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Shape and log ft values of forbidden β decays within the realistic shell model

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We describe the electron energy spectra and half-lives of forbidden β decays for nuclear systems outside the ^{78}Ni core in the framework of the realistic shell model, starting from a realistic interaction. The effective shell-model Hamiltonians and decay operators are derived using many-body perturbation theory. In particular, we examine how sensitive the calculated electron energy spectra and half-lives are to the renormalization of forbidden β-decay operators. The study focuses on the first-forbidden decays of ^{89}Sr and ^{90}Y, on the second-forbidden decays of ^{94}Nb and ^{99}Tc, as well as the fourth-forbidden decays of ^{113}Cd and ^{115}In.

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1. Introduction

The renormalization of electroweak currents is crucial for nuclear structure research, particularly for calculating reliable nuclear matrix elements in single- and double-β decay.

Since most nuclear models rely on the truncation of the Hilbert space, accurate wave function calculations alone do not guarantee reliable nuclear matrix elements. It is, in fact, necessary to evaluate consistently the corresponding effective transition operator to account for excluded configurations. Therefore, the ability of nuclear models to reproduce β-decay observables represents the better way to validate both wave functions and renormalization procedures.

This issue is related to the well-known 'quenching puzzle' of the axial coupling constant, which refers to the necessity for most nuclear structure models to reduce g_A to accurately reproduce observables associated with Gamow-Teller (GT) transitions [1–3]. However, this approach is empirical and cannot be generalized to any β-decay operator that depends on the axial coupling constant.

The realistic shell model (RSM) provides a coherent framework for deriving effective Hamiltonians and decay operators, relying solely on the nuclear force as a parameter. This approach has been effectively applied to investigate allowed single-β and double-β decays in various nuclei, such as ^{48}Ca, ^{76}Ge, and others [4–7].

To further validate our framework, we recently calculated the spectra and half-lives of forbidden β decays in ^{94}Nb, ^{99}Tc, ^{113}Cd and ^{115}In [8] to examine the renormalization sensitivity of forbidden β decay operators. In this manuscript, we present these findings, including new results for the first forbidden β decays of ^{89}Sr and ^{90}Y. These nuclei were selected due to their relevance to various experiments [9–12].

The paper is organized as follows. Section 2 briefly describes the theoretical framework of the work. The results of the shell-model calculations are reported in Sect. 3 and compared with the available experimental data, while finally some concluding remarks are reported in sect. 4.

2. Outline of the theory

2.1 The effective SM Hamiltonian and Operators

The effective Shell Model (SM) Hamiltonian, H_{eff}, which includes the single-particle energies (SPEs) and two-body matrix elements (TBMEs) of the residual interaction, must incorporate the degrees of freedom excluded from the reduced model space. In this case, the model space is spanned by the four proton orbitals, $0f5/2$, $1p3/2$, $1p1/2$, $0g9/2$, and the five neutron orbitals, $0g7/2$, $1d5/2$, $1d3/2$,$2s1/2$, $0h11/2$ outside ^{78}Ni. In this study, H_{eff} is derived from a CD-Bonn potential [13] renormalized by way of the V$_{low-k}$ approach [14, 15]. Specifically, H_{eff} is expressed through the Kuo-Lee-Ratcliff folded-diagram expansion involving the $\hat{Q}$-box vertex function [16–18]. The derivation details of H_{eff} can be found in [18].

By diagonalizing H_{eff}, we obtain projections of the true nuclear wave functions onto the selected model space P. Each transition/decay operator Θ must then be renormalized to account for the degrees of freedom in the excluded Q-space. The derivation of the effective SM transition/decay operators Θ_{eff} follows the same approach used for H_{eff}, employing a perturbative expansion based on a vertex function $\hat{\Theta}$-box, analogous to the $\hat{Q}$-box used for H_{eff}. The procedure, initially introduced by Suzuki and Okamoto [19], is detailed in [19, 20].

2.2 β-decay theory

The theory of β-decay is here briefly outlined. More details can be found in Refs. [8, 21, 22]. In the following we focus on the β^--decay, moreover, we use natural units ($\hbar = c = m_e = 1$).

The total half-life of the β decay is expressed in terms of the k-th partial decay half-life $t^k_{1/2}$ as follows:

$$\frac{1}{T_{1/2}} = \sum_k \frac{1}{t^k_{1/2}} \ . \tag{1}$$

On the other hand, the partial half-life $t^k_{1/2}$ is related to the dimensionless integrated shape function $\tilde{C}$ by way of the relation:

$$t^k_{1/2} = \frac{\kappa}{\tilde{C}} \ , \tag{2}$$

where $\kappa = 6144 \pm 2\ s$ [23].

For a given k-th final state, the integrated shape function $\tilde{C}$ – whose integrand defines the β-decay energy spectrum – is written as

$$\tilde{C} = \int_1^{w_0} C(w_e) p_e w_e (w_0 - w_e)^2 F(Z, w_e) dw_e \ . \tag{3}$$

The quantities on the right-hand side of the above definition are listed as:

a) Z is the atomic number of the daughter nucleus, w_e the adimensional energy of the emitted electron, w_0 the endpoint energy – namely the maximum electron energy for a given transition–, and p_e the electron momentum.

b) The function $F(Z, w_e)$ is the Fermi function that defines the effects of the Coulomb interaction between the electron and the daughter nucleus, whose explicit expressions can be found in Ref. [21].

c) $C(w_e)$ is the so-called nuclear shape function, which depends on the nuclear matrix elements (NMEs). For allowed β transitions, it corresponds to the GT reduced transition probability, and in such a case does not depend on the electron energy.

For nth-forbidden transitions, $C(w_e)$ depends on the electron energy, and the explicit expression can be found in Ref. [8, 21, 22].

3. Results

In this section we present the electron energy spectra obtained within our realistic SM framework. We focus, in particular, on the properties of the β decay between ground states, except for the ^{90}Y, where we have studied the $J_i = 7^-_1$ state of ^{90}Y decaying to the $J_f = 5^+_1$ state of ^{90}Zr.

From inspecting Fig. 1, we observe that the theoretical electron energy spectra calculated using both bare and effective β-decay operators closely match the experimental spectra for the forbidden β-decays of ^{99}Tc, ^{113}Cd, and ^{115}In [9, 24–26]. The theoretical spectra display minimal sensitivity to

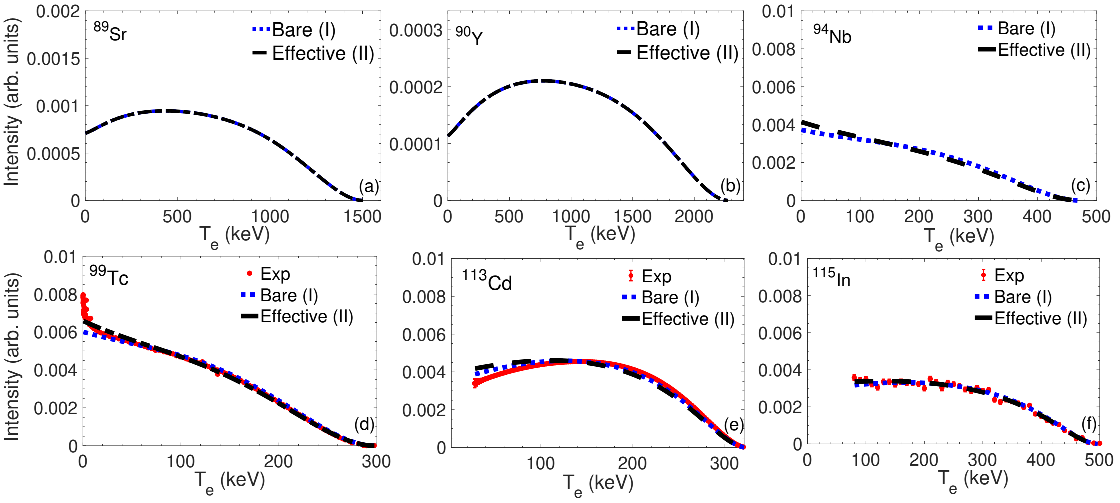

Figure 1: Theoretical and experimental normalized β-spectra of ^{89}Sr (a), ^{90}Y (b)^{94}Nb (c), ^{99}Tc (d),^{113}Cd (e) and ^{115}In (f) as a function of the electron kinetic energy T$_e$. The theoretical spectra are calculated with the bare operator (blue dotted line), the SM effective operator (black dashed line). The red dots corresponds to experimental values, where they are available [9, 24–26].

the renormalization of the β-decay operators, with only minor discrepancies appearing at energies below 100 keV across all decays. Additionally, as expected, the forbidden β-decay of ^{89}Sr and ^{90}Y, being unique, there is only one operator contributing to the transition, and, therefore, the normalized spectrum is not sensitive to the renormalization of the operator. These finding lead to the conclusion that the calculated normalized electron energy spectra are largely unaffected by the renormalization of the forbidden β-decay operator, as implemented through our many-body perturbation approach to the shell model, and show good agreement with current experimental data.

It is also noteworthy that previous studies, including Refs. [24, 27, 28], have investigated the impact of g_A renormalization on calculated spectra and found a significant dependence on the choice of a phenomenological quenching factor q.

At variance with respect to the spectrum, the logfts, reported in Table1 are sensitive to the renormalization.

Table 1: Theoretical and experimental logft values. Data are taken from Ref. [29].

	Bare	Effective	Exp.
^{89}Sr	8.299	8.960	9.432
^{90}Y	7.59	8.40	9.62
^{93}Nb	11.30	11.58	11.95 (7)
^{99}Tc	11.580	11.876	12.325 (12)
^{113}Cd	21.902	22.493	23.127 (14)
^{125}In	21.22	21.64	22.53 (3)

As we can see, for all the cases under investigation the renormalizion of the decay operator always pushes the logfts towards the correspondent experimental data.

4. Conclusions and Outlook

This work report on the attempt to describe the features of forbidden β-decays within the framework of the realistic shell model, without the use of any phenomenological quenching factors for the axial and vector coupling constants.

This study serves not only as a validation of our theoretical framework for assessing the reliability of predicting $0\nu\beta\beta$ nuclear matrix elements [30], but also provides valuable insights for recent experimental studies on the electron energy spectra of forbidden β-decays.

We have calculated both the half-lives and electron energy spectra of the emitted electrons for the first-forbidden unique β-decay of ^{89}Sr and ^{90}Y, the second-forbidden β-decay of ^{93}Nb and ^{99}Tc, and the fourth-forbidden β-decay of ^{113}Cd and ^{125}In.

From the examination of the theoretical $\log ft$s and of the corresponding experimental values shows that the results obtained with bare operators consistently underestimate the data, echoing the issue of g_A quenching observed in allowed β-decay transitions. The effective operators bring the results closer to the experimental values, as expected. Additionally, we find that the calculated electron energy spectrum shapes of the non-unique decays are relatively insensitive to whether a bare or effective β-decay operator is used. As for the unique decays, being determined by a single decay operator, are completely insensitive to the renormalization. For the cases where experimental data are available, the agreement with the observed normalized electron energy spectra is highly satisfactory.

Our findings suggest that the study of forbidden β-decay processes could help to refine the theoretical understanding of transition operator renormalization and exclude models that may be unreliable for predicting nuclear matrix elements in decays such as $\beta\beta$-decays with no neutrino emission ($0\nu\beta\beta$).

On this basis, we plan to extend the present work by studying forbidden β-decays in other mass regions, especially near nuclei that are candidates for $0\nu\beta\beta$ decay detection. Another promising direction involves deriving electroweak currents via chiral perturbation theory—a frontier approach that frames the nuclear many-body problem in terms of the fundamental theory of Quantum Chromodynamics. This method is currently used to investigate Gamow-Teller (GT) transitions across various nuclear approaches [7, 31–36].

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