PROCEEDINGS OF SCIENCE

Using light hypernuclei to constrain hypernuclear interactions

Andreas Nogga,[a,b,*] **Hoai Le,**[a] **Johann Haidenbauer**[a] **and Ulf-G. Meißner**[c,a,b,d]

[a] *Institute for Advanced Simulation (IAS-4), Forschungszentrum Jülich, 52428 Jülich, Germany*

[b] *Centre for Advanced Simulation and Analytics (CASA), Forschungszentrum Jülich, 52428 Jülich, Germany*

[c] *Helmholtz-Institut für Strahlen- und Kernphysik and Bethe Center for Theoretical Physics, Universität Bonn, 53115 Bonn, Germany*

[d] *Tbilisi State University, 0186 Tbilisi, Georgia*

E-mail: `a.nogga@fz-juelich.de`

Hypernuclei provide important information to constrain and test the hyperon-nucleon (YN) and three-baryon (YNN) interactions. In this contribution, we discuss our recent results obtained using chiral YN and YNN interactions for light hypernuclei. Based on the results for different orders of the chiral expansion, the theoretical uncertainty could be reliably determined, which in turn allows a quantitative estimate of the size of the possible YNN contributions. This estimate is then compared to first calculations that explicitly take the leading YNN interactions into account.

10th International Conference on Quarks and Nuclear Physics (QNP2024)
8-12 July, 2024
Barcelona, Spain

---

[*] Speaker

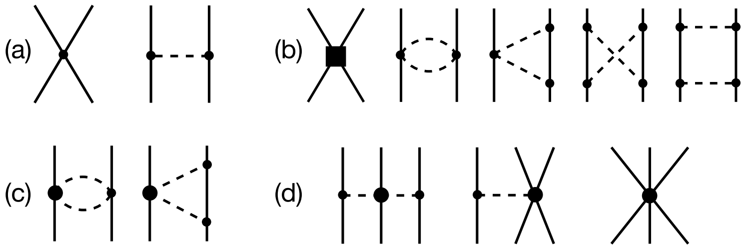

Figure 1: Topologies contributing to LO (a) NLO (b) and N2LO (c) YN interactions. The diagrams contributing to the YNN interaction in N2LO are shown in (d).

1. Introduction

Hypernuclear interactions have gained considerable interest in recent years particularly due to the fact that the nuclear equation of state (EOS) might be softened when hyperons are present. Thereby properties of the hyperon-nucleon (YN) and hyperon-nucleon-nucleon (YNN) interactions significantly influence the densities at which hyperons appear and the extent to which the EOS is modified [1]. Depending on the interaction used, the EOS becomes inconsistent with the recently found neutron stars with masses larger than $2M_\odot$. This is usually referred to as 'hyperon-puzzle'.

Knowledge on the interactions is also desirable because they offer insights into the flavor dependence of baryonic interactions which is linked to explicit chiral symmetry breaking and which can provide a better understanding how symmetries of QCD affect such interactions.

But determining their properties is difficult. Only a few low energy YN scattering data exist that provide constraints on the overall strength of the interaction. The angular dependence is to a large extend unknown and the spin dependence is usually determined using the binding energy of the lightest strange bound system, the ${}^3_\Lambda$H. Also, the longest-ranged part of ΛN interaction is driven by Λ-Σ conversion that is tightly related to the strength of possible contributions of YNN interactions.

Because of this situation, hypernuclei are generally seen as testing ground and as an important source of information on YN and YNN interactions. Over the years several, especially light, hypernuclei have been found and their binding energy has been determined [2–4]. Because the hyperon is not affected by Pauli blocking, there usually exist several bound states with different spins which gives important information on the spin dependence, so that even the lightest hypernuclei provide non-trivial constraints. Unfortunately, there is no bound YN state. Therefore, for all conclusions based on hypernuclei, one has to consider also possible contributions from YNN interactions and maybe even more-baryon interactions.

2. Hyperon-Nucleon interactions

In the past, models of the YN interaction were mostly based on one-boson exchange and used flavor-SU(3) symmetry to relate NN, YN and YY interactions to each other (see e.g. [2] and references therein). Since flavor-SU(3) is broken, all of these interactions require SU(3) symmetry breaking, and especially physical masses of the exchange bosons have been employed. The models have the disadvantage that, once the approach has been formulated, there is little freedom to adjust properties of the interaction when new data appears. It is also not possible to consistently

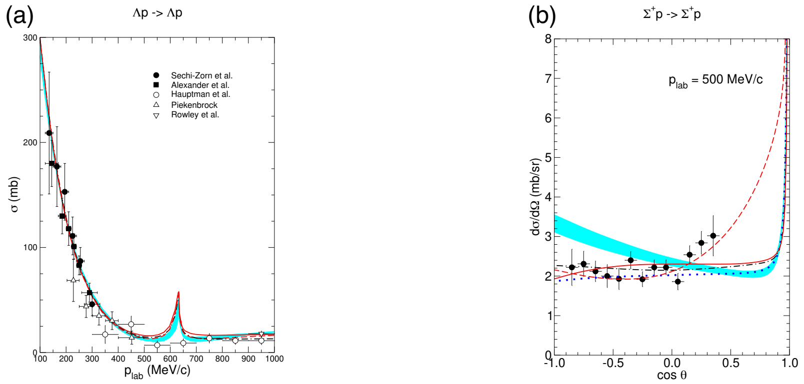

Figure 2: Comparison of the results of the SMS NLO (black, dashed-dotted line), the SMS N^2LO (red, solid line), an alternative version of the SMS N^2LO (red, dashed line) at $\Lambda = 550$ MeV to results for different cutoffs of the older NLO19 interaction (cyan band) for the Λp cross section (a) and the differential $\Sigma^+ p$ cross section (b). In (b), a refitted NLO19 interaction at $\Lambda = 600$ MeV (blue, dotted line) is shown, too.

construct YNN and higher-body forces. This motivates to use chiral effective field theory (ChEFT) to consistently formulate YN and YNN interactions based on the symmetries of QCD. For this, interactions are expanded in terms of the ratio of a typical, low momentum and the chiral symmetry breaking scale $\Lambda_\chi \approx 600 - 700$ MeV. The non-perturbative character of the interaction is taken into account by solving a Schrödinger or Lippmann-Schwinger equation for the chiral interaction. Using symmetries of QCD constrains the couplings of the exchange bosons to the baryons and allows one to systematically expand the interactions. The interactions require regularization which is usually performed using momentum cutoff functions. In the newest version of the interaction, the semi-local momentum space (SMS) regularization has been employed [5]. The size of YNN forces strongly depend on the degrees of freedom taken into account. We explicitly include Λ-Σ conversion. In this case, the YNN force only appears in next-to-next-to-leading order (N^2LO) in the chiral expansion.

In Fig. 2, the results for the new SMS interactions [5] are compared to the ones of the older NLO19 interaction [6]. The agreement with the data is in all cases excellent, similar as for the older interactions. The new data for $\Sigma^+ p$ scattering [7] was not available when the NLO19 interaction was fitted. The slight deviations for the differential cross sections for this interaction can be remedied by a refit mostly affecting the ε_1 phase. This is shown as the alternative NLO19 interaction. Note also that we have devised an alternative fit for the SMS interaction that gives a better description of the $\Sigma^+ p$ cross section at $p_{lab} = 500$ MeV/c but not for other momenta.

Importantly, the available YN data and the ${}^3_\Lambda$H separation energy well constrain the s-wave interactions including its spin dependence. p-waves and higher partial waves and s-d transitions are not that well constrained, but can be easily readjusted to take upcoming data into account. However, since s-wave interactions drive the properties of light hypernuclei, the chiral forces are already well suited for predictions for such hypernuclei.

3. Uncertainty of Λ separation energies

The probably most important advantage of chiral baryon-baryon interactions is the possibility to obtain reliable uncertainty estimates by using the residual cutoff dependence or by using different

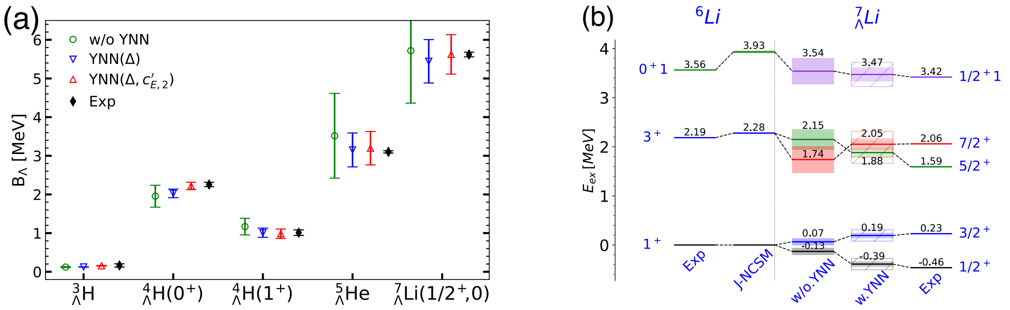

Figure 3: Separation energies for various hypernuclei (a) and spectrum of $^7_\Lambda$Li (b) compared to the spectrum of the core nucleus ^{6}Li.

chiral orders. With the new interactions up to order N^{2}LO available, the latter approach is preferable. It is based on the expectation that any observable X can be expanded in terms of the chiral expansion parameter Q. The K-th order approximation of the observable can then be written as $X_K = X_{ref} \sum_{k=0}^{K} c_k Q^k$ where X_{ref} is a typical value of the observable that in practice can be taken from the LO result, from experiment, or taking other choices. The coefficients c_k are of order 1 and their distribution is assumed to be independent of k. Following Meledenz et al. [8, 9], one can use the available coefficients c_k for $k \leq K$ from calculations to obtain the probability distribution for all c_k and use this to get the distribution for the contribution of higher orders $\delta X_K = X_{ref} \sum_{k=K+1}^{\infty} c_k Q^k$.

For the separation energies of light hypernuclei, this was done in Ref. [10]. Although the N^{2}LO interaction was incomplete in this study, the results allow one to estimate the uncertainties. At order NLO, they are at the same time estimates of possible YNN force contributions. We obtained 68% degree of believe intervals of 15 keV for $^3_\Lambda$H, approximately 240 keV for $^4_\Lambda$He and 900 keV for $^5_\Lambda$He. This indicates that the YNN force contribution is negligible for $^3_\Lambda$H. For $A = 4$ and $A = 5$ hypernuclei, the NLO uncertainty is significant so that it will be necessary to include these interactions in future calculations.

4. Chiral hyperon-nucleon-nucleon interactions

Chiral YNN interactions have been formulated already in [11] and applied to nuclear matter in [12–14]. The leading YNN interactions are shown in Fig. 1(d). Of course, in principle, all mesons from the lightest octet contribute to the 1- and 2-boson exchange diagrams. Because of the large masses of K and η mesons, we do not take these exchanges into account but assume that they can effectively be taken into account by contact interactions. The number of low energy constants (LECs) that need to be determined is nevertheless very large so that they cannot be uniquely fixed from the available data on hypernuclei. Therefore, we use resonance saturation due to decuplet baryons assuming that the corresponding parts of the YNN interactions are enhanced. Under this assumption, the strength of the YNN force can be related to the octet-decuplet-pion coupling constant and the octet-decuplet contact interactions. The former one is constrained by large N_c symmetry and latter one involves two LECs G_1 and G_2 to be determined from data. A detailed calculation reveals that the ΛNN interaction only depends on the linear combination $G_1 + 3G_2$. We have now implemented these interactions [15] and successfully benchmarked them [16].

Attempts to determine the two LECs by fitting the 0^+ and 1^+ states of ${}^4_\Lambda$He failed. Both energies where only sensitive to the linear combination $G_1 + 3G_2$ although also the Σ component was taken into account in the YNN interactions. We have therefore added, additionally to the terms that contribute for decuplet saturation, the term $C_2' \, \vec{\sigma}_\Lambda \cdot (\vec{\sigma}_1 + \vec{\sigma}_2) \, (1 - \vec{\tau}_1 \cdot \vec{\tau}_2)$ to the ΛNN force [15]. The additional constant was then adjusted so that the splitting of the 0^+ and 1^+ states were reproduced. Fig. 3 summarizes the results of the calculations. The figure also includes the chiral uncertainties estimated as described in Sec. 3. For the calculation without chiral YNN interactions, we assume NLO error bars. For the ones including YNN interactions, we take the smaller N^{2}LO error bars. For the separation energies, we show results excluding and including the C_2' term. The additional term improves the description for the 0^+ state of ${}^4_\Lambda$ by construction. For the other hypernuclei, both calculations lead to very similar predictions that are generally in good agreement with experiment. Also for the excitation energies of ${}^7_\Lambda$Li, the results with and without C_2' term are very similar. Therefore, we only show the ones including this term. The spectrum is significantly improved by the YNN force contribution. The splitting of the lowest states becomes larger in agreement with experiment and the ordering of the higher states is corrected. Clearly, the YNN interactions lead to a consistent description of the light hypernuclei.

5. Conclusions

We have presented calculations for light hypernuclei for realistic chiral YN and YNN interactions. Due to the scarcity of the data, unique fits of all available LECs are not possible. The fits however do describe the available YN data very well. Including the YNN interaction also the light nuclei can be well described. Since chiral interactions are now availabe up to order N^{2}LO, we are also able to obtain reliable estimates of our theory uncertainty.

The inclusion of the YNN interaction leads to a significant improvement of separation and excitation energies. In the near future, we plan further studies with different cutoff values to confirm the estimates of the theoretical uncertainties. Also studies of contribution of the fitted YNN interactions to nuclear matter will be important to see whether the complete chiral interactions can help to understand the 'hyperon puzzle'.

Acknowledgments

This project was supported by the ERC Advanced Grant “EXOTIC” supported the European Research Council (ERC) under the European Union’s Horizon 2020 research and innovation programme (grant agreement No. 101018170), the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) and the NSFC through the funds provided to the Sino-German Collaborative Research Center TRR110 “Symmetries and the Emergence of Structure in QCD” (DFG Project ID 196253076 - TRR 110, NSFC Grant No. 12070131001), and by the MKW NRW under the funding code NW21-024-A. The work of UGM was also supported in part by The Chinese Academy of Sciences (CAS) President’s International Fellowship Initiative (PIFI) (grant no. 2025PD0022). The numerical calculations were performed on JURECA of the Jülich Supercomputing Centre, Jülich, Germany.

References

[1] H. Djapo et al., *On the appearance of hyperons in neutron stars*, *Phys. Rev. C* **81** (2010) 035803 [`0811.2939`].

[2] A. Gal, E.V. Hungerford and D.J. Millener, *Strangeness in nuclear physics*, *Rev. Mod. Phys.* **88** (2016) 035004.

[3] D. Davis, *50 years of hypernuclear physics*, *Nuclear Physics A* **754** (2005) 3 .

[4] E. Botta et al., *On the binding energy and the charge symmetry breaking in $A \leq 16$ Λ-hypernuclei*, *Nucl. Phys. A* **960** (2017) 165 [`1608.07448`].

[5] J. Haidenbauer et al., *Hyperon–nucleon interaction in chiral effective field theory at next-to-next-to-leading order*, *Eur. Phys. J. A* **59** (2023) 63 [`2301.00722`].

[6] J. Haidenbauer et al., *Hyperon–nucleon interaction within chiral effective field theory revisited*, *Eur. Phys. J. A* **56** (2020) 91 [`1906.11681`].

[7] K. Miwa et al., *Recent progress and future prospects of hyperon nucleon scattering experiment*, *EPJ Web Conf.* **271** (2022) 04001.

[8] J.A. Melendez et al., *Bayesian truncation errors in chiral effective field theory: nucleon-nucleon observables*, *Phys. Rev. C* **96** (2017) 024003 [`1704.03308`].

[9] J.A. Melendez et al., *Quantifying Correlated Truncation Errors in Effective Field Theory*, *Phys. Rev. C* **100** (2019) 044001 [`1904.10581`].

[10] H. Le et al., *Separation energies of light Λ hypernuclei and their theoretical uncertainties*, *Eur. Phys. J. A* **60** (2024) 3 [`2308.01756`].

[11] S. Petschauer et al., *Leading three-baryon forces from SU(3) chiral effective field theory*, *Phys. Rev. C* **93** (2016) 3.

[12] J. Haidenbauer et al., *Lambda-nuclear interactions and hyperon puzzle in neutron stars*, *Eur. Phys. J. A* **53** (2017) 121 [`1612.03758`].

[13] M. Kohno, *Single-particle potential of the Λ hyperon in nuclear matter with chiral effective field theory NLO interactions including effects of YNN three-baryon interactions*, *Phys. Rev. C* **97** (2018) 035206 [`1802.05388`].

[14] D. Gerstung et al., *Hyperon–nucleon three-body forces and strangeness in neutron stars*, *Eur. Phys. J. A* **56** (2020) 175 [`2001.10563`].

[15] H. Le et al., *Light Λ hypernuclei studied with chiral hyperon-nucleon and hyperon-nucleon-nucleon forces*, `arXiv:2409.18577`.

[16] H. Le et al., *Benchmarking ΛNN three-body forces and first predictions for A=3-5 hypernuclei*, `arXiv:2407.02064`.