PROCEEDINGS OF SCIENCE

$\bar{b}\bar{b}ud$ Tetraquarks with $I(J^P) = 0(1^-)$ and $\bar{b}\bar{c}ud$ Tetraquarks with $I(J^P) = 0(0^+)$ and $I(J^P) = 0(1^+)$ from Lattice QCD Antistatic-Antistatic Potentials

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We study heavy spin effects in $\bar{b}\bar{b}ud$ and $\bar{b}\bar{c}ud$ four-quark systems using the Born-Oppenheimer approximation and existing antistatic-antistatic potentials computed with lattice QCD. We report about a recent refined investigation of the $\bar{b}\bar{b}ud$ system with $I(J^P) = 0(1^-)$, where we predicted a tetraquark resonance slightly above the B^*B^* threshold. Furthermore, we extend our Born-Oppenheimer approach to $\bar{b}\bar{c}ud$ four-quark systems. For quantum numbers $I(J^P) = 0(0^+)$ as well as $I(J^P) = 0(1^+)$ we find virtual bound states rather far away from the lowest meson-meson thresholds.

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1. Introduction

In this talk we discuss our investigations of $\bar{b}\bar{b}ud$ and $\bar{b}\bar{c}ud$ four-quark systems using the Born-Oppenheimer approximation, which is a two-step approach. In the first step antistatic-antistatic potentials in the presence of two light quarks are computed with lattice QCD (see Section 2). In the second step, these potentials are used in appropriate coupled-channel Schrödinger equations, where bound states as well as resonances can be predicted using standard techniques from non-relatvistic quantum mechanics (see Section 3 and Section 4). Using such coupled-channel Schrödinger equations as well as experimental results for B, B^*, D and D^* mesons allows to take into account effects from the heavy quark spins, even though the antistatic-antistatic potentials are degenerate with respect to these spins.

In the following sections we briefly summarize a completed study of a $\bar{b}\bar{b}ud$ tetraquark resonance with quantum numbers $I(J^P) = 0(1^-)$, where details have recently been published in Ref. [8]. We also present theoretical basics and new results for $\bar{b}\bar{c}ud$ four-quark systems with quantum numbers $I(J^P) = 0(0^+)$ as well as $I(J^P) = 0(1^+)$, which have previously not been investigated within the Born-Oppenheimer approximation.

2. The Antistatic-Antistatic-Light-Light Potentials $V_5(r)$ and $V_j(r)$

Theoretical details of antistatic-antistatic potentials as well as their numerical computation with lattice QCD are extensively discussed in Refs. [1–4]. In this work we use existing potentials from Refs. [2], which were computed using $N_f = 2$ flavor ETMC gauge link ensembles [5, 6] and extrapolated to physically light u and d quark masses. Relevant in the context of this work are the two $I = 0$ potentials $V_5(r)$ and $V_j(r)$ representing the interaction of two pseudoscalar and/or vector static light mesons. Suitable parameterizations of lattice QCD results for these potentials are

$$V_X(r) = -\frac{\alpha_X}{r} e^{-(r/d_X)^2} \quad , \quad X = 5, j \tag{1}$$

with $\alpha_5 = 0.34 \pm 0.03$, $d_5 = 0.45^{+0.12}_{-0.10}$ fm (see Ref. [2]) and $\alpha_j = -0.10 \pm 0.07$ and $d_j = (0.28 \pm 0.017)$fm (see Ref. [7]). The parameterizations are shown in Figure 1. For details we refer to Section II of our recent publication [8].

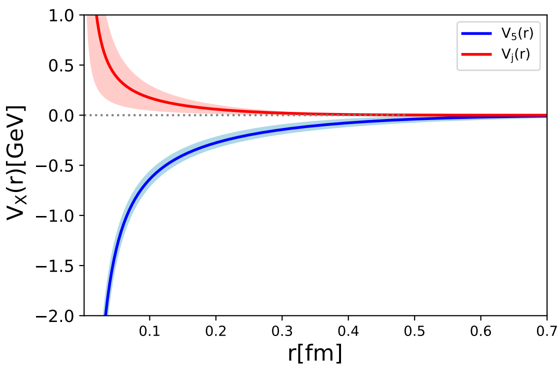

Figure 1: Parametrizations of lattice QCD results from Ref. [2] for the $\bar{Q}\bar{Q}qq$ potentials $V_5(r)$ and $V_j(r)$.

3. Coupled-Channel Schrödinger Equations

3.1 Identical Heavy Flavors: $\bar{b}\bar{b}ud$ with $I(J^P) = 0(1^-)$

In Ref. [8] we have derived the coupled-channel Schrödinger equation relevant for the $\bar{b}\bar{b}ud$ system with $I(J^P) = 0(1^-)$. It is given by

$$\left(\begin{pmatrix} 2m_B & 0 \\ 0 & 2m_{B^*} \end{pmatrix} - \frac{\nabla^2}{2\mu_{bb}} + H_{\text{int},S=0}\right) \vec{\varphi}_{L=1,S=0}(r) = E\vec{\varphi}_{L=1,S=0}(r) \tag{2}$$

with

$$\nabla^2 = \frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - \frac{L(L+1)}{r^2}\bigg|_{L=1} = \frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - \frac{2}{r^2} \tag{3}$$

(L represents the orbital angular momentum of the heavy antiquarks) and

$$H_{\text{int},S=0} = \frac{1}{4}\begin{pmatrix} V_5(r) + 3V_j(r) & \sqrt{3}(V_5(r) - V_j(r)) \\ \sqrt{3}(V_5(r) - V_j(r)) & 3V_5(r) + V_j(r) \end{pmatrix}, \tag{4}$$

where $\mu_{bb} = m_b/2$ is the reduced b quark mass, $S = 0$ denotes the heavy spin and r is the radial coordinate of the heavy antiquark separation. The 2 components of the wave function represent the following meson-meson combinations:

$$\vec{\varphi}_{L=1,S=0} \equiv \left(BB\ ,\ \frac{1}{\sqrt{3}}\vec{B}^*\vec{B}^*\right)^T = \left(BB\ ,\ \frac{1}{\sqrt{3}}\left(B^*_x B^*_x + B^*_y B^*_y + B^*_z B^*_z\right)\right)^T. \tag{5}$$

3.2 Different Heavy Flavors: $\bar{b}\bar{c}ud$ with $I(J^P) = 0(0^+)$ and $I(J^P) = 0(1^+)$

To derive the coupled-channel Schrödinger equations for the $\bar{b}\bar{c}ud$ systems one can closely follow Refs. [7, 8] as sketched in the following.

$\bar{b}\bar{c}ud$ systems at large $\bar{b}\bar{c}$ separations r are meson pairs, where one of the two mesons is a B or B^* meson and the other meson is a D or D^* meson. Consequently, the free Hamiltonian describing non-interacting meson pairs in the center of mass frame has a 16×16 matrix structure,

$$H_0 = M_B \otimes \mathbb{1}_{4\times4} + \mathbb{1}_{4\times4} \otimes M_D + \frac{\vec{p}^2}{2\mu_{bc}}, \tag{6}$$

where $M_B = \text{diag}(m_B, m_{B^*}, m_{B^*}, m_{B^*})$ and $M_D = \text{diag}(m_D, m_{D^*}, m_{D^*}, m_{D^*})$ are diagonal matrices containing the meson masses and $\mu_{bc} = m_b m_c/(m_b + m_c)$ is the reduced mass of a b and a c quark. This Hamiltonian acts on a 16-component wave function for the relative coordinate of the heavy quarks $\vec{r}$, where the components can be interpreted as

$$\vec{\Psi} \equiv \Big(BD, BD^*_x, BD^*_y, BD^*_z\ ,\ B^*_xD, B^*_xD^*_x, B^*_xD^*_y, B^*_xD^*_z\ ,\ B^*_yD, B^*_yD^*_x, B^*_yD^*_y, B^*_yD^*_z\ ,$$
$$B^*_zD, B^*_zD^*_x, B^*_zD^*_y, B^*_zD^*_z\Big)^T. \tag{7}$$

To include interactions, one has to add the the potentials $V_5(r)$ and $V_j(r)$ discussed in Section 2. One can show that these potentials do not correspond to simple meson pairs, as represented by the components of $\vec{\Psi}$, but to linear combinations containing all four types of mesons, B, B^*, D and D^*.

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These linear combinations can be expressed in terms of a 16×16 matrix T using Fierz identities. The interacting part of the Hamiltonian is then

$$H_{\text{int}} = T V_{\text{diag}} T^{-1} \quad , \quad V_{\text{diag}} = \text{diag}(\underbrace{V_5(r), \ldots, V_5(r)}_{4\times}, \underbrace{V_j(r), \ldots, V_j(r)}_{12\times}). \tag{8}$$

Combining Eq. (6) and Eq. (8) leads to the Schrödinger equation

$$H\vec{\Psi}(\vec{r}) = \Big(H_0 + H_{\text{int}}\Big)\vec{\Psi}(\vec{r}) = E\vec{\Psi}(\vec{r}). \tag{9}$$

Because H_{int} only depends on the radial coordinate $r = |\vec{r}|$, but not on the direction of $\vec{r}$, the orbital angular momentum L is conserved. Since the total angular momentum is a conserved quantity, the total spin S is also conserved. Consequently, both L and S can be used as quantum numbers. The former allows to reduce the partial differential equation (9) to an ordinary differential equation in r, while the latter allows to decompose the 16×16 Hamiltonian into smaller blocks.

We are particularly interested in $\bar{b}\bar{c}ud$ systems with quantum numbers $I(J^P) = 0(0^+)$ and $I(J^P) = 0(1^+)$, since recent full lattice QCD computations indicate the existence of shallow bound states for these systems [9–11]. After working out the $I(J^P)$ quantum numbers for each block of the decomposed Hamiltonian using QCD symmetries and the Pauli principle (for details see e.g. Section III.III in Ref. [8]), one can read off the relevant coupled channel Schrödinger equations.

Schrödinger Equation for $\bar{b}\bar{c}ud$ with $I(J^P) = 0(0^+)$

The coupled-channel Schrödinger equation for the $\bar{b}\bar{c}ud$ system with $I(J^P) = 0(0^+)$ is

$$\left(\begin{pmatrix} m_B + m_D & 0 \\ 0 & m_{B^*} + m_{D^*} \end{pmatrix} - \frac{1}{2\mu_{bc}}\frac{d^2}{dr^2} + H_{\text{int},S=0}\right)\vec{\varphi}_{L=0,S=0}(r) = E\vec{\varphi}_{L=0,S=0}(r) \tag{10}$$

with $H_{\text{int},S=0}$ as defined in Eq. (4). The 2 components of the wave function represent the following meson-meson combinations:

$$\vec{\varphi}_{L=0,S=0} \equiv \left(BD \ , \ \frac{1}{\sqrt{3}}\vec{B}^*\vec{D}^*\right)^T = \left(BD \ , \ \frac{1}{\sqrt{3}}\Big(B^*_x D^*_x + B^*_y D^*_y + B^*_z D^*_z\Big)\right)^T. \tag{11}$$

Schrödinger Equation for $\bar{b}\bar{c}ud$ with $I(J^P) = 0(1^+)$

The coupled-channel Schrödinger equation for the $\bar{b}\bar{c}ud$ system with $I(J^P) = 0(1^+)$ is

$$\left(\begin{pmatrix} m_{B^*} + m_D & 0 & 0 \\ 0 & m_B + m_{D^*} & 0 \\ 0 & 0 & m_{B^*} + m_{D^*} \end{pmatrix} - \frac{1}{2\mu_{bc}}\frac{d^2}{dr^2} + H_{\text{int},S=1}\right)\vec{\varphi}_{L=0,S=1,S_z}(r) = E\vec{\varphi}_{L=0,S=1,S_z}(r) \tag{12}$$

with

$$H_{\text{int},S=1} = \frac{1}{4}\begin{pmatrix} V_5(r) + 3V_j(r) & V_j(r) - V_5(r) & \sqrt{2}(V_5(r) - V_j(r)) \\ V_j(r) - V_5(r) & V_5(r) + 3V_j(r) & \sqrt{2}(V_j(r) - V_5(r)) \\ \sqrt{2}(V_5(r) - V_j(r)) & \sqrt{2}(V_j(r) - V_5(r)) & 2(V_5(r) + V_j(r)) \end{pmatrix}. \tag{13}$$

The 3 components of the wave function represent the following meson-meson combinations:

$$\vec{\varphi}_{L=0,S=1,S_z} \equiv \Big(B^*_{S_z} D, B D^*_{S_z}, T_{1,S_z}(\vec{B}^*, \vec{D}^*)\Big)^T \tag{14}$$

with T_{1,S_z} denoting a spherical tensor coupling the three spin orientations of a B^* and of a D^* meson to a total spin $S = 1$ with z component S_z.

4. Scattering Formalism and the T matrix

Possibly existing bound states and resonances can be studied within the same formalism, by writing the wave function as a sum of an incident plane wave and an emergent spherical wave and by carrying out a partial wave expansion. Then, one can read off the T matrix and determine its poles in the complex energy plane, which signal bound states or virtual bound states (for $\mathrm{Re}(E_{\mathrm{pole}}) < 2m_B$ and $\mathrm{Im}(E_{\mathrm{pole}}) = 0$) or resonances (for $\mathrm{Re}(E_{\mathrm{pole}}) > 2m_B$ and $\mathrm{Im}(E_{\mathrm{pole}}) < 0$). For details we refer to Section IV of our recent publication [8].

4.1 The T matrix for the $\bar{b}\bar{b}ud$ system with $I(J^P) = 0(1^-)$

After the aforementioned partial wave expansion the $L = 1$ wave function (5) becomes

$$\vec{\varphi}_{L=1,S=0}(r) = \begin{pmatrix} A_{BB} j_1(k_{BB} r) + \chi_{BB}(r)/r \\ A_{B^*B^*} j_1(k_{B^*B^*} r) + \chi_{B^*B^*}(r)/r \end{pmatrix}, \tag{15}$$

where A_{BB} and $A_{B^*B^*}$ are the prefactors of the incident BB and B^*B^* waves, respectively, $j_1(k_{BB}r)$ and $j_1(k_{B^*B^*}r)$ denote spherical Bessel functions with scattering momenta $k_{BB} = \sqrt{2\mu(E - 2m_B)}$ and $k_{B^*B^*} = \sqrt{2\mu(E - 2m_{B^*})}$ representing the $L = 1$ contribution to these incident plane waves and $\chi_{BB}(r)/r$ and $\chi_{B^*B^*}(r)/r$ are the radial wave functions of the emergent BB and B^*B^* spherical waves. Inserting $\vec{\varphi}_{L=1,S=0}(r)$ from Eq. (15) into the Schrödinger equation (2) leads to

$$\left(\begin{pmatrix} 2m_B & 0 \\ 0 & 2m_{B^*} \end{pmatrix} - \frac{1}{2\mu_{bb}}\left(\frac{d^2}{dr^2} - \frac{2}{r^2}\right) + H_{\mathrm{int},S=0} - E\right)\begin{pmatrix} \chi_{BB}(r) \\ \chi_{B^*B^*}(r) \end{pmatrix} = -H_{\mathrm{int},S=0}\begin{pmatrix} A_{BB} r j_1(k_{BB} r) \\ A_{B^*B^*} r j_1(k_{B^*B^*} r) \end{pmatrix}. \tag{16}$$

As usual, the boundary conditions for the wave functions close to the origin are $\chi_\alpha(r) \propto r^{L+1}|_{L=1} = r^2$. For large r the wave functions $\chi_\alpha(r)$ exclusively describe emergent spherical waves and, thus, are proportional to spherical Hankel functions,

$$\chi_\alpha(r) \propto irt_{BB;\alpha} h_1^{(1)}(k_\alpha r) \qquad \text{for } r \to \infty \text{ and } (A_{BB}, A_{B^*B^*}) = (1, 0) \tag{17}$$

$$\chi_\alpha(r) \propto irt_{B^*B^*;\alpha} h_1^{(1)}(k_\alpha r) \qquad \text{for } r \to \infty \text{ and } (A_{BB}, A_{B^*B^*}) = (0, 1), \tag{18}$$

where $t_{\alpha;\beta}$ denote entries of the T matrix. Thus, Eq. (17) and Eq. (18) allow to determine the 2×2 T matrix,

$$\mathrm{T} = \begin{pmatrix} t_{BB;BB} & t_{BB;B^*B^*} \\ t_{B^*B^*;BB} & t_{B^*B^*;B^*B^*} \end{pmatrix}. \tag{19}$$

4.2 T Matrices for the $\bar{b}\bar{c}ud$ Systems with $I(J^P) = 0(0^+)$ and $I(J^P) = 0(1^+)$

One can proceed as sketched in Section 4.1. Because $L = 0$ in both cases one has to replace j_1 by j_0. Moreover, scattering momenta have to be defined according to the meson types associated with each channel. At the end one arrives at a 2×2 T matrix for $I(J^P) = 0(0^+)$ and at a 3×3 T matrix for $I(J^P) = 0(1^+)$. Because of the page limit, we refrain from providing the corresponding equations.

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5. Numerical Results

The following numerical results were generated using quark masses $m_b = 4977\,\text{MeV}$ and $m_c = 1628\,\text{MeV}$ taken from a quark model [12]. For the meson mass splittings we use $m_{B^*} - m_B = 45\,\text{MeV}$ and $m_{D^*} - m_D = 138\,\text{MeV}$ as quoted by the PDG [13]. We solved the coupled-channel radial Schrödinger equations for the wave functions of the emergent wave $\chi_\alpha(r)$ for given complex energy E using a standard fourth order Runge-Kutta integrator (e.g. Eq. (16) in the case of the $\bar{b}\bar{b}ud$ system with $I(J^P) = 0(1^-)$). Then we read off the corresponding T matrix elements from the behavior of the resulting $\chi_\alpha(r)$ at large r, using e.g. Eq. (17) and Eq. (18) for the $\bar{b}\bar{b}ud$ system with $I(J^P) = 0(1^-)$. Finally, we determine the poles of the T matrix by numerically searching for roots of $1/\det(\text{T})$. For details we refer again to our recent publication [8].

5.1 $\bar{b}\bar{b}ud$ Tetraquark Resonance with $I(J^P) = 0(1^-)$

Numerical results for the $\bar{b}\bar{b}ud$ system with $I(J^P) = 0(1^-)$ are extensively discussed in Ref. [8]. Our main findings are the following:

(1) We found a pole of the T matrix on the $(-,-)$-Riemann sheet [1] indicating a tetraquark resonance with mass $2m_B + 94.0^{+1.3}_{-5.4}\,\text{MeV} = 2m_{B^*} + 4.0^{+1.3}_{-5.4}\,\text{MeV}$, i.e. slightly above the B^*B^* threshold, and decay width $\Gamma = 140^{+86}_{-66}\,\text{MeV}$.

(2) The coupled channel Schrödiger equation (16), in particular the potential matrix (4), led to a solid physical understanding, why there is a tetraquark resonance close to the B^*B^* threshold, but not in the region of the BB threshold, as naively expected from our previous work [14] using a simplified single-channel approach. The reason is that the attractive potential $V_5(r)$ dominates the B^*B^* channel, but is strongly suppressed in the BB channel, whereas the situation is reversed for the repulsive potential $V_j(r)$.

(3) This theoretical result is supported by our computation of branching ratios, where we found $\text{BR}_{BB} = 26^{+9}_{-4}\%$ and $\text{BR}_{B^*B^*} = 74^{+4}_{-9}\%$, implying that a decay of the tetraquark resonance is around three times more likely to a B^*B^* pair than to a BB pair.

5.2 $\bar{b}\bar{c}ud$ virtual bound states with $I(J^P) = 0(0^+)$ and $I(J^P) = 0(1^+)$

Virtual Bound States

Using the same techniques as for the $\bar{b}\bar{b}ud$ tetraquark resonance with $I(J^P) = 0(1^-)$, we also searched for poles of the T matrix in the complex energy plane for the two $\bar{b}\bar{c}ud$ systems. These pole searches were carried out on all four Riemann sheets for $I(J^P) = 0(0^+)$ and on all eight Riemann sheets for $I(J^P) = 0(1^+)$. For both systems we did neither find bound states nor resonances, but virtual bound states, indicated by poles on the negative real axis on the $(-,+)$-sheet and on the

[1] For n scattering channels there are n scattering momenta k_α and 2^n Riemann sheets for the complex energy E. These sheets are labeled by the signs of the imaginary parts of the scattering momenta, e.g. by $(\text{sign}(k_{BB}), \text{sign}(k_{B^*B^*}))$ for the $\bar{b}\bar{b}ud$ system with $I(J^P) = 0(1^-)$. There is a one-to-one correspondence between bound states and poles on the negative real axis of the physical Riemann sheet, which is characterized by having exclusively positive signs, e.g. the $(+,+)$ sheet in the case of 2-channel scattering.

$(-,+,+)$-sheet, respectively. These poles are, however, rather far away from the lowest meson-meson thresholds, $\text{Re}(E)-(m_B+m_D) = -106^{+65}_{-148}$ MeV and $\text{Re}(E)-(m_{B^*}+m_D) = -100^{49}_{-212}$ MeV. Thus, it is questionable, whether they have a sizable effect on physical observables like scattering rates or cross sections. We plan to investigate this in more detail in the near future.

Dependence on the Charm Quark Mass for $I(J^P) = 0(1^+)$

In Ref. [7] we used the same potentials and formalism discussed in Section 2 and Section 3 to predict a $\bar{b}\bar{b}ud$ bound state with quantum numbers $I(J^P) = 0(1^+)$ and binding energy $(m_B + m_{B^*}) - E = 59^{+30}_{-38}$ MeV. This system, which has a BB^* channel and a B^*B^* channel is conceptually similar to the $\bar{b}\bar{c}ud$ system with the same quantum numbers. In particular, one can show that, when setting $m_c = m_b$, $m_D = m_B$ and $m_{D^*} = m_{B^*}$ in the coupled channel Schrödinger equation (12), one equation decouples and the remaining two equations are essentially identical to those solved in Ref. [7]. We have studied this numerically by starting with Eq. (12) and physical quark masses m_b and m_c as provided at the beginning of Section 5 and then continously increasing m_c from its physical value 1628 MeV to the value of the b quark mass. At the same time we reduce the mass splitting between the D and the D^* meson according to

$$m_{D^*} - m_D = \frac{m_b}{m_c}\Big(m_{B^*} - m_B\Big), \tag{20}$$

which is the leading order in Heavy Quark Effective Theory [15]. The resulting pole energy as function of m_c is shown in Figure 2. One can see the expected transition from a virtual bound state corresponding to a pole on the $(-,+,+)$-Riemann sheet to a bound state corresponding to a pole on the physical $(+,+,+)$-Riemann sheet. The transition between the two sheets takes place at $m_c \approx 2930$ MeV, where the pole is located at $E = 0$. For $m_c = m_b$ we recover the binding energy $(m_B + m_{B^*}) - E = 59^{+30}_{-38}$ MeV predicted in Ref. [7], which is an excellent cross check and shows consistency of this work and Ref. [7].

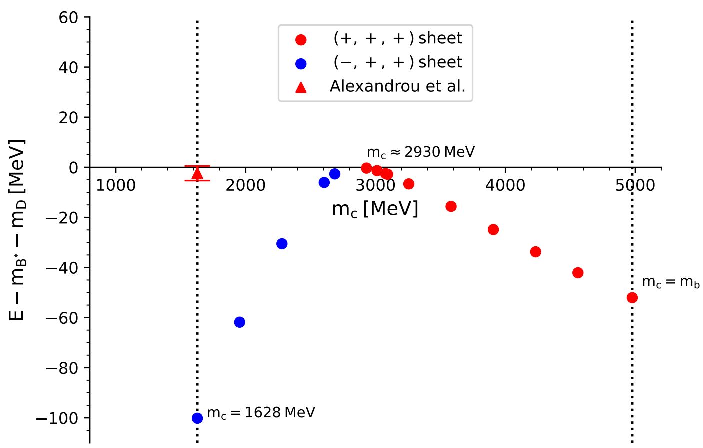

Figure 2: The energy of the T matrix pole as function of the charm quark mass m_c for the $\bar{b}\bar{c}ud$ system with quantum numbers $I(J^P) = 0(1^+)$. The red triangular data point represents the full lattice QCD result from Ref. [10].

Comparison to Full Lattice QCD Results

Recent full lattice QCD studies of $\bar{b}\bar{c}ud$ systems with quantum numbers $I(J^P) = 0(0^+)$ and $I(J^P) = 0(1^+)$ have predicted shallow bound states rather close to the BD threshold and the B^*D threshold, respectively [9–11] (the result from Ref. [10] for $I(J^P) = 0(1^+)$ is plotted in Figure 2). It is interesting to note that the study from Ref. [10], which uses a very advanced lattice QCD setup (large symmetric correlation matrices including both local and scattering interpolating operators, Lüschers finite volume method to carry out a scattering analysis), cannot rule out the existence of shallow virtual bound states, even though genuine bound states are strongly favored. In any case there is a sizable quantitative difference of these full lattice QCD results and our $\bar{b}\bar{c}ud$ predictions from this work, which are based on lattice QCD potentials and the Born-Oppenheimer approximation. A possible reason for that could be that the attraction of the potential $V_5(r)$ was underestimated in Refs. [1, 2]. To check this, we have recently started a recomputation of these potentials using a significantly improved up-to-date lattice QCD setup [3, 4].

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