PROCEEDINGS OF SCIENCE

Effect of hadron-quark phase transition on neutron star $f-$mode oscillations in general relativistic calculations

Sarmistha Banik,[a,*] **Debanjan Guha Roy,**[a] **Tuhin Malik**[b] **and Swastik Bhattacharya**[a]

[a] *BITS Pilani, Hyderabad Campus, Secunderabad, Telangana 500078, India*

[b] *CFisUC, Department of Physics, University of Coimbra, Coimbra 3004-516, Portugal*

E-mail: sarmistha.banik@hyderabad.bits-pilani.ac.in

Characteristic oscillations of neutron stars (NSs) known as quasinormal modes (QNMs) are dependent on the state of matter inside the star. We construct a set of NS matter equations of state (EOSs) with hadron-quark phase transitions (PTs) combining two models - a relativistic mean-field (RMF) model for nucleonic regime and a mean-field theory of quantum chromodynamics (MFTQCD) for quark regime. We investigate the influence of a possible hadron-quark PT on $f-$mode oscillations, a class of QNM oscillations that contributes the most during gravitational wave (GW) emission from a perturbed NS. A fully general relativistic formalism for studying NS QNMs allows us to probe the influence of phase transition on the GW damping times of the oscillations. We employ constraints from nuclear physics and recent astrophysical observations to narrow down the allowed EOSs through Bayesian inference. We find that resulting EOSs manifest a smooth hadron-quark PT based on the set of constraints imposed and the model employed during Bayesian analysis. Additionally, the mass-radius, $f-$mode frequencies, and corresponding GW damping times vary in the presence of deconfined quark matter to such an extent that future observations are needed for a thorough exploration.

10th International Conference on Quarks and Nuclear Physics (8-12 July, 2024)
Institute of Cosmos Sciences, University of Barcelona
Avinguda Diagonal, 643 08028 - Barcelona, Spain

*Speaker

PoS(QNP2024)150

1. Introduction

Quark matter in neutron stars (NSs) represents a potential phase of matter that may emerge under extreme densities[1], surpassing the conditions found in ordinary nuclear matter with saturation density $\rho_{B,0} = 0.16$ fm^{-3}. The MIT bag model[2] and the Nambu-Jona-Lasinio[3] model are used extensively to study the effect of quarks on the NS structure. MIT bag model, however, is unable to reproduce NSs with recent mass observations of $\approx 2\ M_\odot$. This led to exploring modified versions of the MIT bag model by including interaction terms in the Lagrangian for quarks. One such improvement over the original MIT bag model is the mean-field theory of quantum chromodynamics (MFTQCD) [4], which incorporates perturbative and non-perturbative effects from gluon fields. This quark model has successfully produced hybrid stars with masses relevant observationally.

Apart from the mass and radius of a static spherically symmetric NS, asteroseismology, i.e. the study of oscillations within these dense objects, offers an avenue / a tool for inspecting their internal structure and composition [5]. We expect to infer the existence of exotic matter like free quarks inside the NS core through its frequency spectrum. The oscillation modes can be determined using different theoretical frameworks, viz. Cowling approximation and full general relativistic (GR) methods. But it is well-known that Cowling approximation overestimates the frequency of $f-$mode oscillations[6] In this work, we prepare a set of EOSs with hadron-quark phase transitions and compare how complex $f-$mode oscillation frequencies within a fully general relativistic formalism change in the presence of quark matter inside NSs.

2. Methodology

2.1 Hadronic and quark models

We employ the relativistic mean-field (RMF) model — with multiple non-linear meson interaction terms [7] added to the Walecka Lagrangian to represent the hadronic EOS. The MFTQCD formalism provides us with the quark EOSs [8] from the Lagrangian

$$\mathcal{L} = -b\phi_0^4 + \frac{m_G^2}{2}\alpha_0^a\alpha_0^a + \bar{\psi}_i^q(i\delta_{ij}\gamma^\mu\partial_\mu + g_h\gamma^0 T_{ij}^a\alpha_0^a - \delta_{ij}m)\psi_j^q \tag{1}$$

where $\xi_{quark} = g/m_G$, g is strong coupling constant, m_G is dynamical gluon mass, and $b\phi_0^4$ account for the bag constant of MIT Bag model. We use the analytical expressions of Ref. [9, 10] for the pressure and energy density of the quark EOS.

2.2 Phase transition

There are two different ways to simulate a hadron-quark phase transition - Maxwell and Gibbs construction. Maxwell construction manifests with an abrupt jump in the energy density or baryon number density at the onset of the phase transition at a definite number density. This is a signature of a first order change to the quark phase. In contrast to the Maxwell construction, the EOS is smooth in the Gibbs construction. However, there will be discontinuities in its derivatives concerning baryon density, which will be evident in the speed of sound at the densities where the mixed phase starts and ends.

g_σ	g_ω	g_ρ	B	C	ξ	Λ_ω	$\rho_{B,0}$
(6.5, 15.5)	(6.5, 15.5)	(5.5, 16.5)	(0.5, 9.0)	(-5.0, 5.0)	(0.0, 0.04)	(0.0, 0.12)	0.16 ± 0.004

Table 1: Priors for RMF model parameters. $\rho_{B,0}$ in fm^{-3} is normal prior, other being uniform.

In order to avoid any preference in favour of Gibbs or Maxwell transitions, we choose to construct our hybrid EOSs using $(P - P_Q)(P - P_H) = \delta(\mu_B) = \delta_0 \exp[-(\mu_B/\mu_c)]$, where P_H and P_Q are the pressures for hadronic and quark phases respectively[9, 11]. The two free parameters, δ_0 and μ_C control the smoothness of the phase transition; $\delta_0 = 0$ (MeV/fm^3)^2 and/or $\mu_C \approx 0 - 200$ MeV corresponds to EOSs with a sharp discontinuity in energy density; higher values of δ_0 result in EOSs with smoother transitions [9]. Ideally, we would like our analysis to be as general as possible by defining prior ranges for the parameters controlling the smoothness of phase transitions. But some preliminary studies during the preparation of our latest work[10] suggest that our likelihood evaluations do not constrain δ_0 and μ_C well enough and have slight preference for $\delta_0 = 50$ (MeV/fm^3)^2 and $\mu_C = 700$ MeV. This choice allows our Bayesian analysis to be computationally less expensive, without any loss of generality.

2.3 Bayesian inference

We perform Bayesian inference with `PyMultiNest`[12] to sample a large number of EOSs. For the nucleonic EOSs, we specify priors for the RMF model parameters as shown in Table 1. To build hybrid EOSs with phase transitions, we sample the MFTQCD quark EOS parameters within the uniform priors - ξ_{quark}: (0.0, 0.003) and B_{quark}: (40.0, 180.0) along with the RMF model parameters as given in Table 1.

RMF model parameters are constrained at low densities by constraints listed (except the last one) under 'Nuclear Physics Regime' in Table A.2. in the supplementary material of the Ref. [10]. Phase transition is such that we can observe the impact of imposing recent observational data from NICER collaboration for pulsars - PSR J0030+0451 and PSR J0470+6620 in addition to data for the GW170817 event (see 'Astrophysical Constraints' listed in Table A.2. in the supplementary material of the Ref. [10] for details on implementation.)

2.4 Quasinormal mode frequencies and damping times

We determine the equilibrium configuration of non-rotating NSs with our EOSs by solving the Tolman-Oppenheimer-Volkoff equations. Then, we follow the formalism of Lindblom-Detweiler[13], as done in Ref. [7] to calculate the frequencies for a particular class of stellar oscillations known as the *fundamental* or f−modes. We consider quadrupolar ($\ell = 2$) f−mode oscillations only. A fully general relativistic formalism allows us to obtain the complex-valued frequencies - the real part being the oscillation frequency (ν_f) and the inverse of the imaginary part being the GW damping time (τ_f).

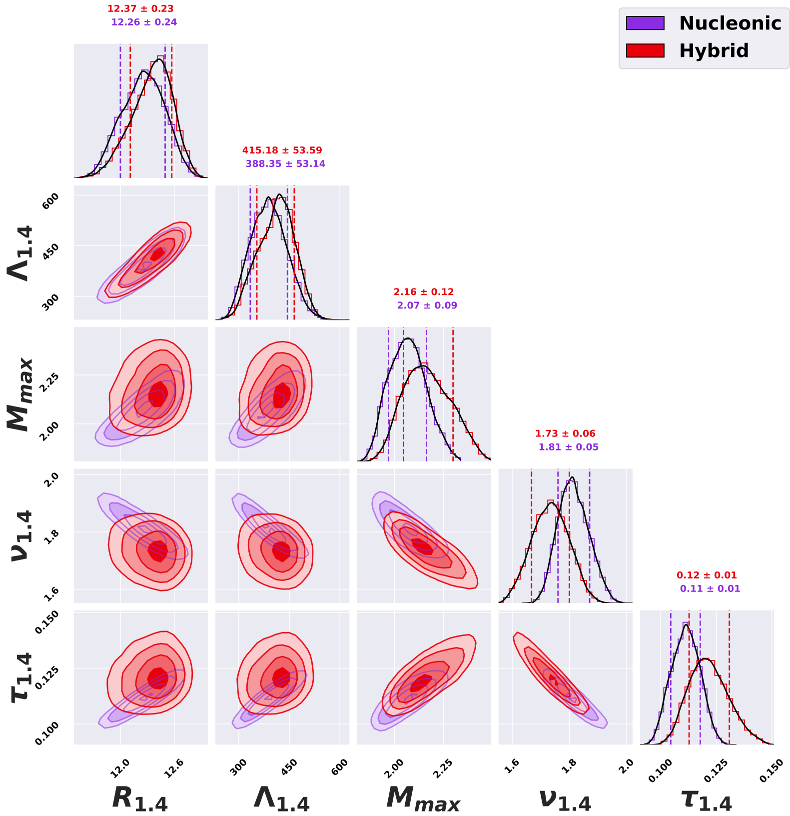

Figure 1: Corner plot showing the correlations among NS observables: radius (R), tidal deformability (Λ), $f-$mode oscillation frequency (ν_f), GW damping time (τ) and maximum mass M_{max} for nucleonic (blue) and hybrid (red) EOSs. Apart from M_{max}, the rest of the quantities are calculated for 1.4 $M_\odot$ NS.

3. Results

We calculate NS observables like mass, radius, $f-$mode oscillation frequency and corresponding GW damping times to look for the effect of hadron-quark phase transitions inside NSs.

In Fig. 1, the corner plot displays correlations among various NS observables. The panels along the diagonal show the marginalized 1D distributions of the observables. The median value of a specific observable is reported along with the bounds (shown with dashed vertical lines within a panel) for the 1σ (68%) credible interval (CI) above the panels. For nucleonic EOSs, the median value for maximum mass $M_{\rm max}$ for each EOS is 2.07 $M_\odot$. The observable $R_{1.4}$ has a median value of 12.26 km, while $\Lambda_{1.4}$ has a median value of 389.

We obtain a large number of hybrid EOSs in the posterior with phase transition beginning at transition number density $\rho_{B,tr}$ lying between 1.2 - 2.5 times the nuclear saturation density $\rho_{B,0}$. They appear to be stiffer than nucleonic EOSs. The slightly higher median values of $R_{1.4}$ = 12.37 km and $\Lambda_{1.4}$ = 416 for hybrid EOSs in the posterior support this observation. Stiffer hybrid EOSs produce a median $M_{\rm max}$ = 2.16 $M_\odot$, higher than that for nucleonic EOSs. Hybrid EOSs seem to predict tidal deformabilities beyond the 90% CI of tidal deformabilities for a 1.36 $M_\odot$ from the binary NS merger event GW170817. This might be a consequence of the inclusion of NICER data for PSR J0030+0451 and PSR J0740+6620 within the Bayesian analysis setup for this work.

We can assess the impact of hadron-quark phase transitions on the oscillation frequencies and corresponding GW damping times from the shift in the median values of the quantities $\nu_{f_{1.4}}$ and $\tau_{f_{1.4}}$. They are the real part of complex $f-$mode oscillation frequency and the inverse of the imaginary part of the same for a 1.4 $M_\odot$ NS, respectively. The median value of $\nu_{f_{1.4}}$ is 1.73 kHz for the nucleonic EOSs and 1.81 kHz for the hybrid EOSs. The corresponding GW damping times $\tau_{f_{1.4}}$ increases from 0.11 s to 0.12 s. In a recent work [7], we have shown how ν_f for an NS of given mass increases with the corresponding radius. We can observe a similar trend here - the median value of $\nu_{f_{1.4}}$ increases with an increase in the median value of $R_{1.4}$ when we compare the results between two EOS models. A visual comparison of the correlation, however, tells us that the trend weakens in the case of hybrid EOSs. Moreover, the correlations between $R_{1.4}$-$\nu_{f_{1.4}}$, $R_{1.4}$-$\tau_{f_{1.4}}$ and $R_{1.4}$-M_{max} also notably decrease for hybrid EOSs as do the correlations between $\Lambda_{1.4}$ and the same quantities.

In order to demonstrate the dependence of quasinormal modes on the quark EOS parameter ξ_{quark}, in Fig. 2, we plot the $f-$mode oscillation frequencies and the GW damping times for four different NS masses with the hybrid EOSs. The colour gradient represents the range of the quark EOS parameter ξ_{quark} with darker shades representing higher magnitude. Interestingly, for a given mass of NS, the complex oscillation frequency is strongly dependent on ξ_{quark}. The increase in the spread of data points for lower mass NSs illustrates that the relation between the complex frequency and the quark EOS parameter ξ_{quark} becomes more precise for higher mass NSs. Specifically, a lower value of ξ_{quark} corresponds to a higher value of the real as well as the imaginary part of the complex frequency. Since the GW damping time of an oscillation mode is the inverse of the imaginary part, the damping time for a given mass decreases with an increase in ξ_{quark}. Considering the linear association of the real part of $f-$mode frequency with the radius as demonstrated in Ref. [7], the trend in Fig. 2 implies that the EOSs become stiffer with an increase in ξ_{quark}, which in turn increases the radius and ν_f for a particular mass of NS.

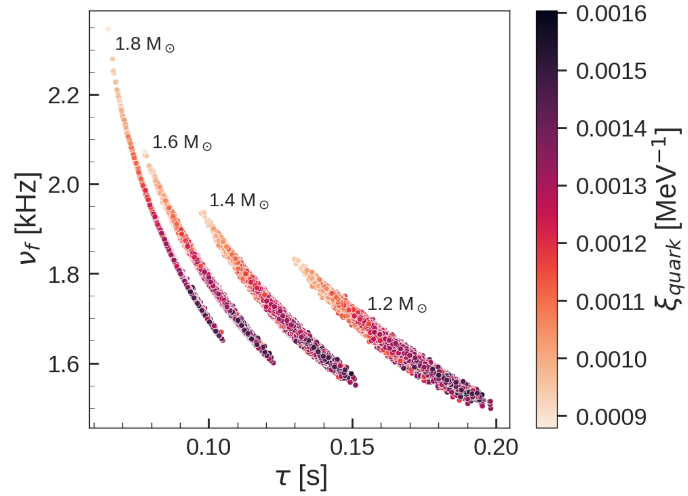

Figure 2: Plot of $f-$mode oscillation frequency ν_f and damping time τ_f. Each bunch of points are generated for a given mass of NS for all the hybrid EOSs.

4. Conclusions

Our Bayesian framework based on the hybrid EOS model constrained by nuclear physics calculations and astrophysical observations leads to EOSs with hadron-quark transition beginning within 1.2 - 2.5 times $\rho_{B,0}$. We find the transitions to be smooth in the pressure-energy density plane. In comparison to the EOSs within the nucleonic model, a greater median value of maximum mass implies that hybrid EOSs are stiffer. NSs with hadron-quark phase transition show relatively larger radii for a 1.4 $M_\odot$ NS. $f-$mode oscillation frequencies and GW damping times also exhibit the signature of the presence of quarks. The weakening of correlation between radius, tidal deformability and complex-valued $f-$mode frequency for hybrid EOSs indicates that correlations are dependent on densities above those inside the core of a 1.4 $M_\odot$ NS. Further model-independent studies, unlike ours, might be more conclusive. We can leverage the effect of phase transition on quasinormal modes of NSs in order to probe the possibility of quark deconfinement inside NS cores through future GW observations. One can also test the suitability of our hybrid EOS model over nucleonic EOS models by comparing how well one particular model is in agreement with various sets of data from astrophysical observations.

5. Acknowledgment

The authors would like to thank Mr. Anagh Venneti for his valuable insights and discussions for the development of this work.

References

[1] E. Annala, T. Gorda, A. Kurkela, et al. *Nat. Phys.*, 16:907, 2020.

[2] A. Chodos, R. L. Jaffe, K. Johnson, and C. B. Thorn. *Phys. Rev. D*, 10:2599, 1974.

[3] K. Schertler, S. Leupold, and J. Schaffner-Bielich. *Phys. Rev. C*, 60:025801, 1999.

[4] D. A. Fogaça and F. S. Navarra. *Phys. Lett. B*, 700:236, 2011.

[5] N. Andersson and K. D. Kokkotas. *Mon. Not. Roy. Astron. Soc.*, 299:1059, 1998.

[6] B. K. Pradhan, D. Chatterjee, M. Lanoye, and P. Jaikumar. *Phys. Rev. C*, 106:015805, 2022.

[7] D. Guha Roy, T. Malik, S. Bhattacharya, et al. *Astrophys. J*, 968:124, 2024.

[8] M. B. Albino, T. Malik, M. Ferreira, et al. *arXiv:2406.15337 [nucl-th]*, 2024.

[9] M. B. Albino, F. S. Navarra, et al. *Jour. of Phys.: Conf. Series*, 2340:012015, 2022.

[10] D. Guha Roy, A. Venneti, T. Malik, S. Bhattacharya, and S. Banik. *Phys. Lett. B*, 859:139128, 2024.

[11] Y. Hama, R. P. G. Andrade, F. Grassi, et al. *Nuc. Phys. A*, 774:169, 2006.

[12] J. Buchner, A. Georgakakis, K. Nandra, et al. *Astron. & Astrophys.*, 564:A125, 2014.

[13] L. Lindblom and S. L. Detweiler. *Astrophys. J. Supp. Series*, 53:73, 1983.