A}_M{}^{\bar{A}}$	6	18	$\mathcal{A}_\mu{}^{\tilde{\mathcal{A}}}$	3	9
						$\mathcal{A}_\mu{}^{\bar{\mathcal{A}}}$	3	9
						$\mathcal{A}_5{}^{\tilde{\mathcal{A}}}$	3	–
						$\mathcal{A}_5{}^{\bar{\mathcal{A}}}$	3	–
			$\mathcal{A}_6{}^{\bar{A}}$	6	6	$\mathcal{A}_6{}^{\tilde{\mathcal{A}}}$	3	3
						$\mathcal{A}_6{}^{\bar{\mathcal{A}}}$	3	3
$P_{5\,\alpha}$	5	5	$P_{5\,\hat{A}}$	4	4	$P_{5\,\hat{\mathcal{A}}}$	3	3
						P_{54}	1	1
			P_{55}	1	1	P_{55}	1	1

2. Gravity model and background

The bottom-up holographic model described in Ref. [77] consists of gravity in six dimensions coupled to a bulk scalar field, $\mathcal{X}$, transforming in the (real) vector representation, **5**, of a gauged $SO(5)$ symmetry, with gauge field $\mathcal{A}_{\hat{M}}$, summarised in Table 1. One of the non-compact spacetime dimensions, ρ, serves as the holographic direction. We focus attention on background solutions

Table 2: Table II from Ref. [77]. Summary table associating the fields in five-dimensional language to their fluctuations in the four-dimensional, ADM formalism.

Field	Fluctuation	Field	Fluctuation
g_{MN}	$\mathfrak{e}_{\mu\nu}$	$\left(\mathcal{B}_M{}^{\hat{\mathcal{A}}}, \mathcal{B}_M{}^{\tilde{\mathcal{A}}}\right)$	$\left(\mathfrak{v}_\mu{}^{\hat{\mathcal{A}}}, \mathfrak{v}_\mu{}^{\tilde{\mathcal{A}}}\right)$
χ_M	$\mathfrak{v}_\mu$	$\mathcal{A}_M{}^4$	$\mathfrak{v}_\mu{}^4$
(ϕ, χ)	$(\mathfrak{a}^\phi, \mathfrak{a}^\chi)$	$\mathcal{A}_M{}^{\bar{\mathcal{A}}}$	$\mathfrak{v}_\mu{}^{\bar{\mathcal{A}}}$
$\left.\begin{matrix}\mathcal{B}_6^{\hat{\mathcal{A}}} \\ \mathcal{A}_6^4\end{matrix}\right\}$	$\mathfrak{a}^{\hat{A}} = \begin{cases}\mathfrak{a}^{\hat{\mathcal{A}}} \\ \mathfrak{a}^4\end{cases}$	$\left.\begin{matrix}\pi^{\hat{\mathcal{A}}} \\ \Pi^4\end{matrix}\right\}$	$\mathfrak{p}^{\hat{A}} = \begin{cases}\mathfrak{p}^{\hat{\mathcal{A}}} \\ \mathfrak{p}^4\end{cases}$
$\left.\begin{matrix}\mathcal{B}_6^{\tilde{\mathcal{A}}} \\ \mathcal{A}_6^{\bar{\mathcal{A}}}\end{matrix}\right\}$	$\mathfrak{a}^{\bar{A}} = \begin{cases}\mathfrak{a}^{\tilde{\mathcal{A}}} \\ \mathfrak{a}^{\bar{\mathcal{A}}}\end{cases}$		

with asymptotically AdS_6 geometry for large values of ρ, corresponding to the ultraviolet (UV) regime of the putative dual field theory. Another spatial dimension is compactified on a circle that smoothly shrinks to zero size at a finite value of the radial direction, $\rho = \rho_o$, marking the infrared (IR) regime. The termination of the space at this point introduces a mass gap in the dual field theory, mimicking the effects of confinement in the four-dimensional field theory [97].

In the background solutions, $\mathcal{X}$ develops a non-trivial profile, spontaneously breaking the $SO(5)$ gauge symmetry to its $SO(4)$ subgroup. Furthermore, a boundary-localised (spurion) field, P_5, itself transforming as a **5**, acquires a non-trivial vacuum expectation value (VEV), misaligned with $\langle \mathcal{X} \rangle$, so that the symmetry breaks to $SO(3)$. We report here only the information needed to keep the presentation self-contained and clarify the notation, referring for details to Refs. [79, 80]. The bulk action is

$$\mathcal{S}_6^{(bulk)} = \int \frac{\mathrm{d}^6 x}{2\pi} \sqrt{-\hat{g}_6} \left\{ \frac{\mathcal{R}_6}{4} - \frac{1}{2} \hat{g}^{\hat{M}\hat{N}} \left(D_{\hat{M}} \mathcal{X}\right)^T D_{\hat{N}} \mathcal{X} - \mathcal{V}_6 - \frac{1}{2} \mathrm{Tr} \left[\hat{g}^{\hat{M}\hat{P}} \hat{g}^{\hat{N}\hat{Q}} \mathcal{F}_{\hat{M}\hat{N}} \mathcal{F}_{\hat{P}\hat{Q}} \right] \right\}. \quad (1)$$

Here, $\hat{M} = 0, 1, 2, 3, 5, 6$ denote the six-dimensional spacetime indices. The metric in six dimensions, $\hat{g}_{\hat{M}\hat{N}}$, has a determinant $\hat{g}_6$ and signature mostly '+'. The Ricci scalar for the six-dimensional spacetime is denoted by $\mathcal{R}_6$. The covariant derivatives are denoted as $D_{\hat{N}}$, and $F_{\hat{M}\hat{N}}$ is the $SO(5)$ field strength. The scalar potential reads:

$$\mathcal{V}_6 = -5 - \frac{\Delta(5-\Delta)}{2} \phi^2 - \frac{5\Delta^2}{16} \phi^4 , \quad (2)$$

in terms of ϕ, which appears in the parametrisation of the scalar field, $\mathcal{X}$, as

$$\mathcal{X} \equiv \exp\left[2i \sum_{\hat{A}} \pi^{\hat{A}} t^{\hat{A}}\right] \mathcal{X}_0\, \phi, \qquad \text{where} \qquad \mathcal{X}_0 \equiv (0, 0, 0, 0, 1)^T,$$

with $\hat{A} = 1, \ldots, 4$, indexing the generators of the $SO(5)/SO(4)$ coset. The four PNGBs, $\vec{\pi} = (\pi^1, \pi^2, \pi^3, \pi^4)$, span the $SO(5)/SO(4)$ coset [77].

We dimensionally reduce the action to five dimensions, $\mathcal{S}_5^{(bulk)}$, and introduce boundaries at finite values of the radial direction, $\rho = \rho_i$ for $i = 1, 2$, which act as regulators. Our calculations are carried out within the constrained range $\rho_1 \leq \rho \leq \rho_2$, while physical results are recovered in the limits $\rho_1 \to \rho_o$ and $\rho_2 \to \infty$. Boundary spacetime indices are denoted as $\mu = 0, 1, 2, 3$—see Table 1. The complete five-dimensional action, $\mathcal{S}_5$, contains also boundary terms [77]:

$$\mathcal{S}_5 = \mathcal{S}_5^{(bulk)} + \sum_{i=1,2} \left(\mathcal{S}_{\text{GHY},i} + \mathcal{S}_{\lambda,i} \right) + \mathcal{S}_{P_5,2} + \mathcal{S}_{\mathcal{V}_4,2} + \mathcal{S}_{\mathcal{A},2} + \mathcal{S}_{\chi,2} + \mathcal{S}_{\mathcal{X},2} \,. \tag{3}$$

where the terms in bracket make the variational problem well defined, while

$$\begin{aligned}
\mathcal{S}_{P_5,2} &= \int \mathrm{d}^4x \sqrt{-\tilde{g}} \left\{ -\frac{1}{2} K_5\, \tilde{g}^{\mu\nu} \left(D_\mu P_5 \right) D_\nu P_5 - \lambda_5 \left(P_5^T P_5 - v_5^2 \right)^2 \right\} \Big|_{\rho=\rho_2} \,, \\
\mathcal{S}_{\mathcal{V}_4,2} &= - \int \mathrm{d}^4x \sqrt{-\tilde{g}}\, \mathcal{V}_4(\mathcal{X}, \chi, P_5) \Big|_{\rho=\rho_2} \,, \\
\mathcal{S}_{\mathcal{A},2}\Big|_{P_5=\overline{P_5}} &= \int \mathrm{d}^4x \sqrt{-\tilde{g}} \left\{ -\frac{1}{4} \hat{D}_2\, \tilde{g}^{\mu\rho} \tilde{g}^{\nu\sigma} \mathcal{F}_{\mu\nu}^{\hat{A}} \mathcal{F}_{\rho\sigma}^{\hat{A}} - \frac{1}{4} \bar{D}_2\, \tilde{g}^{\mu\rho} \tilde{g}^{\nu\sigma} \mathcal{F}_{\mu\nu}^{\bar{A}} \mathcal{F}_{\rho\sigma}^{\bar{A}} \right\} \Big|_{\rho=\rho_2} \,. \\
\mathcal{S}_{\mathcal{X},2} &= \int \mathrm{d}^4x \sqrt{-\tilde{g}} \left\{ -\frac{1}{2} K_{\mathcal{X},2}\, \tilde{g}^{\mu\nu} (D_\mu \mathcal{X})^T D_\nu \mathcal{X} \right\} \Big|_{\rho=\rho_2} \,.
\end{aligned}$$

We set $P_5 = \overline{P_5}$ for simplicity [77]. The parameters K_5, λ_5, $\hat{D}_2$, $\tilde{D}_2$, $K_{\mathcal{X},2}$ are discussed later.

2.1 Model parameters and $SO(4)$ gauging

The boundary terms in the action, Eq. (3), are used in the regularisation process, implemented along the lines of holographic renormalisation [98–100]. Their finite parts are physical parameters in our analysis. The spurion field, P_5, is introduced so that all symmetry-breaking effects have spontaneous origin in the gravity formulation [77]. The boundary term $\bar{D}_2$ contains the free parameter, $\bar{\varepsilon}^2$, that controls the strength of the gauging of the $SO(4)$ in the field theory. In the next section we comment on parameters, m_4^2 and v, appearing in the boundary potential, $\mathcal{V}_4(\mathcal{X}, \chi, P_5)$. The symmetry breaking pattern $SO(5) \to SO(4)$ is controlled by $k_{\mathcal{X}} \equiv K_{\mathcal{X},2} e^{\rho_2(8/3-\Delta)}$ [77].

The presence of boundary localised terms breaks the $SO(5)$ symmetry to a gauged $SO(4)$ subgroup, which may or may not be aligned to the unbroken $SO(4)$ subgroup, depending on the value of the vacuum misalignment angle, v. In the background, this parameterises the non-zero value of $\pi^4 = v$, and leads to the spontaneous breaking of the gauged $SO(4)$ to $SO(3)$. We introduce indices adapted to $SO(3)$, specifically $\hat{\mathcal{A}} = 1, 2, 3$, $\tilde{\mathcal{A}} = 5, 6, 7$, and $\bar{\mathcal{A}} = 8, 9, 10$. These are chosen so that $t^{\bar{\mathcal{A}}}$ represents the unbroken generators of $SO(3)$. The fluctuations of the fourth component of $\pi^{\hat{A}}$ is written as $\pi^4 = v + \Pi^4$. In the spin-1 sector, there is mixing between the two triplets, denoted by the indices $\hat{\mathcal{A}}$ and $\tilde{\mathcal{A}}$. We define the following linear combinations:

$$\begin{aligned}
\mathcal{B}_6^{\hat{\mathcal{A}}} &\equiv \cos(v) \mathcal{A}_6^{\hat{\mathcal{A}}} + \sin(v) \mathcal{A}_6^{\hat{\mathcal{A}}+4} \,, && (4) \\
\mathcal{B}_6^{\tilde{\mathcal{A}}} &\equiv -\sin(v) \mathcal{A}_6^{\tilde{\mathcal{A}}-4} + \cos(v) \mathcal{A}_6^{\tilde{\mathcal{A}}} \,, && (5) \\
\mathcal{B}_M{}^{\hat{\mathcal{A}}} &\equiv \cos(v) \mathcal{A}_M{}^{\hat{\mathcal{A}}} + \sin(v) \mathcal{A}_M{}^{\hat{\mathcal{A}}+4} \,, && (6) \\
\mathcal{B}_M{}^{\tilde{\mathcal{A}}} &\equiv -\sin(v) \mathcal{A}_M{}^{\tilde{\mathcal{A}}-4} + \cos(v) \mathcal{A}_M{}^{\tilde{\mathcal{A}}} \,. && (7)
\end{aligned}$$

We adopt this basis for the fields (excluding the metric) that fluctuate around the backgrounds:

$$\Phi^a = \{\phi, \chi\}, \quad (8)$$
$$\Phi^{(0)a} = \{\mathcal{B}_6^{\hat{\mathcal{A}}}, \mathcal{A}_6^4, \mathcal{B}_6^{\tilde{\mathcal{A}}}, \mathcal{A}_6^{\tilde{\mathcal{A}}}\}, \quad (9)$$
$$V_M{}^A = \{\chi_M, \mathcal{B}_M{}^{\hat{\mathcal{A}}}, \mathcal{A}_M{}^4, \mathcal{B}_M{}^{\tilde{\mathcal{A}}}, \mathcal{A}_M{}^{\tilde{\mathcal{A}}}\}, \quad (10)$$
$$\mathcal{H}_M^{(1)A} = \left\{0, \frac{\sin(v)}{v}\partial_M \pi^{\hat{\mathcal{A}}} + \frac{g}{2}\mathcal{B}_M{}^{\hat{\mathcal{A}}}, \partial_M \Pi^4 + \frac{g}{2}\mathcal{A}_M{}^4, 0, 0\right\}. \quad (11)$$

We use different symbols to distinguish the original fields in the action from the gauge-invariant combinations of fluctuations associated with them—see Table 2.

The family of backgrounds of interest is characterised by two parameters: Δ, which is linked to the dimension of the dual field-theory operator responsible for breaking $SO(5)$ to $SO(4)$, and $\phi_I = \phi(\rho = \rho_o)$, that controls the size of symmetry breaking effects. We impose the upper bound $\phi_I \leq \phi_I(c)$, with $\phi_I(c)$ the critical value at which a first-order phase transition occurs—see Ref. [77]—and beyond which these solutions would be metastable, and eventually unstable.

The strength of the $SO(4)$ gauge coupling in the dual field theory is approximately $g_4 \equiv \bar{\varepsilon}g$, where g represents the bulk $SO(5)$ coupling. We restrict attention to small values of the renormalization constant, $\bar{\varepsilon}$, to justify the use of perturbation theory.

We can dial the symmetry breaking parameters, v and m_4^2, to values that induce the spontaneous breaking of the gauged $SO(4)$ to $SO(3)$, while also producing a separation between the mass scales of parametrically light states and other heavier resonances. The light states, in the four dimensional language, are three massless gauge fields and three massive (but light) vectors, associated with the Higgsing $SO(4) \to SO(3)$, and one additional scalar singlet.

3. Numerical results: the mass spectrum

We provide examples of the mass spectrum of fluctuations and how they depend on the model parameters. For concreteness, we hold fixed $\Delta = 2$, $\phi_I = \phi_I(c) \approx 0.3882$, $\rho_2 - \rho_0 = 5$, and $\rho_1 - \rho_0 = 10^{-9}$ in this part. Figures 1 and 2 demonstrate how the mass spectrum varies with $\bar{\varepsilon}$, g, v, m_4^2, and k_X. The spectra in Fig. 1 are normalized to the lightest $SO(3)$-singlet spin-2 fluctuation, $\mathfrak{e}_{\mu\nu}$, while those in Fig. 2, to the lightest scalar singlet.

Figure 1 displays the spectra for a representative choice of g, k_X, Δ, ϕ_I, and m_4^2, while varying v and $\bar{\varepsilon}^2$, respectively. The figures reveal several important general characteristics. Only a few states are light: these include the massless vectors that correspond to zero modes in the unbroken, gauged $SO(3)$ sector, the lightest $\mathfrak{p}^4$ pseudoscalar, and the lightest vectors within the $SO(4)/SO(3)$ coset. All other states have larger masses, demonstrating the opening of a small hierarchy between these two sets of states. Additionally, the lightest vector states mass increases when either v or $\bar{\varepsilon}^2$ increases, and it approaches zero when either of these parameters is zero. These behaviours are expected on the basis of the fact that these light vectors acquire a mass via the Higgs mechanism.

In Figure 2, we present three examples, to illustrate what the spectrum of this semi-realistic implementation of CHM looks like. To this purpose, we denoted by H the lightest mode of the

PoS(LATTICE2024)142

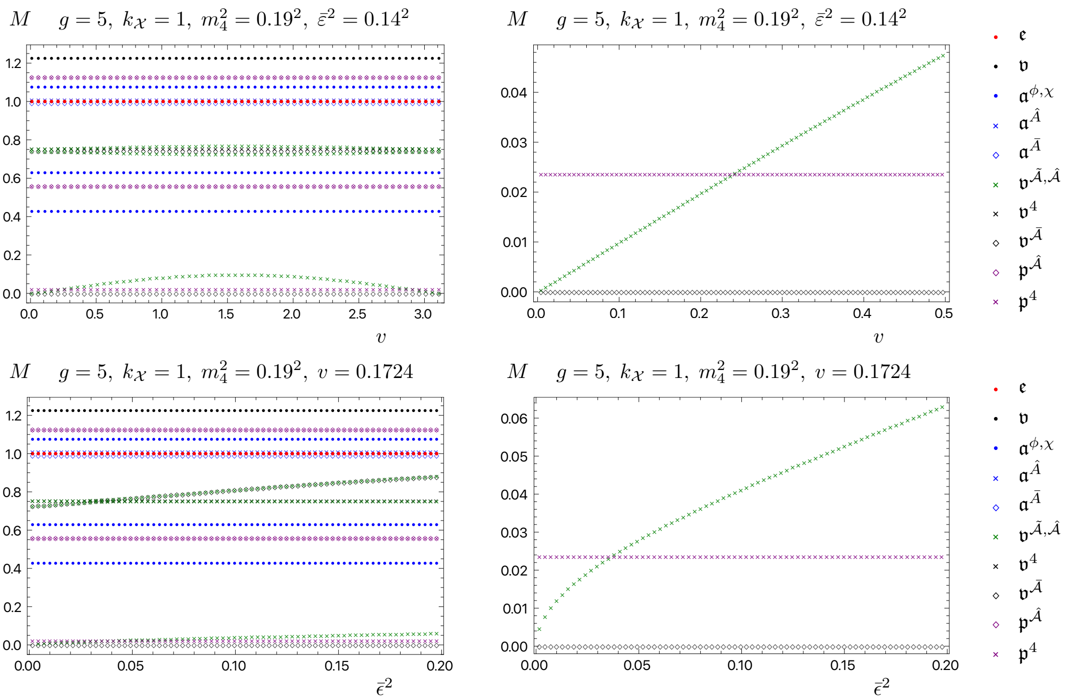

Figure 1: Figures 3 and 4 of Ref. [77]. Mass spectra, M, of (gauge-invariant) bosonic fluctuations, as functions of the parameter v (top) and $\bar{\varepsilon}^2$ (bottom). The right panels are details of the left ones.

$\mathfrak{p}^4$ fluctuations, and its mass as m_H, and normalised the other masses against it. We impose the condition $g_4 = \bar{\varepsilon} g = 0.7$, to obtain a coupling strength for the $SO(4)$ gauge fields comparable to the $SU(2)_L$ coupling in the standard model. We then adjust the remaining parameters so that the mass ratio between the lightest fluctuations in the spin-1 sector and the spin-0 sector is approximately $M_Z/m_H \simeq 0.73$, reflecting the experimental mass ratio between the Z and Higgs bosons. We show three examples of bosonic spectra that meet qualitative model-building requirements: if we identify the lighter states with experimentally established particles, with the obvious caveats, we find a rich spectroscopy of new particles appearing with large masses, after a gap in the energy range in which direct and indirect searches for new physics, so far, yield negative results.

4. Outlook

We displayed a semi-realistic implementation of vacuum misalignment that meets all the requirements of a CHM, in a context in which calculability extends to include the main properties of heavy, composite states. The framework we developed, within gauge-gravity dualities, combines the strongly coupled dynamics, captured by the bulk physics of the gravity description, with weak coupling effects, captured by the boundary terms in gravity. The two next steps of our research programme will take us in opposite, but equally important, directions. First, to make the phenomenology fully realistic, we would replace the weak gauging of $SO(4)$ with the SM

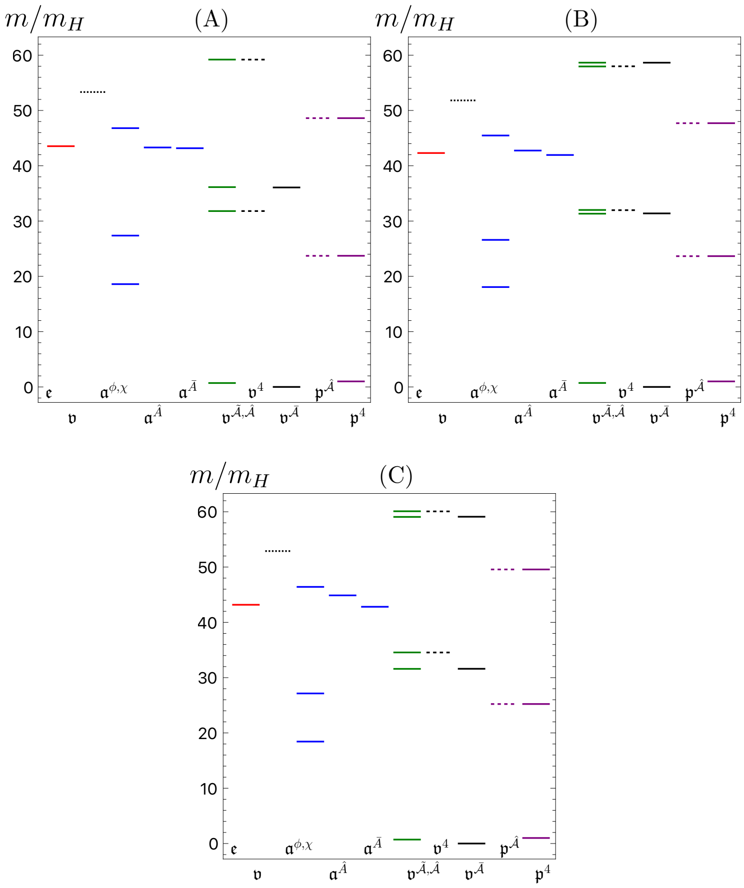

Figure 2: Figure 5 of Ref. [77]. Illustrative examples of mass spectra, showing the suppression of the scale of the three lightest states. The masses as normalised to the mass, m_H, of the lightest pseudoscalar.

group, $SU(2) \times U(1)$. It would be desirable to also include a treatment of top-quark partial compositeness [101], by extending the bulk theory to include fermions in its field content. Second, as suggested in Ref. [78], we envision replacing the current, simplified bottom-up gravity action with that of a known supergravity theory, that can be argued to derive from a theory of quantum gravity. These challenging, but realistic tasks, are left for the future.

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Research Data Access Statement—The data generated for these proceedings and the extended publication in Ref. [77] can be downloaded from Ref. [102].

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