PROCEEDINGS OF SCIENCE

Quark-antiquark pair production in a strong color field: toward precision

Jamal Jalilian-Marian[a,b,*] **and Filip Bergabo**[a,b]

[a] *Baruch College*
17 Lexington Ave., New York, USA
[b] *The Graduate Center, City University of New York,*
365 Fifth Ave., New York, USA

E-mail: jamal.jalilian-marian@baruch.cuny.edu

We consider scattering of a quark-antiquark pair from the strong color field of a high energy proton or nucleus target in the Color Glass Condensate formalism where due to the high energies the target proton or nucleus is modeled as a shock wave of color. This scattering cross section can then be convoluted with the wave function of a virtual photon squared to give the differential cross section for production of a quark-antiquak dijet (or dihadron) in Deep Inelastic Scattering (DIS). We then discuss the latest progress made toward calculating corrections to this process which aim to increase the precision of theoretical calculations in time for the upcoming Electron-Ion Collider (EIC).

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*Speaker

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1. Introduction

High p_t particle production in high energy collisions involving at least one hadron/nucleus based on leading twist collinear factorization formalism [1–3] is very well-understood in perturbative QCD where some ingredients of this formalism are now known to 4-loop accuracy. In this formalism only one parton from a given hadron/nucleus participates in the scattering since the projectile and target hadron are dilute system of partons and double (multiple) scatterings are suppressed by powers of the large momentum transfer. On the other hand the small $x = \frac{p_t}{\sqrt{s}}$ limit of QCD is just starting to be explored in depth. In this kinematics one expects collinear factorization based on twist expansion to break down due to high gluon density effects since contributions of higher twist corrections are comparable to the leading twist one. In this kinematics the leading QCD dynamics is the so called gluon saturation [4].

It is known experimentally that at small x gluon distribution function rises very fast and that gluons become the most abundant parton species at small x. Due to this increase the number of gluons per unit area becomes large (this is the so-called gluon saturation phenomenon) so that at some point one must treat them as a collective rather than individual degree of freedom. In this kinematics where x is small it is appropriate to treat small x gluons as a classical field rather than individual partons. In CGC formalism these classical fields are generated by the larger x degrees of freedom generically called color charges. In the McLerran-Venugopalan model [5–7] of CGC one computes observables at fixed color charge and then averages over these color charges with a Gaussian distribution. Quantum corrections can then be computed and lead to a renormalization group equation known as the JIMWLK equation [8–10]. This approach is generically known as the Color Glass Condensate formalism and it has been applied to many processes involving collisions of high energy protons and nuclei [11–15]. Dihadron production in DIS at small x is believed to cleanest environment in which to explore QCD dynamics in the kinematics where one expects to see large gluon density effects. As a matter of fact azimuthal angular correlations in dihadron production in proton-nucleus collisions at RHIC exhibit properties compatible with what is expected from gluon saturation dynamics; i.e. the away side peak disappears when the transverse momentum is low and appear again when the momentum is high. If gluon saturation dynamics is indeed responsible for the disappearance of the away side peak in dihadron azimuthal angular correlations at RHIC one expects a qualitatively similar behavior for dihadron azimuthal angular correlations in DIS at small x [16–22].

2. Dihadron production in DIS at small x

Quark-antiquark production in DIS at small x is essentially a two stage process; the virtual photon decays into a quark-antiquark pair (called a dipole) long before it reaches the target proton/nucleus, after which the quark-antiquark dipole multiply scatters on the dense target. The later part contains all the QCD dynamics while the decay of virtual photon into a quark-antiquark pair is governed by QED. The produced quark antiquark pair will propagate through the target and multiply scatter from it. The multiple scattering of a quark (or antiquark) on the target is treated in the eikonal approximation appropriate for high energy scattering. In this kinematics the projectile parton has a (infinitely) high energy (p^+ for a right moving parton) so that the predominant coupling to the target gluon field A^μ is of the form $p \cdot A(x) \simeq p^+ A^-(x)$ and contributions of the other components of the gluon field are suppressed by powers of center of mass energy. Furthermore, due to the high energy of the collision the target gluon field $A^-(x)$ becomes independent of x^- coordinate so that $A^-(x) = A^-(x^+, x_\perp)$ (this is the usual time dilation effect). With these approximations multiple scatterings of the parton on a target can be resummed and leads to the following effective propagator $S_F(p, q)$ for a quark (and similarly for a antiquark),

$$S_F(p,q) \equiv (2\pi)^4 \, \delta^4(p-q) \, S^0_F(p) + S^0_F(p) \cdot \tau_F(p,q) \cdot S^0_F(q) \tag{1}$$

where $S^0_F(p)$ is the free Fermion propagator and dot represents a contraction of color indices. Furthermore, the interaction part of the propagator τ_F is defined as

$$\tau_F(p,q) \equiv (2\pi)\delta(p^+ - q^+)\not{n} \int d^2 z_t \, e^{i(p_t - q_t)\cdot z_t} \left[\theta(p^+) \left[V(z_t) - 1\right] - \theta(-p^+) \left[V^\dagger(z_t) - 1\right]\right] \tag{2}$$

and $V(z_t)$ is the path-ordered exponential known as the Wilson line which resums all the ekional multiple scatterings of the quark from the target. Using these effective propagators for the quark and antiquark the production cross section (for one quark flavor) can be written as

$$\begin{aligned} \frac{d\sigma^{\gamma^* A \to q\bar{q}X}}{d^2\mathbf{p}\, d^2\mathbf{q}\, dy_1\, dy_2} &= \frac{e^2 Q^2 (z_1 z_2)^2 N_c}{(2\pi)^7} \delta(1 - z_1 - z_2) \int d^8\mathbf{x} \left[S_{122'1'} - S_{12} - S_{1'2'} + 1\right] \\ & e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}} e^{i\mathbf{q}\cdot\mathbf{x}_{2'2}} \Bigg[4 z_1 z_2 K_0(|\mathbf{x}_{12}|Q_1) K_0(|\mathbf{x}_{1'2'}|Q_1) + \\ & (z_1^2 + z_2^2) \frac{\mathbf{x}_{12} \cdot \mathbf{x}_{1'2'}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2'}|} K_1(|\mathbf{x}_{12}|Q_1) K_1(|\mathbf{x}_{1'2'}|Q_1)\Bigg] \end{aligned} \tag{3}$$

where Q^2 is virtuality of the photon, $(\mathbf{p}, y_1)$ and $(\mathbf{q}, y_2)$ are the transverse momentum and rapidity of the produced quark and antiquark while $z_{1,2}$ are the longitudinal momentum fractions of the quark and antiquark with respect to the photon. The first (second) term inside the square bracket gives the contribution of the longitudinally (transversely) polarized photon. S_{12} and $S_{122'1'}$ are the so called dipole and quadrupole which contain the dynamics of gluon saturation and satisfy the JIMWLK evolution equation. This equation has been used to study dihadron correlations in $e\,p$ and $e\,A$ collisions which will take place in the proposed Electron-Ion Collider. Using this expression with suitable initial conditions for the dipole and quadrupole one can show that the away side peak does disappear as expected from gluon saturation physics. While Leading Order approximations in QCD may be expected to yield qualitatively accurate results a truly quantitative study of gluon saturation requires going beyond Leading Order accuracy and including higher order corrections.

Next to Leading order (NLO) corrections to the Leading Order (LO) results require radiation of a gluon by either the quark or antiquark. The radiated gluon may be absorbed by the quark or antiquark in the amplitude already or be produced as a final state. The first case corresponds to virtual corrections while the second case corresponds to real corrections. In principle this radiation and absorption can be done by the same or different quark (antiquark) and either before and after scattering from the target. Here we show some of the real corrections

$$
\begin{aligned}
\frac{d\sigma^L_{1\times 1}}{d^2\mathbf{p}\, d^2\mathbf{q}\, dy_1\, dy_2} =& \frac{2e^2g^2Q^2N_c^2z_2^3(1-z_2)^2(z_1^2+(1-z_2)^2)}{(2\pi)^{10}z_1}\int\frac{dz}{z}\int d^{10}x\, K_0(|\mathbf{x}_{12}|Q_2)K_0(|\mathbf{x}_{1'2'}|Q_2) \\
& \Delta^{(3)}_{11'}\, [S_{122'1'} - S_{12} - S_{1'2'} + 1]\, e^{i\mathbf{p}\cdot(\mathbf{x}'_1-\mathbf{x}_1)}e^{i\mathbf{q}\cdot\mathbf{x}_{2'2}}e^{i\frac{z}{z_1}\mathbf{p}\cdot\mathbf{x}_{1'1}} \\
\frac{d\sigma^L_{2\times 2}}{d^2\mathbf{p}\, d^2\mathbf{q}\, dy_1\, dy_2} =& \frac{2e^2g^2Q^2N_c^2z_1^3(1-z_1)^2(z_2^2+(1-z_1)^2)}{(2\pi)^{10}z_2}\int\frac{dz}{z}\int d^{10}\mathbf{x}\, K_0(|\mathbf{x}_{12}|Q_1)K_0(|\mathbf{x}_{1'2'}|Q_1) \\
& \Delta^{(3)}_{22'}[S_{122'1'} - S_{12} - S_{1'2'} + 1]e^{i\mathbf{q}\cdot\mathbf{x}_{2'2}}e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}}e^{i\frac{z}{z_2}\mathbf{q}\cdot\mathbf{x}_{2'2}} \\
& + \cdots\cdots
\end{aligned}
\tag{4}
$$

where the radiation kernel $\Delta^{(3)}_{ij}$ is

$$
\Delta^{(3)}_{ij} = \frac{\mathbf{x}_{3i}\cdot\mathbf{x}_{3j}}{\mathbf{x}^2_{3i}\mathbf{x}^2_{3j}} \tag{5}
$$

with similar expressions for the virtual corrections. In general these expressions contain divergences which must be dealt with before one can extract meaningful predictions from them. There are 4 categories of divergences; UV divergences when the loop momenta $k^\mu \to \infty$, these divergences appear in the virtual corrections and cancel among each other. The second category is soft divergences which happen when $k^\mu \to 0$, these divergences appear in both real and virtual contributions and cancel between real and virtual corrections. The next category is rapidity divergences which appear at finite $\mathbf{k}$ when $z \to 0$. These can be shown to lead to rapidity evolution of the dipoles and quadrupoles appearing in the cross section according to JIMWLK evolution equation. The last category of divergences is collinear divergences which appear when the radiated gluon becomes collinear to its parent, i.e. when the radiation angle $\theta \to 0$ at finite $\mathbf{k}$. These are absorbed into the bare parton-hadron fragmentation function and lead to its DGLAP evolution which can also be recast as renormalization of the bare fragmentation function. The final result can be symbolically written as

$$
d\sigma^{\gamma^* A\to h_1 h_2 X} = d\sigma_{LO}\otimes \text{JIMWLK} + d\sigma_{LO}\otimes D_{h_1/q}(z_{h_1},\mu^2)D_{h_2/\bar{q}}(z_{h_2},\mu^2) + d\sigma^{\text{finite}}_{NLO} \tag{6}
$$

where the first term means the dipoles and quadrupoles appearing in the Leading Order cross section evolve with JIMWLK equation while in the second term the fragmentation functions appearing in the Leading Order cross section evolve with DGLAP evolution equation. The last part is the genuine Next to Leading Order correction to the cross section. The resulting expressions can then be used to investigate inclusive dihadron production in DIS at small x. Nevertheless it is well-known that in the back to back limit of dihadron production one becomes sensitive to so called Sudakov effects. Its is not too difficult to include these effects in general, specially for single inclusive hadron production [23–26].

While our results can be used to investigate dihadron (dijet) production in DIS at small x it is important to note that the proposed Electron-Ion Collider (EIC) [27] will have a large phase space in x. It is therefore important to formulate a more general approach which includes both collinear factorization at large x (i.e. high p_t) and the Color Glass Condensate formalism at small x (i.e. low p_t). Some preliminary steps in this direction are taken for example in [28–31].

References

[1] G. Altarelli and G. Parisi, *Asymptotic Freedom in Parton Language*, *Nucl. Phys. B* **126** (1977) 298.

[2] V.N. Gribov and L.N. Lipatov, *Deep inelastic e p scattering in perturbation theory*, *Sov. J. Nucl. Phys.* **15** (1972) 438.

[3] Y.L. Dokshitzer, *Calculation of the Structure Functions for Deep Inelastic Scattering and e+ e- Annihilation by Perturbation Theory in Quantum Chromodynamics.*, *Sov. Phys. JETP* **46** (1977) 641.

[4] F. Gelis, E. Iancu, J. Jalilian-Marian and R. Venugopalan, *The Color Glass Condensate*, *Ann. Rev. Nucl. Part. Sci.* **60** (2010) 463 [1002.0333].

[5] L.D. McLerran and R. Venugopalan, *Computing quark and gluon distribution functions for very large nuclei*, *Phys. Rev. D* **49** (1994) 2233 [hep-ph/9309289].

[6] L.D. McLerran and R. Venugopalan, *Gluon distribution functions for very large nuclei at small transverse momentum*, *Phys. Rev. D* **49** (1994) 3352 [hep-ph/9311205].

[7] J. Jalilian-Marian, A. Kovner, L.D. McLerran and H. Weigert, *The Intrinsic glue distribution at very small x*, *Phys. Rev. D* **55** (1997) 5414 [hep-ph/9606337].

[8] J. Jalilian-Marian, A. Kovner, A. Leonidov and H. Weigert, *The BFKL equation from the Wilson renormalization group*, *Nucl. Phys. B* **504** (1997) 415 [hep-ph/9701284].

[9] J. Jalilian-Marian, A. Kovner, A. Leonidov and H. Weigert, *The Wilson renormalization group for low x physics: Towards the high density regime*, *Phys. Rev. D* **59** (1998) 014014 [hep-ph/9706377].

[10] J. Jalilian-Marian, A. Kovner and H. Weigert, *The Wilson renormalization group for low x physics: Gluon evolution at finite parton density*, *Phys. Rev. D* **59** (1998) 014015 [hep-ph/9709432].

[11] A. Dumitru and J. Jalilian-Marian, *Forward quark jets from protons shattering the colored glass*, *Phys. Rev. Lett.* **89** (2002) 022301 [hep-ph/0204028].

[12] A. Dumitru, A. Hayashigaki and J. Jalilian-Marian, *The Color glass condensate and hadron production in the forward region*, *Nucl. Phys. A* **765** (2006) 464 [hep-ph/0506308].

[13] J. Jalilian-Marian, *Production of forward rapidity photons in high energy heavy ion collisions*, *Nucl. Phys. A* **753** (2005) 307 [hep-ph/0501222].

[14] J. Jalilian-Marian, *Electromagnetic signatures of the color glass condensate: Dileptons*, *Nucl. Phys. A* **739** (2004) 319 [nucl-th/0402014].

[15] J. Jalilian-Marian, *Photon + hadron production in high energy deuteron (proton)-nucleus collisions*, *Nucl. Phys. A* **770** (2006) 210 [hep-ph/0509338].

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[16] A. Ayala, M. Hentschinski, J. Jalilian-Marian and M.E. Tejeda-Yeomans, *Polarized 3 parton production in inclusive DIS at small x*, *Phys. Lett. B* **761** (2016) 229 [1604.08526].

[17] A. Ayala, M. Hentschinski, J. Jalilian-Marian and M.E. Tejeda-Yeomans, *Spinor helicity methods in high-energy factorization: efficient momentum-space calculations in the Color Glass Condensate formalism*, *Nucl. Phys. B* **920** (2017) 232 [1701.07143].

[18] P. Caucal, F. Salazar and R. Venugopalan, *Dijet impact factor in DIS at next-to-leading order in the Color Glass Condensate*, *JHEP* **11** (2021) 222 [2108.06347].

[19] P. Taels, T. Altinoluk, G. Beuf and C. Marquet, *Dijet photoproduction at low x at next-to-leading order and its back-to-back limit*, *JHEP* **10** (2022) 184 [2204.11650].

[20] F. Bergabo and J. Jalilian-Marian, *One-loop corrections to dihadron production in DIS at small x*, *Phys. Rev. D* **106** (2022) 054035 [2207.03606].

[21] F. Bergabo and J. Jalilian-Marian, *Dihadron production in DIS at small x at next-to-leading order: Transverse photons*, *Phys. Rev. D* **107** (2023) 054036 [2301.03117].

[22] E. Iancu and Y. Mulian, *Dihadron production in DIS at NLO: the real corrections*, *JHEP* **07** (2023) 121 [2211.04837].

[23] T. Altinoluk, J. Jalilian-Marian and C. Marquet, *Sudakov double logs in single-inclusive hadron production in DIS at small x from the color glass condensate formalism*, *Phys. Rev. D* **110** (2024) 094056 [2406.08277].

[24] P. Caucal, E. Ferrand and F. Salazar, *Semi-inclusive single-jet production in DIS at next-to-leading order in the Color Glass Condensate*, 2401.01934.

[25] F. Bergabo and J. Jalilian-Marian, *SIDIS at small x at next-to-leading order: gluon contribution*, 2401.06259.

[26] F. Bergabo and J. Jalilian-Marian, *Single inclusive hadron production in DIS at small x: next to leading order corrections*, *JHEP* **01** (2023) 095 [2210.03208].

[27] M. Hentschinski et al., *White Paper on Forward Physics, BFKL, Saturation Physics and Diffraction*, *Acta Phys. Polon. B* **54** (2023) 3 [2203.08129].

[28] J. Jalilian-Marian, *Elastic scattering of a quark from a color field: longitudinal momentum exchange*, *Phys. Rev. D* **96** (2017) 074020 [1708.07533].

[29] J. Jalilian-Marian, *Quark jets scattering from a gluon field: from saturation to high p_t*, *Phys. Rev. D* **99** (2019) 014043 [1809.04625].

[30] J. Jalilian-Marian, *Rapidity loss, spin, and angular asymmetries in the scattering of a quark from the color field of a proton or nucleus*, *Phys. Rev. D* **102** (2020) 014008 [1912.08878].

[31] T. Altinoluk, G. Beuf and J. Jalilian-Marian, *Renormalization of the gluon distribution function in the background field formalism*, 2305.11079.

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