PROCEEDINGS OF SCIENCE

Dipion transitions between heavy hybrids and heavy quarkonium excitations

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We report on preliminary results for dipion transitions involving heavy quarkonium and heavy hybrid states of size much larger than the typical hadronic scale. For these states the usual QCD multipole expansion does not hold. As an alternative, we propose an interaction Lagrangian for pions and the QCD string. It allow us to calculate the light quark mass dependence of the string tension, elastic pion scattering off the string, and the decay of string excitations by pion emission. We then introduce an interaction Lagrangian of pions with heavy quarkonium and heavy hybrid states such that it reproduces the previous results in the static limit. We calculate a couple of selected transitions with this Lagrangian for illustration purposes.

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1. Introduction

Dipion transitions in heavy quarkonium are usually calculated through a two step procedure. First the multipole expansion is used [1]. This is expanding soft gluon operators about the heavy quarkonium center of mass. Next the soft gluon operators are hadronized in terms of pions. For this approach to be valid the typical size of the bound states r must be smaller than the typical hadronic scale $1/\Lambda_{\rm QCD}$ and smaller than the time scale of the transition $1/\Delta E$, where ΔE is the energy difference between initial and final quarkonium states. Often an additional expansion is used, the so called twist expansion [2] , which localizes at the same time soft gluon operators at different times. This approximation assumes that the typical binding energies are larger than $\Lambda_{\rm QCD}$ and ΔE, and it is difficult to justify in practise [3]. For quarkonium states with large principal quantum number even the multipole expansion may fail. For instance, typical values of $1/r$ for $\psi(nS)$ ($\Upsilon(nS)$) are 520 (1030) MeV, 260 (430) MeV, 190 (300) MeV and 150 (230) MeV for $n = 1, 2, 3, 4$ [4]. Except for the $1S$ states, the remaining figures for $1/r$ are of the order or smaller than $\Lambda_{\rm QCD}$. It is then worth exploring the consequences of giving up the multipole expansion. In order to do so we focus on the opposite limit, $r\Lambda_{\rm QCD} \gg 1$. In this limit, the QCD effective string theory (EST) can be used [5]. We propose an interaction Lagrangian of the QCD string with pions, which respects both the symmetries of the EST and chiral symmetry. With this Lagrangian, we then calculate the quark mass dependence of the string tension, elastic pion scattering off the string and the decay of string excitations into pion pairs. We put forward interaction Lagrangians for quarkonium to quarkonium and hybrid to quarkonium dipion transitions which reproduce the string result in the static limit. We present preliminary results for quarkonium/hybrid to quarkonium dipion transitions.

2. The interaction of pions with the QCD string

The effective QCD string (EST) provides an accurate description of the static potential at long distances ($r\Lambda_{\rm QCD} \gg 1$), both for quarkonium and hybrids. This is also the case for a number of $1/m$ suppressed quarkonium potentials. The Nambu-Goto action provides the leading terms of the EST.

$$S_{NG} = -\sigma \int d^2\xi \sqrt{-\det(\partial_a x^\mu \partial_b x_\mu)}\,, \tag{1}$$

σ is the string tension. This action enjoys reparameterization invariance, and Poincaré invariance. We choose the frame $\xi^1 = x^0 = t$, $t \in \mathbb{R}$, $\xi^2 = x^3 = z$, $z \in [-r/2, r/2]$, $x^i = x^i(t, z)$, $i = 1, 2$, and assume $x^i \sim 1/\Lambda_{\rm QCD}$ and $\partial_t \sim \partial_r \sim 1/r$. For $r\Lambda_{\rm QCD} \gg 1$, we then have,

$$S_{NG} \simeq -\sigma \int dtdz \left[1 - \frac{1}{2}\partial_0 x^i \partial_0 x^i + \frac{1}{2}\partial_z x^i \partial_z x^i\right] = -\sigma \int dtdz \left[1 - \partial_0\varphi^*\partial_0\varphi + \partial_z\varphi^*\partial_z\varphi\right]\,, \tag{2}$$

$\varphi = \varphi(z, t) = \left(x^1(z, t) + ix^2(z, t)\right)/\sqrt{2}$.

At low energies ($p \sim m_\pi \ll \Lambda_{\rm QCD}$), pion physics is described by the Chiral Lagrangian [6],

$$\mathcal{L}^{LO}_{\rm Ch} = \frac{f_\pi^2}{4}{\rm Tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{f_\pi^2 m_\pi^2}{4m_q}{\rm Tr}(U^\dagger \mathcal{M} + \mathcal{M}^\dagger U) \tag{3}$$

which is also Poincaré invariant, and approximately invariant under $SU_L(2) \otimes SU_R(2)$ chiral symmetry. The latter symmetry is spontaneously broken by the ground state to the diagonal $SU(2)$ (isospin) and explicitly broken by the light quark masses, $\mathcal{M} = m_q \mathbb{I}$, $m_q = (m_u + m_d)/2$.

The guidelines to build the interaction of pions with the string in an EFT framework are symmetries and power counting. We shall write down a local effective Lagrangian that respects the symmetries of both (2) and (3). For the power counting we are going to assume $1/r$, p, $m_\pi \ll \Lambda_{\rm QCD}$, which holds both for the EST and the Chiral Lagrangian. This means an expansion in $\partial x^i(\xi)/\partial \xi^a \sim 1/r\Lambda_{\rm QCD}$, $\partial_\mu/4\pi f_\pi \sim p/\Lambda_{\rm QCD}$, $\mathcal{M}/4\pi f_\pi \sim m_\pi/\Lambda_{\rm QCD}$. Since the QCD string does not transform under chiral symmetry, the simplest lower dimensional local operator we may build corresponds to the embedding of the string into the Chiral Lagrangian.

$$S_{\rm int} = \int d^2\xi \sqrt{-\det(\partial_a x^\mu \partial_b x_\mu)}\,\mathcal{L}_{\rm ChS}(x(\xi)) \tag{4}$$
$$\mathcal{L}_{\rm ChS} = \lambda {\rm Tr}(\partial_\mu U^\dagger \partial^\mu U) + \lambda' {\rm Tr}(U^\dagger \mathcal{M} + \mathcal{M}^\dagger U) + \lambda''' {\rm Tr}(\mathcal{M}^\dagger \mathcal{M}) + \lambda'''' {\rm Tr}(U^\dagger \mathcal{M} U^\dagger \mathcal{M} + {\rm h.c.})\,.$$

We have displayed in $\mathcal{L}_{\rm ChS}$ the leading order (LO) terms, and the next-to-leading (NLO) terms needed for renormalization.

2.1 The quark mass dependence of the string tension

Note that in the subspace of zero pions, the Lagrangian (5) reduces to the usual Nambu-Goto action, and hence it redefines the string tension. This redefinition introduces a light quark mass dependence through $\mathcal{M}$ in (5) and the pion mass. Indeed, space-time translational invariance implies $\langle 0|\, \mathcal{L}_{\rm ChS}(x(\xi))\, |0\rangle = \langle 0|\, \mathcal{L}_{\rm ChS}(0)\, |0\rangle$. At leading order in the light quark mass, there is a tree-level linear contribution from the second term in the LO Lagrangian (5). At NLO there is a one-loop contribution from the first and second terms of the LO Lagrangian and a tree-level contribution from the NLO Lagrangian. The final result reads,

$$\sigma \longrightarrow \sigma - \left(4\lambda' m_q + \left[\frac{6B_0}{8\pi^2 f_\pi^2}(2B_0\lambda - \lambda')\left(\ln\frac{2B_0 m_q}{\mu^2} - 1\right) + 2\lambda''\right] m_q^2\right)\,, \tag{5}$$

$\lambda'' = \lambda''' + 2\lambda''''$ is necessary to absorb the UV divergences of the one-loop calculation. Dimensional regularization in the $\overline{\rm MS}$ scheme has been used. We have also used $m_\pi^2 = 2m_q B_0$ ($B_0 \sim \Lambda_{\rm QCD}$) in the higher order terms. This result is expected to be useful to understand the differences in the heavy quarkonium spectrum observed in lattice calculations at different pion masses (see for instance [7] and [8] for charmonium), which are more pronounced for hybrids and highly excited quarkonium states. The light quark mass dependence of the string tension has been recently observed in [9].

2.2 Pion scattering off the string

The simplest process involving pions is the elastic scattering of a pion off any string state. The LO contribution arises at tree level, with $\sqrt{-\det(\partial_a x^\mu \partial_b x_\mu)} \simeq 1$. We have

$$\langle \pi(\vec{q})|\mathcal{L}^{LO}_{\rm ChS}(x)|\pi(\vec{q}\,')\rangle = \frac{4}{f_\pi^2}\left(\lambda q_\mu q'^\mu - \lambda' \frac{m_\pi^2}{2B_0}\right)e^{i(q-q')x}\,, \tag{6}$$

$x = x(\xi)$. This expression must now be sandwitched between the given string state. At LO the exponential can be approximated by $e^{i((E_q - E_{q'})t - (q_z - q'_z)z)}$, which leads to,

$$\langle \pi(\vec{q})|S_{\rm int}|\pi(\vec{q}\,')\rangle \;=\; \frac{16\pi}{f_\pi^2}\left(\lambda q_\mu q'^\mu - \lambda' \frac{m_\pi^2}{2B_0}\right)\frac{\sin\left[(q_z - q'_z)\frac{r}{2}\right]}{(q_z - q'_z)}\delta(E_q - E'_q)\,, \tag{7}$$

$q^\mu = (E_q, \mathbf{q})$, $\mathbf{q} = (q^1, q^2, q_z)$, and analogously for q′. The LO expression above does not depend on the particular string state the pion scatters off. Notice the non-trivial interplay between the third component of the pion momentum transfer $q_z - q'_z$ and the string length r.

2.3 Decay of string excitations through pion emission

The string excitations may decay into the string ground state by a two pion (dipion) emission. Let us discuss the amplitude corresponding to the lowest lying excitations $N = 1$, $E_N = \pi N/r$ at LO. The calculation for the $N = 2$ excitations can be found in [10]. The $N = 1$ excitations consist of two degenerate Π_u states, a clockwise (R) and an anticlockwise (L) rotation of $|L_z| = 1$. The part of the calculation involving pions, can be obtained from (6) by crossing, $q' \to -q'$. Now we need the second term in the expansion of the exponential, $e^{i((E_q+E_{q'})t-(q_z+q'_z)z)}(1-i((q+q')^i x^i(z,t)))$. $\sqrt{-\det(\partial_a x^\mu \partial_b x_\mu)} \simeq 1$ still holds. We obtain,

$$\langle \pi(\vec{q}), \pi(\vec{q}\,'); 0_{\rm EST}|S_{\rm int}|0_\pi; 1\Pi_u^R\rangle = \langle \pi(\vec{q}), \pi(\vec{q}\,'); 0_{\rm EST}|S_{\rm int}|0_\pi; 1\Pi_u^L\rangle^* \tag{8}$$
$$= \frac{16\pi\sqrt{\pi}}{f_\pi^2\sqrt{\sigma}r}\left(\lambda q_\mu q'^\mu + \lambda'\frac{m_\pi^2}{2B_0}\right)\frac{\cos\left[(q_z+q'_z)\frac{r}{2}\right]}{\frac{\pi^2}{r^2}-(q_z+q'_z)^2}\frac{i}{\sqrt{2}}\left(q^1+q'^1+i(q^2+q'^2)\right)\delta\left(E_q+E'_q-\frac{\pi}{r}\right),$$

3. The long distance interaction of pions with heavy quarkonium and heavy quarkonium hybrids

In order to establish the long distance interactions of pions with heavy quarkonium and heavy quarkonium hybrids, we just write down interaction Lagrangians that reproduce the amplitudes of the pion scattering off the string and of the decay of string excitations, calculated in sec. 2.2 and in sec. 2.3 respectively, in the static limit.

In the case that only quarkonia are involved it reads,

$$\mathrm{L}_{\rm int} = \int d^3\mathbf{R}\int d^3\mathbf{r}\,{\rm tr}\left[S^\dagger(\mathbf{R},\mathbf{r},t)S(\mathbf{R},\mathbf{r},t)\right]\int_{-r/2}^{r/2} dz\, g(r,z)\mathcal{L}_{\rm ChS}(t,\mathbf{R}+z\hat{\mathbf{r}}), \tag{9}$$

where $S(\mathbf{R},\mathbf{r},t)$ is the quarkonium wave function field. This expression matches (7) if $g(r,z) = 1$.

In the case of a hybrid with 1^{+-} light degrees of freedom (LDF) in the initial state, it reads,

$$\mathrm{L}_{\rm int} = \int d^3\mathbf{R}\int d^3\mathbf{r}\,{\rm tr}\left[S^\dagger(\mathbf{R},\mathbf{r},t)\mathbf{H}(\mathbf{R},\mathbf{r},t)\right]\int_{-r/2}^{r/2} dz\, g(r,z)\hat{\mathbf{r}}\times\nabla_{\mathbf{R}}\,\mathcal{L}_{\rm ChS}(t,\mathbf{R}+z\hat{\mathbf{r}}) \tag{10}$$

where $\mathbf{H}(\mathbf{R},\mathbf{r},t)$is the wave function field for the hybrid. $g(r,z) = i\cos(\pi z/r)/\sqrt{\pi\sigma}$ matches the string theory result (8).

3.1 Dipion transitions between highly excited quarkonium states

We then obtain from (9) for the spin zero case,

$$\mathcal{M}(i=\{nLM\}\to f=\{n'L'M'\}\,\pi(q)\pi(q')) = -\frac{8\pi}{f_\pi^2}\left(\lambda q_\mu q'^\mu + \lambda'\frac{m_\pi^2}{2B_0}\right)I_{i\to f}(s)$$
$$I_{i\to f}(s) = \int dr\int d\Omega\, S_{n'L'}(r)S_{nL}(r)Y^*_{L'M'}(\hat{\mathbf{r}})Y_{LM}(\hat{\mathbf{r}})\frac{\sin\left(sr\frac{\cos\theta}{2}\right)}{s\cos\theta}, \tag{11}$$

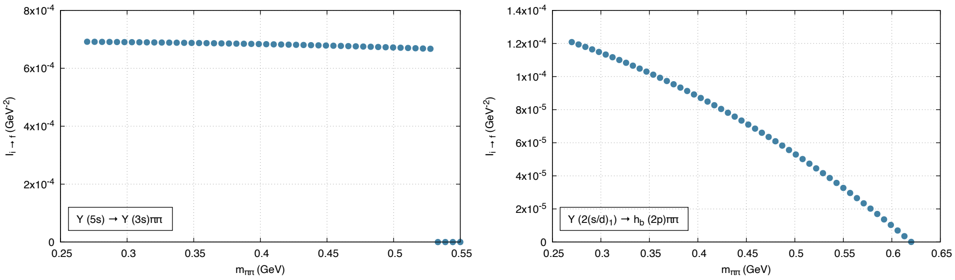

Figure 1: Dipion invariant mass distribution for $\Upsilon(10860)$ transitions assuming it corresponds to a $5s$ quarkonium state (left panel) and to a $2(s/p)_1$ (H_1') hybrid state (right panel).

nLM ($n'L'M'$) are the quarkonium principal quantum number, orbital angular momentum and its third component in the initial (final) state, $S_{nL}(r)$ the quarkonium reduced radial wave function, $Y_{LM}(\hat{\mathbf{r}})$ the spherical harmonics ($\hat{\mathbf{r}} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$), and $s = |\mathbf{q} + \mathbf{q}'|$.

3.2 Dipion transitions between hybrids and highly excited quarkonium states

If the initial state is a hybrid with 1^{+-} LDF, we obtain from (10) for the spin zero case,

$$\begin{aligned}
\mathcal{M}(i = \{nJLM\} \to f = \{n'L'M'\}\,\pi(q)\pi(q')) &= \frac{8\sqrt{\pi}}{f_\pi^2\sqrt{\sigma}}\left(\lambda q_\mu q'^\mu + \lambda' \frac{m_\pi^2}{2B_0}\right) I_{i\to f}(s) \\
I_{i\to f}(s) = \int dr S_{n'L'}(r) P^L_{nJ}(r) \int d\Omega Y^*_{L'M'}(\hat{\mathbf{r}}) &\left(C(L1J; M-1\,1) e^{i\phi} Y_{LM-1}(\hat{\mathbf{r}})\right. \\
&\left. + C(L1J; M+1\,-1) e^{-i\phi} Y_{LM+1}(\hat{\mathbf{r}})\right) \frac{-is\sin\theta\cos\left(sr\frac{\cos\theta}{2}\right)}{r\sqrt{2}\left(\frac{\pi^2}{r^2} - (s\cos\theta)^2\right)}
\end{aligned} \tag{12}$$

$P^L_{nJ}(r)$ is the reduced radial wave function of the hybrid. J is the sum of the orbital angular momentum and the angular momentum of the LDF and $C(LL_{\rm LDF}J; MM_{\rm LDF})$ are Clebsch-Gordan coefficients. For a given nJ there are two states, one with $L = J$, and one with $L = J \pm 1$. In the second case, the contributions from both L to $I_{i\to f}(s)$ must be added.

3.3 Dipion invariant mass distribution

The dipion invariant mass distribution reads,

$$\frac{d\Gamma}{dm^2_{\pi\pi}} = \frac{2}{f_\pi^4 (2\pi)^3}\left[\frac{\lambda}{2} m^2_{\pi\pi} + \left(\frac{\lambda'}{2B_0} - \lambda\right) m_\pi^2\right]^2 \sqrt{1 - \frac{4m_\pi^2}{m^2_{\pi\pi}}}\, s|\bar{I}_{i\to f}(s)|^2, \tag{13}$$

$m^2_{\pi\pi} = (q+q')^2 \simeq \Delta E - s^2$. ΔE is the mass difference between the quarkonium/hybrid in the initial state and the quarkonium in the final state once recoil corrections are neglected. $|\bar{I}_{i\to f}(s)|^2$ stands for the spin average of $|I_{i\to f}(s)|^2$. Note that the dependence on the unknown low energy constants λ and λ' is universal, namely independent of the initial and final states. As an example, we show in fig. 1 a preliminary estimate of $|\bar{I}_{i\to f}(s)|^2$ assuming that $\Upsilon(10860)$ is a $5s$ quarkonium and a $2(s/d)_1$ hybrid (H_1'). We observe that the shapes are qualitatively different.

4. Discussion

In [4], it was found that $\Upsilon(10860)$ could be a mixture of a $5s$ quarkonium and a $2(s/d)_1$ hybrid. The analysis of the dipion spectrum may help elucidating the mixing pattern. Note that from (13) the ratio of spectra is independent of the low energy constants λ and λ', and only depends on the initial and final states.

If $rs \ll 1$, as it is the case if the masses of the initial and final states differ by about $2m_\pi$, (9) and (10) simplify as the pion fields can be expanded in z and the integrals over z reduce to functions of r. In the EFT spirit, a systematic analysis of the operators contributing at each order of the expansion $rs \ll 1 \ll r\Lambda_{\rm QCD} \sim \Lambda_{\rm QCD}/m_\pi$ is necessary and under way.

Finally, let us mention that the dipion decay of hydrids has been considered before, assuming the multipole expansion to hold [11].

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References

[1] K. Gottfried, Phys. Rev. Lett. **40** (1978), 598

[2] M. B. Voloshin, Nucl. Phys. B **154** (1979), 365-380

[3] A. Pineda and J. Tarrús Castellà, Phys. Rev. D **100** (2019) no.5, 054021 [arXiv:1905.03794 [hep-ph]].

[4] R. Oncala and J. Soto, Phys. Rev. D **96** (2017) no.1, 014004 [arXiv:1702.03900 [hep-ph]].

[5] M. Luscher and P. Weisz, JHEP **07** (2004), 014 [arXiv:hep-th/0406205 [hep-th]].

[6] J. Gasser and H. Leutwyler, Annals Phys. **158** (1984), 142

[7] L. Liu *et al.* [Hadron Spectrum], JHEP **07** (2012), 126 [arXiv:1204.5425 [hep-ph]].

[8] G. K. C. Cheung *et al.* [Hadron Spectrum], JHEP **12** (2016), 089 [arXiv:1610.01073 [hep-lat]].

[9] J. Bulava, F. Knechtli, V. Koch, C. Morningstar and M. Peardon, Phys. Lett. B **854** (2024), 138754 [arXiv:2403.00754 [hep-lat]].

[10] Sandra Tomàs Valls, *Interactions of Pions with the QCD String and Light Quark Mass Dependencies*, M.Sc. Thesis, Universitat de Barcelona, 2023.

[11] J. Tarrús Castellà and E. Passemar, Phys. Rev. D **104**, no.3, 034019 (2021) [arXiv:2104.03975 [hep-ph]].