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Prospects for lattice field theory beyond the Standard Model

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This talk summarizes the explorations of theories beyond the Standard Model with lattice simulations. After a brief comment on the current status of the Standard Model extensions, the essential contribution made by numerical simulations in various approaches is discussed. This poses new challenges for simulation methods. The interplay with new theories gives rise to more general theoretical considerations that establish a close relationship with investigations of fundamental concepts such as gauge/gravity duality, confinement, or renormalization group flow. Recent results presented at this conference are discussed, which include in particular composite Dark Matter models, Sp(4) gauge theory, gauge theories at the lower end of the conformal window, and supersymmetric gauge theories.

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1. Theories beyond the Standard Model and lattice field theory

There is already a substantial history of the search for extensions of the Standard Model of particle physics, but the primary motivation still remains unchanged. In essence, the objective is to identify and incorporate missing parts, either to explain observations or to ameliorate aesthetic imperfection.

Phenomenological indications for incompleteness are the missing explanations for Dark Matter and Dark Energy, gravitational interactions, neutrino masses, or baryon asymmetry.

Aesthetic imperfections are more controversial but should still be acknowledged, as such considerations have always been a driving force behind important theoretical developments. In this sense, the role of the Standard Model as a parameterisation rather than an explanation could be questioned. Unification of fundamental forces or additional symmetry might provide a more comprehensible picture. One might hope to explain structures like the flavour hierarchy or the strong CP problem from a more fundamental view point. The hierarchy Problem has been a seminal concept in theoretical developments, manifesting as either a fine-tuning issue for the mass of the Higgs particle in view of its substantial quantum corrections or as an unresolved explanation for the considerable gap between the Planck scale and the scale of electroweak physics.

Another issue with the Standard Model is our lack of understanding of some core mechanisms like confinement or, in a more fundamental sense, quantum gravity.

Despite of its remarkable success, all of these imperfections have led to a large variety of proposals for theories beyond the Standard Model. These activities have stimulated new developments of theoretical concepts and tools which have lead to a better general understanding of quantum field theory. New concepts like supersymmetry have emerged and our general comprehension of the theories was subjected to scrutiny when, for example, additional non-QCD like strong interactions were introduced.

The search for physics beyond the Standard Model has stimulated a large experimental program. Over several decades the energy range accessible by detectors has exponentially increased, an achievement of experimental development, which coincided with the theoretical development of the Standard Model. The large hadron collider already marked the end of a period of substantial growth in accelerator energies. At the same time it has lead to the discovery of the last missing piece of the Standard Model, the Higgs boson, which makes it a self-contained theory. Hence there is no guarantee for a further new discovery up to very high energy scales. It might be that there are signs for new physics just within the reach of next experiments, but it might also be that there is no new discovery up to energies close to the Planck scale [1].

For phenomenology this implies that the collider discoveries are primarily constraints of models. New physics needs to be weakly coupled to the Standard Model or its energy scale should be sufficiently large. Some improvements on of experiments at the energy frontier might still be expected. Precision measurements provide an alternative way to connect theory and observations even in case the coupling between Standard Model and new physics is very weak or hidden at high energies. Astrophysics and cosmology can provide additional inputs without the constraints of collider experiments. The disadvantage of these alternatives are more ambiguities in the interpretation and additional assumptions which need to be controlled. The recent investigations of the anomalous magnetic moment of the muon are an examples of the resulting challenges.

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What can lattice field theory provide at this general state of high energy physics? There are different directions that are currently followed. An important input are high precision data for QCD corrections relevant in experiments. Precision matters for phenomenology without direct access to very high energies and therefore even subleading contributions are relevant. The lattice can hence explain observations or confirm mismatch with the Standard Model. Investigations along these lines are part of other contributions at this conference.

Numerical simulations of lattice field theory can also challenge the assumptions of approaches for an extension of the Standard Model based on strong interactions. The phenomenological considerations rely on certain assumption about the low energy effective theories and the bound states of the strong interactions. Like an experiment, lattice field theory can probe these assumptions provides additional constraints even beyond theories testable by experiments.

Such kind of investigations lead us to consider the more general theory space of strong interactions. In fact, the understanding of this theory space provides a further motivation for numerical investigations of strong interactions beyond the Standard Model. This motivation has gained more interest in recent years since it is independent of the experimental input and currently favored models. Our general understanding of strong interactions is biased by our knowledge of QCD and it is very interesting to search for strongly insteracting theories which might have a completely different behavior.

Many connections have been established from the research field of strong interactions beyond the Standard Model to other research topics. Our understanding of confinement and the running coupling benefits from the considerations of theories with larger number of fermions in different representations. Analytic methods like the large N limit or semiclassical expansions derived in certain special limits of theory space can be connected to QCD via explorations of beyond Standard Model theories. Quantum gravity is connected to investigations of supersymmetric gauge theories via gauge/gravity duality. In addition, the non-standard considerations of theories on the lattice might even lead to new approaches towards quantum computing of lattice gauge theories [2, 3].

2. Concepts of Standard Model extensions

The number of lattice investigations beyond the Standard Model presented at this conference is quite large and not everything can be covered here. For example, it is impossible to cover the strong CP problem and axion searches, which is related to the field of QCD vacuum, or the many approaches of numerical quantum gravity covered in separate conferences and workshops.

In order to guide the reader, an introduction of some underlying basic concepts is provided in this first section. Specific theories considered in different contributions are discussed afterwards.

2.1 Supersymmetry

Supersymmetry has been primarily introduced in the context of beyond Standard Model physics as a solution to the hierarchy problem. The symmetry relates fermions and bosons, which implies cancellation between fermionic and bosonic quantum corrections to the Higgs mass. Hence the Higgs mass becomes naturally small due to approximate supersymmetry like small quark masses appear natural due to approximate chiral symmetry. Some time ago, supersymmetry has been considered to have the highest potential for discovery in collider experiments. It provides also a

Dark Matter scenario in terms of weakly interacting massive particles, which fits nicely into models of early universe evolution. However, the simplest supersymmetric extensions are disfavored by experiments [4]. Note that there is always an implicit assumption about an underlying mechanism for supersymmetry breaking requiring some non-perturbative dynamics.

Supersymmetry is a general concept and not only a tool to solve the hierarchy problem. Supersymmetry has allowed many interesting insights into mechanisms of strong interactions such as fermion condensation and chiral symmetry breaking. It has also stimulated other theoretical developments like gauge/gravity dualities. Therefore the focus of numerical studies of strongly interacting supersymmetric gauge theories has shifted from phenomenological applications to conceptual theoretical questions. Overall supersymmetry remains a very active topic of current research.

2.2 Composite Higgs and technicolor

The Higgs particle is quite exceptional in the Standard Model. It is the only scalar field, while all other parts are composed of fermions interacting via gauge boson exchange. Hence, it seems an attractive idea to replace the Higgs particle with a bound state of a new strong interaction of fermions and gauge fields, sometimes called hypercolor. This extension solves the hierarchy problem since an additional cutoff is introduced by the energy scale Λ_{HC} of the new interactions. Like in QCD, this scale is dynamically generated. The Standard Model has to emerge as a low energy effective theory. In a simplification of this picture, the hypercolor sector can be considered separately to generate a low energy effective theory. To complete the Standard Model, this effective theory is coupled weakly to a second sector. Suppression of the interactions between theses sectors can be due to small couplings or higher dimensional operators suppressed by an additional even higher scale. It is required that the theory replaces all effects of the Higgs in the Standard Model, which are primarily the generation of W, Z, and fermion masses. In addition, the appearance of a scalar bound state without further resonances in the particle spectrum, as indicated by the experiments, should follow from the underlying theory. Reviews on this topic can be found in [5–7].

Masses of W and Z bosons can be generated from effective coupling between hypercolor and weak sector following the original idea of technicolor [8]. The generation of fermion masses requires the assumption of an extended technicolor theory at an even higher energy scale Λ_{EHC}. Suppressed by this scale, effective interactions are introduced, which couple the fermion condensate of the technicolor theory to the Standard Model fermions [9]. Masses are generated via these effective couplings. However, generically other four fermion operators are also generated in this process which would lead to conflicts with electroweak precision data. In consequence, a QCD-like version of strong interactions would not be a viable candidate for technicolor.

To arrive at a possible solution, it is necessary to consider strong interactions that differ significantly from those of QCD [10, 11]. An increase in the number of fermions, N_f, results in a change in the running of the strong coupling. In a theory similar to QCD, this running is described by an exponential scaling relating asymptotic freedom at high energies to confinement at low energies. As the number of fermions, N_f, increases, screening by fermions becomes dominant, resulting in the loss of asymptotic freedom, and the complete beta function takes on the opposite sign compared to QCD. Between these two regimes, a so-called "conformal window" is expected, in which the beta function changes sign at a certain point [12]. This is an infrared fixed point with a zero beta function. Just below the conformal window, the strong coupling runs slower as a function of the energy scale, implying that the theory remains in a strongly coupled regime over a large range of scales. The conjectured scenario of walking technicolor is that a large mass anomalous dimension in this regime leads to an enhancement of the scalar condensate and the related effective coupling compared other operators, which leads to fermion masses without tensions with experimental observations. Furthermore, the approximate scale invariance close to the conformal fixed point might imply a dilaton-like scalar bound state separated from the rest of the particle spectrum resembling the Higgs particle.

In the dilaton-like Higgs scenario, breaking of scale invariance is related to the lightness of the Higgs. In a natural way, a light particle compared to Λ_{HC} can also be achieved by chiral symmetry breaking if the Higgs appears as a pseudo Nambu-Goldstone boson (pNGb). The chiral symmetry breaking pattern of QCD leads to pseudoscalar pNGbs, the pions. A pseudoreal gauge group with two fermions in the fundamental representation, on the other hand, leads to a breaking pattern of $\text{SU}(4) \to \text{Sp}(4)$ inducing scalar pNGbs. Hence the low energy effective theory can in this case resemble a light Higgs well separated from the rest of the spectrum.

In order to complete a candidate composite Higgs theory, further considerations are required. The coupling to Standard Model gauge fields requires an unbroken subgroup of the chiral group consistent with their gauge symmetries. Fermion mass generation, especially for the top quark, is implemented by additional higher dimensional operators. One main suggestion is known as partial compositeness [13]: effective couplings between the Standard Model fermions and composites of the new strong sector are added. It requires a composite operator combining three fermion fields coupled to Standard Model fermions. It can be composed from fermions in different representations of the gauge group. Overall, the search for candidate composite Higgs theories with these requirements is a question related to group theory: which theories contain the correct chiral symmetry breaking pattern and a sufficient fermion content for partial compositeness. This question has been addressed for example in [14]. With this classification of possible candidates, lattice simulations can test the phenomenological relevance of this approach.

While composite Higgs and Technicolor seem to be different approaches to a Standard Model extension, there are significant similarities. Theories can combine both effects and the Higgs can be a mixture of a dilaton-like state and a pNGb [15]. The fermion mass generation might require more generally effective operators with large anomalous dimensions, which are enhanced in a walking scenario.

In all of these approaches, one is lead to a more general consideration of strong interactions in particular regarding theories much different from QCD. With any kind of strongly interacting Standard Model extension, additional bound states appear quite naturally. Some of them might be stable and lead to Dark Matter candidates.

2.3 Composite Dark Matter

The theories introduced in previous sections explain in a natural way the appearance of possible Dark Matter particles. In a far more general sense one might search for Dark Matter candidates introduced by strong interactions beyond the Standard Model. It might seem that a large variety of theories could serve as a candidate for strongly interacting Dark Matter as long as the couplings to the Standard Model are sufficiently small to hide it from current detections. However considering scenarios in more detail leads to relevant additional constraints limiting the theory space. A review of this topic is presented, for example, in [16].

The current relic abundance and density of Dark Matter, which is around five times the density of Standard Model matter, is a constraint for models describing the evolution of the early universe. In famous weakly interacting massive particle scenario the desired conditions are provided by new weak-scale particles. This WIMP scenario has been suggested in particular for supersymmetric extensions of the Standard Model [17]. One possible alternative scenario based on strongly interacting massive particles (SIMP) is obtained by a possible decay of Dark Matter density in 3 to 2 processes introduced by a Wess-Zumino-Witten term [18, 19]. Theories with Sp(N_c) gauge group have been suggested for this scenario, but it also applies to a more general class of theories.

The approaches of WIMP and SIMP Dark Matter are not the only possibilities. Alternatively one might think of more general processes reducing the particle number. In a different approach the matter/anti-matter asymmetry of the Standard Model might be connected to an asymmetry in the Dark Matter sector leading to a relation of the observed relic abundance (asymmetric Dark Matter) [20]. While it is hard to summarize all of the proposed scenarios, a main common requirement is a non-zero coupling between Dark Matter and the Standard Model, which is in particular relevant for the evolution of the early universe.

Another input are constraints on Dark Matter self interactions, which are supposed to be small but non-zero. There are considerable uncertainties in these findings, but they still provide relevant constraints for composite Dark Matter. If the Dark Matter candidate is a bound state of strong interactions, a further constraint might be obtained from astronomical observations. Confinement of the strong forces can induce a phase transition in the early universe. If this transition is of first order, it might have produced a gravitational wave signal. The presence or absence of such a signal in future measurements leads to constraints of the parameter space. In particular, its absence might provide an upper limit for masses of fundamental fermions since the transition needs to be weakened by these fields.

3. Lattice investigations of strongly interacting Dark Matter

Several aspects of theories with strongly interacting Dark Matter have recently been investigated as a continuation and extension of earlier studies. In the following a selection of new results is discussed.

3.1 Stealth Dark Matter

Experimental inputs suggest that a strongly interacting Dark Matter theory should be hidden, but the cosmological evolution requires certain interactions in the early universe. These two aspects can be matched if a theory is hidden by confinement such that the stable bound state of the theory is a neutral scalar particle. A natural stable bound state describing Dark Matter is of baryonic type, like the stable baryons of QCD. Such a theory could also be considered as candidate for asymmetric Dark Matter.

The simplest realization of this theory is an SU(4) gauge theory with four fermions in the fundamental representation [21]. A Higgs coupling can be added provided it is sufficiently small. The only coupling to the Standard Model would hence be via polarizability and Higgs coupling.

This is a short summary of the stealth Dark Matter model. One main advantage of this model is that it allows relevant mass scales close to experimental detection, but still sufficiently hides Dark Matter components.

Lattice investigations and studies of this theory have revealed several new insights. Direct detection limits have been derived from electromagnetic polarizability and Higgs coupling [21, 22]. It has been concluded that a lower bound for the mass of the Dark Matter particle is $M_{DM} > 0.2$ TeV and further bound state masses have been obtained. In order to provide also an upper limit on the Dark Matter mass, gravitational wave signatures from the deconfinement transition have been considered [23]. An open challenge, which is nevertheless very important for phenomenology, are estimates of self interactions. First steps towards a determination have been done in [24], but the completion still requires further efforts. Self interactions are a crucial ingredient for a comparison with Dark Matter models and cosmological evolution.

Recently the theory space has been extended. The four fermions can have different masses, which leads to a wider range of possibilities. One particular choice has been called hyper stealth Dark Matter [25] and effectively separates out one significantly lighter fermion. This reduces experimental constraints and allows Dark Matter masses down to a few GeV. Effectively one can reduce the theory in this case to just one fermion flavor. In a one flavor theory, chiral symmetry is broken by the anomaly and therefore the signal for the chiral limit is less obvious. It is hence desirable to have a chiral symmetric lattice realization such as provided by domain-wall fermions. First investigations with this lattice action have been provided in contributions to this conference [26]. These studies have been mainly concerned with estimates of the phase diagram. Once completed, they can provide information about gravitational wave signatures from the transitions in the early universe and constrain in this way the upper limit of fermion masses.

3.2 Lattice input for gravitational wave signals

Gravitational wave signals have a considerable potential for observations of beyond Standard Model physics. Detectors such as Laser Interferometer Space Antenna (LISA) will provide new data even in the absence of new collider experiments, see e. g. [27] for an overview. These can indicate or constrain strongly interacting extensions of the Standard Model. Lattice simulations are essential to connect properties of the transition to an underlying theory. Combinations of effective theories and lattice input to derive possible experimental signatures can, for example, be found in [28]. Further results from lattice simulations are required to obtain reliable estimates.

The parameter range for a first order transition is a first ingredient, which has been studied in particular as a function of the fermion masses. Recently techniques for a much more detailed analysis have been developed and tested in pure gauge theory. The LLR method provides quite accurate results for the density of states and hence detailed insights into thermodynamic properties [29]. In addition studies of nucleation rate have been presented highlighting the limitations of perturbative studies [30, 31].

4. Sp(2), Sp(4) composite Higgs and SIMP Dark Matter

The simplest theory investigated along the lines of the SIMP Dark Matter paradigm has been SU(2), equivalent to Sp(2), Yang-Mills theory with two fermions. Several aspects of this theory

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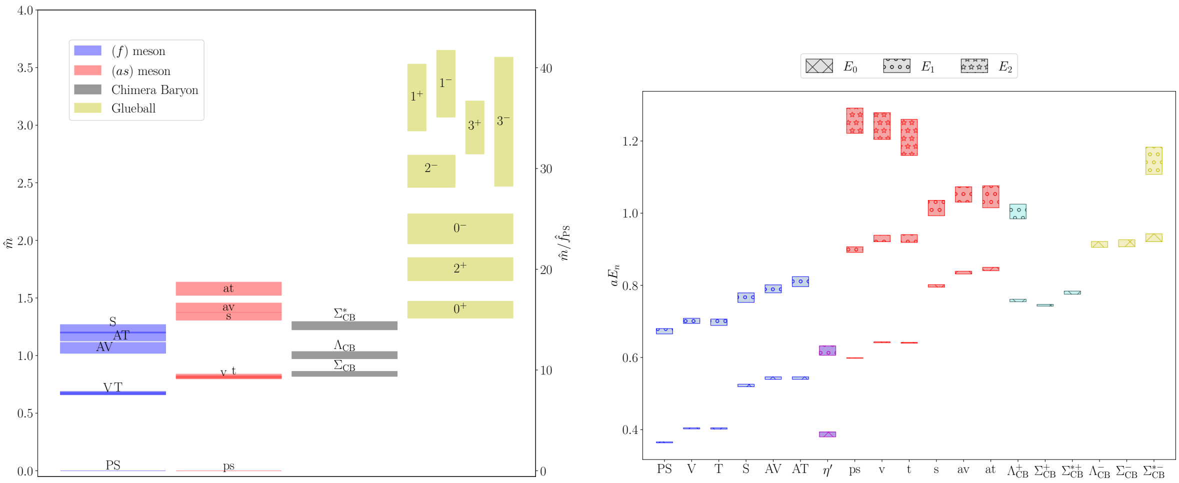

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Figure 1: Particle spectrum of Sp(4) gauge theory: a) Continuum and massless limit of the quenched theory in gradient flow units. Fundamental mesons (blue), antisymmetric mesons (red), chimera baryons (gray), glueballs(yellow). b) Results with dynamical fermions on a single ensemble. Same color coding, except for parity-even and parity-odd chimera baryons distinguished by cyan (even) and yellow (odd) colors. Added in magenta: flavour singlet pseudoscalar η' ground and excited state. Detail can be found in [32, 33].

are hence known from numerical lattice simulations. The particle spectrum and also scattering cross-sections have been analyzed. This extensive analysis shows how the data obtained from lattice simulations can reduce the space of candidate theories.

The balance between 3 to 2 and 2 to 2 processes is key to the SIMP paradigm since it determines the generation and annihilation of Dark Matter and its self interactions. This has been studied in the framework of chiral perturbation theory leading to additional constraints of the theory space. According to the findings of this study, Sp(N_c) with larger N_c are more likely to fulfill the requirements and even $N_c = 16$ has been considered. A first step towards larger N_c is Sp(4) with two fermions.

The Sp(N_c) theories (N_c even) are relevant for several different approaches to Standard Model extensions. They are also natural candidates for composite Higgs theories since they provide scalar pNGbs. Most lattice investigations have considered Sp(4), while there are also results for pure gauge with $N_c = 2, 4, 6, 8$ available [34, 35]. To add partial compositeness in case of Sp(4), three additional (Dirac) fermions transforming in the antisymmetric representation are added besides the two fundamental ones. With these ingredients, baryonic operators combining fermions in different representations, usually called “chimera baryons”, can be formed. These transform according to the requirements of partial compositeness. Very recently remarkable new results have been obtained for the spectrum of Sp(4) gauge theories, which are also reported in the contributions to this conference [36]. The method of spectral densities has been applied to extract a large number of particle masses [32]. The particle spectrum has been determined in Sp(4) with mixed fermion representation (2 fundamental and 3 antisymmetric) [33]. Furthermore, the pseudoscalar flavor singlet states are discussed in [37]. Fig. 1 shows an illustration of the results.

Partial compositeness has been analyzed earlier for a different composite Higgs candidate:

SU(4) with fermions in the fundamental and antisymmetric representation [38, 39]. It has been found that a sufficiently large anomalous dimension and a near conformal scenario would be required to fulfill phenomenological constraints. Results from lattice simulations indicate that this is not likely the case. The phenomenological relevance of the Sp(N_c) models hence might depend on the outcome of similar considerations for this theory.

Composite Higgs models are in some respects closely related to walking technicolor theories. The relevant fermion content of the Sp(N_c) theories in this scenario, which is larger than for the presented SIMP Dark Matter candidates, brings them close to the conformal window. As discussed, walking might also be preferred due to phenomenological constraints, which might require to increase the fermion content even further. For example, in the case of SU(2) with two fundamental fermions, extensions with adjoint fermions lead to a minimal walking theory [40, 41].

Some parts of the analysis might require further scrutiny and the methods are constantly updated. For example, in SU(2) gauge theory with two fundamental fermions there are recent efforts to achieve a small mass regime using an improved fermion action [42].

5. The conformal window on the lattice

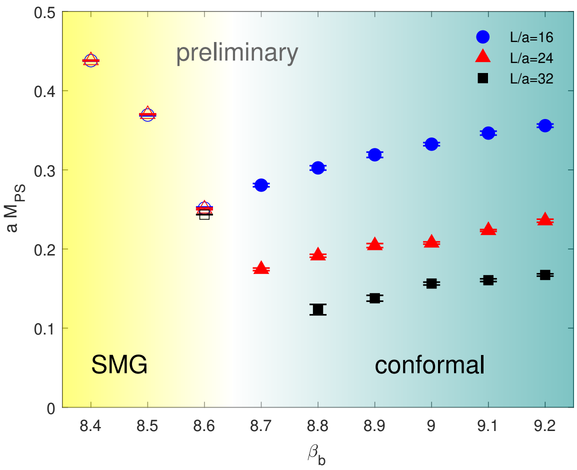
The conformal window and the landscape of gauge theories with different fermion content has been investigated on the lattice for several years. In a large effort, the community has worked on a classification of a large space of theories, as summarized in review talks at previous conferences [43–51]. Important insights have been gained and methods have been significantly improved in these investigations. Our general understanding of the renormalization group (RG) flow and its realization on the lattice has been greatly improved. Nevertheless there are still important unsolved puzzles, differing perspectives, and unexpected new results.

The determination of the lower end of the conformal window requires non-perturbative tools and is therefore an important target for numerical studies on the lattice. There are considerable challenges in such kind of investigations since only a limited range of scales is available to investigate RG scaling and the parameter range is limited by a bulk transition caused by lattice artefacts. Our intuition about the approach to a universal continuum limit has been shaped by a QCD-like scaling and might not generalize to all of these theories.

Another fundamental assumption on which our analysis of lattice data is based, is the validity of effective theories such as chiral perturbation theory. In a walking scenario, it is reasonable to extend the effective theory with a part originating from broken scale invariance. It is not obvious how such a generalization is implemented in a systematic way. The derivation of an effective theory is still part of ongoing research and results are presented in [52] at this conference and a recent discussion can also be found in [53]. In the following, two theories at the lower end of the conformal window are discussed. This illustrates some of the particular challenges and unexpected results.

5.1 SU(3) with eight flavors and the edge of the conformal window

In several investigations the lower end of the conformal window of SU(3) gauge theory was found to be close to 8 fundamental flavors [54–59]. Detailed studies of the 8 flavor theory with staggered fermions have revealed a much richer phase space than initially expected [60]. Two phases have been identified separated by a continuous phase transition. The phase at weak coupling seems



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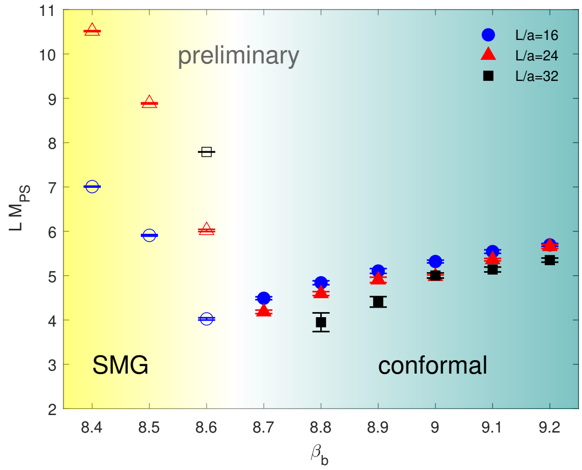


(b)

Figure 2: SU(3) $N_f = 8$ QCD phases identified by their different scaling of the pseudoscalar meson mass a) in units of the lattice spacing a; b) in units of the lattice size L. Further details can be found in [61].

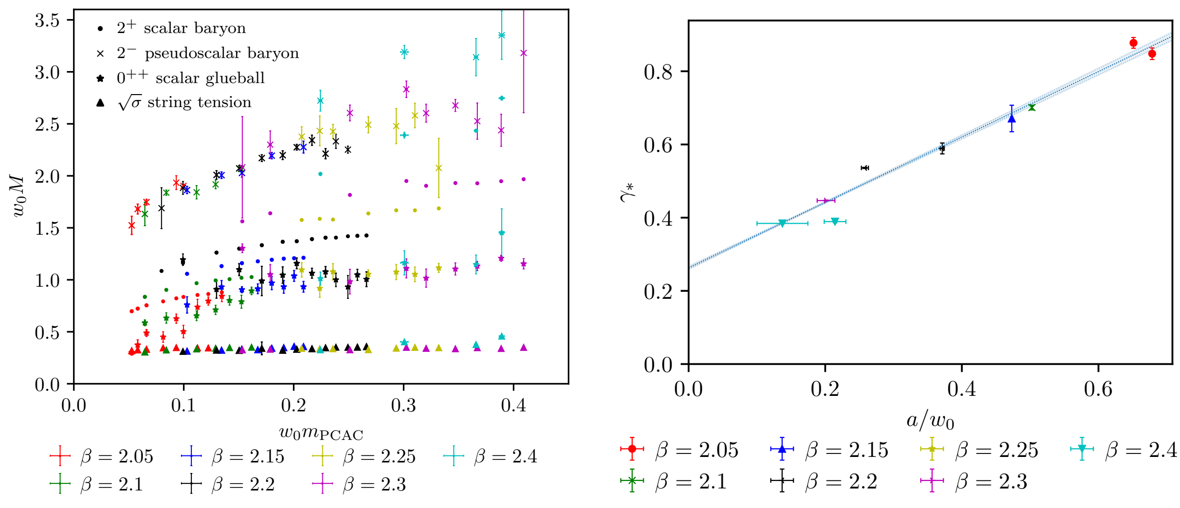
to be conformal, while the strong coupling phase indicates confinement without chiral symmetry breaking. The strong coupling phase has been associated with the mechanism of symmetric mass generation (SMG). It is important to note that the fermion content leads to 't Hooft anomaly cancellation since such a scenario would be otherwise excluded. The realization on the lattice requires Pauli-Villars fields and smearing to approach large enough gauge couplings without a bulk transition. These heavy boson fields are expected to reduce UV fluctuations, while keeping the IR properties intact. A possible picture brought forward in this analysis is a merger of two fixed points at the lower end of the conformal window.

New results as an extensions of these initial investigations have been presented in [61], see Fig. 2. The particle spectrum shows a distinct behavior in the two different phases. The strong coupling SMG phase provides a mass gap independent of the volume. In the conformal phase at weak coupling, on the other hand, the masses scale according to the volume. In all of the cases, independent of the bar coupling, parity doublets are observed. The investigations have also been extended to SU(2) gauge theory, where anomaly matching requires a minimum of $N_f = 4$ fermions for SMG [62]. One might speculate about a possible connection to the lower end of the conformal window, which seems to be close to $N_f = 4$ in case of SU(2) and close to $N_f = 8$ in the case of SU(3) gauge theory.

A possible caveat of these studies is their reliance on staggered fermion discretization and it is not clear whether the observed phases are a more universal feature. It is also important to keep lattice artefacts under control at the considered strong couplings, which are usually affected by a bulk transition.

5.2 SU(2) with one adjoint Dirac flavor

Another example of a theory close to the lower end of the conformal window is SU(2) with $N_f = 1$ adjoint Dirac fermion. The theory with $N_f = 2$ flavors has been investigated in several studies since it is a possible walking technicolor candidate. The results indicate that SU(2) $N_f = 2$



(a) **(b)**

Figure 3: SU(2) gauge theory with $N_f = 1$ adjoint Dirac fermion. a) The particle spectrum of the theory in units of the gradient flow scale w_0 combining different gauge couplings, fermion masses, and states. For comparison, the scale of the string tension has been added. b) The gauge coupling dependence of fixed point mass anomalous dimension obtained from the mode number. Details can be found in [65].

adjoint QCD is inside the conformal window with a mass anomalous dimension likely smaller than desired for the walking scenario. There are, however, still some tensions in the data. A review about gauge theories with adjoint fermions and possible applications can be found in [63].

The SU(2) $N_f = 1$ adjoint QCD has been considered for several reasons. Determining the lower end of the conformal window and the landscape of gauge theories with adjoint fermions has been the first objective. The mass anomalous dimension is expected to rise towards the lower end of the conformal window. The investigation might hence provide an estimate for the upper bound of the anomalous dimension. Later on, it has been observed that anomaly matching conditions allow in principle for an exotic phase in this theory with light fermionic bound states [64]. In addition, it resembles $\mathcal{N} = 2$ supersymmetric Yang-Mills theory without scalar fields.

Lattice investigations of this theory have shown challenges expected at the lower end of the conformal window. First studies with Wilson fermions have indicated a conformal theory [66], while chiral symmetry breaking has been deduced from simulations with overlap fermions [67]. Further investigations have not been able to distinguish conformal and chiral symmetry breaking scenarios [68]. In order to improve the results, the simulations have been extended towards much larger volumes and a range of gauge couplings. Even in this case, the results are not completely conclusive [65]: the lightest state is a scalar glueball, different from a chiral symmetry breaking scenario. On the other hand, the mass ratios are not approximately constant, which would be expected in a conformal theory. Possible improvements are discussed in a contribution to this conference [69]. It might be that similar techniques like applied for $N_f = 8$ fundamental flavors can also be used here. It is also important to investigate in further detail the difference between Wilson and overlap fermion disctretisation.

6. Supersymmetric gauge theories

$\mathcal{N} = 1$ supersymmetric Yang-Mills (SYM) theory corresponds to adjoint QCD with one Majorana fermion. It is clearly below the conformal window and features a spontaneous breaking of a discrete chiral symmetry with the formation of a gluino condensate.

Supersymmetry is broken by any lattice discretization of a non-trivial theory. Like in chiral symmetry, there is a contradiction between the symmetry and locality on the lattice. In case of supersymmetry it can not be simply resolved in terms of a Ginsparg-Wilson relation. Details about supersymmetry on the lattice can be found in reviews like [70, 71].

Generically fine-tuning is required to approach the supersymmetric continuum limit. Exceptions are $\mathcal{N} = 1$ SYM with Ginsparg-Wilson fermions and certain lattice formulations of $\mathcal{N} = 4$ SYM. Especially in supersymmetric theories with scalar fields, the tuning becomes challenging since a large number of operators has to be considered.

The difficulties of fine-tuning are illustrated by the recent perturbative calculation presented in a contribution to this conference [72]. In a large effort, the tuning has been computed for supersymmetric QCD involving quartic and Yukawa couplings. In a second contribution another aspect of supersymmetric QCD on the lattice has been considered, which are possible improvements in the bosonic sector to match fermionic discretization [73].[1] Combining perturbative estimates and improved lattice formulation simulations of the theory might soon be possible.

Recent developments of $\mathcal{N} = 4$ SYM on the lattice have shown promising results. Initial data suggests that certain instabilities in the simulations which have restricted the investigations to small couplings have been overcome with an improved lattice formulation [75]. Therefore it seems that further studies are already possible.

The prospects of lattice simulations of such theories can be already seen in results from lower dimensional theories. In this case the fine-tuning is less involved and simulation methods are less restricted. In that way remarkable consistency between lattice results and gauge/gravity predictions have been achieved for example in [76]. Closing the gap to four dimensions, consistency between holography and lattice data in three dimension has been presented at this conference [77].

A further very interesting new result presented at the conference is the gluino condensate of $SU(N_c)$ $\mathcal{N} = 1$ SYM. It is remarkable that this quantity has been computed analytically in this theory. However, there has been a long standing puzzle concerning the obtained value. More specifically, one can find the following values in the literature:

$$\Sigma = \begin{cases} 2e\Lambda^3/N_c & \text{strong coupling [79]} \\ \Lambda^3 & \text{weak coupling [80]} \\ N_c\Lambda^3 & \text{twisted bc [81]} \end{cases}$$

$\mathcal{N} = 1$ SYM has already been investigated on the lattice, including data for the bound state particle spectrum [82, 83]. The condensate has been already computed using methods like domain-wall fermions, overlap fermions, and gradient flow to avoid additive renormalization, see [84] for a comparison of the different results. The missing piece in these studies has been the multiplicative

[1] Note that without fermions supersymmetric QCD is similar to the two Higgs doublet model presented in [74], which has generically also a large parameter space.

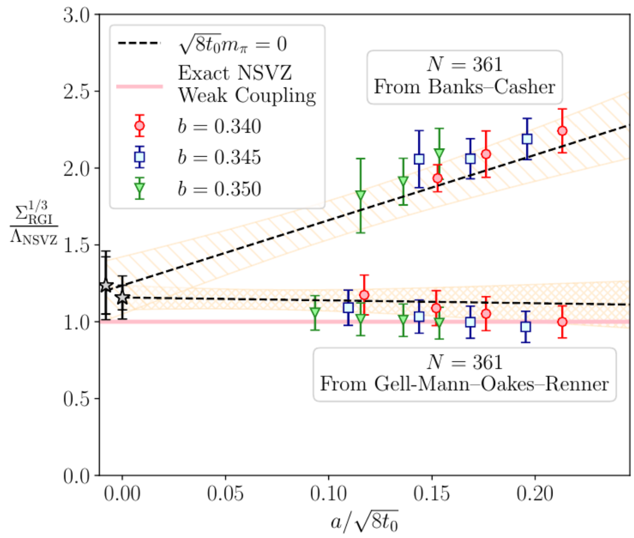


Figure 4: Gluino condensate obtained from simulation of a twisted Eguchi-Kawai large N_c model of $\mathcal{N} = 1$ SYM. The final extrapolation of two different methods shows agreement with the weak coupling result. Details are presented in [78].

renormalization factor, which has been recently added. The determination relies in this case on data from a large N_c twisted Eguchi-Kawai lattice theory. The result is consistent with the weak coupling prediction [78], see Fig. 4.

7. Conclusions

This short summary has provided an overview about the basic status of lattice simulations for physics beyond the Standard Model. Experimental input from colliders are currently basically only constraints and future experiments on the energy frontier might be limited. Lattice simulations can provide additional constraints and reduce the conjectured theory space. They can also help to concretize suggested models and point out observable signatures of theories.

Motivations for a investigations of theories beyond the Standard Model go, however, much further than the considerations about experimental relevance. The objective is a more general understanding of fundamental mechanisms and landscape of strongly interacting gauge theories. It even leads up to new approaches for quantum computing of lattice field theories.

Concepts for extensions of the Standard Model that are of major interest for studies on the lattice are supersymmmetry, composite Higgs, and composite Dark Matter. Related to these concepts are fundamental questions about symmetry realization on the lattice, the RG flow, and the landscape of strongly interacting theories with different gauge groups and fermion representations.

Several updates on this topic have been presented recently, but it is evident that there are still important tasks to be addressed. In case of composite Dark Matter, the theory space is in general not much constrained. Most interesting are theories, which are below current detection limits but still accessible to future experiments. The stealth Dark Matter model is an example of such a theory.

Lattice simulations have helped to adjust the parameter space of the model according to detection constraints. The limits of possible particle masses can be lowered in the hyper stealth Dark matter model. An important missing input from lattice simulations are estimates of Dark Matter self interactions for these candidates.

Another approach for Dark Matter is based on the strongly interacting massive particle (SIMP) approach. It provides a specific scenario for the evolution of the early universe. The scenario can be realized in certain gauge theories and assumes certain 3 to 2 processes. These have to be compared to other processes in order to provide a realistic candidate [85].

Gravitational wave signals induced by first order transitions might provide an additional independent way to detect strong interactions beyond the Standard Model. A detailed understanding of the dynamics of the transition is necessary to draw conclusions about observations in future experiments. Lattice simulations can improve our knowledge in this case, but further developments of the methods are required.

A larger number of new data has been obtained for Sp(4) gauge theory with different fermion content, in particular regarding the particle spectrum of the theory. An interesting aspect for further investigations are the anomalous dimensions and the RG flow of the theory.

Recent investigations of SU(3) gauge theory with 8 fundamental flavors or SU(2) gauge theory with 1 adjoint fermion have demonstrated the challenges inherent in determination of the lower end of the conformal window. A recent aspect of these studies are possible new phases in this regime, featuring symmetric mass generation. This finding illustrates our still limited understanding of the general landscape of theories with strong interactions.

Recent progress has also been reported for lattice simulations of supersymmetric theories. Improvements of methods and perturbative estimates are promising. Possible applications include checks and extensions of gauge/gravity duality predictions, which have been already obtained for lower dimensional theories.

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