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Long distances effects in the hyperfine splittings of heavy quarkonium hybrids

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We report on a calculation of the hyperfine splitting of heavy quarkonium hybrids, which takes into account the long-distance form of the heavy-quark spin-dependent potentials. These potenetials are estimated using an interpolation between the known short-distance form and the long distance form calculated in the QCD effective string theory. We correct an error in the long distance form of one of the potentials used before, which leads to very small modifications of the previous results.

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1. Introduction

In QCD, studying heavy quarkonium systems has been a long-standing effort. In heavy hadrons, heavy quarks with masses exceeding the QCD energy scale move slowly and can be described using non-relativistic effective field theories (EFT) [1, 2]. Analogous EFTs can be built for exotic heavy quarkonium [3]. A non-relativistic bound state is formed by the heavy quarks subject to an interaction potential depending on the gluon and light quark content, collectively known as light degrees of freedom (LDF). Three scales characterize these bound states: the heavy quark mass, m_Q ($m_Q \gg \Lambda_{QCD}$), the relative momentum $m_Q v$ ($m_Q v \sim 1/r \sim \Lambda_{QCD}$), with relative velocity $v \ll 1$, and the binding energy $m_Q v^2$ ($\Lambda_{QCD} \gg m_Q v^2$). Whereas the LDF states are characterized by the typical hadronic scale Λ_{QCD}. The leading order (LO) of the adiabatic expansion between the dynamics of the heavy degrees of freedom, the heavy quarks, and the LDF due to $\Lambda_{QCD} \gg m_Q v^2$ is the so-called Born-Oppenheimer approximation used in a non-relativistic EFT framework (BOEFT) in [4, 5]. The BOEFT has also been extended to include spin-dependent operators up to LO $1/m_Q$ [6, 7] and up to next-to-leading order (NLO) $1/m_Q^2$ [8].

We shall focus here on the calculation of the hyperfine splitting for hybrids as an example of a NLO calculation. In order to compute the hyperfine splittings (HFS) for the lower lying charmonium and bottomonium hybrids at LO in the BOEFT [7], we used an interpolation between the known form at short distances of the spin-dependent potentials given in [8] and the long-distance estimation using the QCD effective string theory (EST) [9]. We provide the results of the spectrum of the lower lying static hybrid states (Σ_u and Π_u). We use the charmonium spectrum from [10] to determine the unspecified parameters in the short-distance form of the potentials and to evaluate the interpolation dependence. Thereafter, we are able to predict the HFS of higher multiplets and bottomonium hybrids.

2. The potentials

In Ref. [3, 7], for $\kappa^p = 1^{+-}$ as the quantum numbers of the LDF, we find that only two independent potentials describe the hyperfine splitting at LO. Which can be arranged as follows,

$$V_{hf}(r) = \frac{1}{6}V^{sa}_{1^+11}(r) - \frac{1}{3}V^{sb}_{1^+10}(r)\,, \quad V_{hf2}(r) = -\frac{1}{2}\left(V^{sa}_{1^+11}(r) + V^{sb}_{1^+10}(r)\right)\,. \tag{1}$$

The long-distance potentials $V^{sa}_{1^+11}(r)$ and $V^{sb}_{1^+10}(r)$ are obtained using QCD EST [11], following the mapping in [12]. Note that the sign convention for $V^{sa}_{1^+11}(r)$ is the opposite to that of its general expression [3]. However, this choice does not affect the final result. Thus, we obtain

$$\frac{V^{sa}_{1^+11}(r)}{m_Q} = -\frac{2c_F\pi^2 g\Lambda'''}{m_Q\kappa r^3} \equiv V^{sa}_{ld}(r)\,, \quad \frac{V^{sb}_{1^+10}(r)}{m_Q} = \mp\frac{c_F g\Lambda' 2\pi^2}{m_Q\sqrt{\pi\kappa}}\frac{1}{r^2} \equiv V^{sb}_{ld}(r)\,. \tag{2}$$

Parameters $g\Lambda' \sim -59$ MeV and $g\Lambda''' \sim \pm 230$ MeV as in [4]. $\kappa \simeq 0.187\ \text{GeV}^2$ is the string tension and $c_F(m_b) \equiv c_F(\nu = 1\,\text{GeV}, m_c) = 1.12155$ and $c_F(m_b) \equiv c_F(\nu = 1\,\text{GeV}, m_b) = 0.87897$. Here, in (2) we correct a factor of two in $V^{sb}_{1^+10}(r)$ overlooked in [7]. Due to the fact that $g\Lambda'$ is about four times smaller than $g\Lambda'''$, this correction has a very small impact on the results. In any case, we discuss the differences in the corrected spectrum compared to [7].

The short distance behavior is described in Refs. [8, 13], so that

$$V_{hf}(r)/m_Q = A + O(r^2)\,, \qquad V_{hf2}(r)/m_Q = Br^2 + O(r^4)\,. \tag{3}$$

$A = c_F\, k_A/m_Q$ and $B = c_F\, k_B/m_Q$ are the unknown real constants in the interpolation that are to be fitted to lattice data. $k_A \sim \Lambda^2_{\rm QCD}$ and $k_B \sim \Lambda^4_{\rm QCD}$ as in [13].

Interpolating between the short- and long-distance behavior with

$$\begin{aligned} \frac{V_{hf}(r)}{m_Q} &= \frac{A + \left(\frac{r}{r_0}\right)^2 \left(\frac{1}{6} V_{ld}^{sa}(r_0) - \frac{r}{3r_0} V_{ld}^{sb}(r_0)\right)}{1 + \left(\frac{r}{r_0}\right)^5} \\ \frac{V_{hf2}(r)}{m_Q} &= \frac{Br^2 - \left(\frac{r}{r_0}\right)^5 \left(\frac{r_0}{2r} V_{ld}^{sa}(r_0) + \frac{1}{2} V_{ld}^{sb}(r_0)\right)}{1 + \left(\frac{r}{r_0}\right)^7} \end{aligned} \tag{4}$$

where $r_0 \sim 1/\Lambda_{\rm QCD}$ is the matching scale. This scale is estimated from the short- and long-distance behavior of the static hybrid potentials giving $r_0 \simeq 3.96\ \text{GeV}^{-1}$.

3. Charmonium hybrids spectrum

We fix the parameters A and B by comparing the spectrum results obtained with our interpolations (4) with the correct long-distance behavior (2) to the lattice data of [10] for the lower lying hybrid states. Using $m_c = 1.47$ GeV, scanning values of A and B in the intervals $[-0.3\,, 0.3]$ GeV and $[-0.06\,, 0.06]$ GeV^3, respectively, and searching for the values with the lowest χ^2/dof, we obtain similar results to those reported in [7]. The best fit corresponds to a negative $V_{ld}^{sa}(r)$, which implies $g\Lambda''' < 0$, and a positive $V_{ld}^{sb}(r)$. If we allow $g\Lambda'$ and $g\Lambda'''$ to move within the errors quoted in [4], the lowest χ^2/dof corresponds to $g\Lambda' = -0.0796$ GeV and $g\Lambda''' = 0.3105$ GeV. These values lead to $A = 0.1509$ GeV and $B = -0.0015$ GeV^3. The interpolation dependence is estimated by moving $r_0 \in [3\,, 5]$ GeV^{-1}. The χ^2/dof is the lowest for the default value $r_0 = 3.96$ GeV^{-1}, in contrast to [7] where the χ^2/dof improved marginally around $r_0 \sim 3.5$ GeV^{-1}. Regarding the errors of A and B due to the input data and the error due to higher orders in the $1/m_Q$ expansion, we obtain $A = 0.124 \pm 0.034$ GeV and $B = 0.004 \pm 0.016$ GeV^3. The fit parameters dependence on sign ambiguities, on r_0 and the corrected hybrid charmonium spectrum with its errors are displayed in the first tables of [14].

4. Bottomonium hybrids spectrum

With A and B fixed, the corresponding parameters A' and B' to predict the hyperfine bottomonium splittings are also fixed by a relation on the masses of charm and bottom. Using $m_b = 4.88$ GeV and computing the spectrum for the central values of these parameters $A' = 0.027 \pm 0.004$ GeV, $B' = 0.001 \pm 0.002$ GeV^3 we obtain results that change marginally compared with the ones obtained in [7]. Corrected table with the hybrid bottomonium spectrum displayed in [14].

5. Conclusions

The benefits of interpolating between short- and long-distance potentials are outlined in Ref. [7]. In summary, significant improvement in describing the hyperfine splittings of charmonium hybrids from lattice data is observed when long-distance contributions computed with QCD EST are included in LO spin-dependent potentials within BOEFT. Comparing with the fit using the NLO short-distance potentials [13], the χ^2/dof moves from 0.999 in that fit to 0.619 in ours. With the unknown parameters fixed, we computed the hyperfine splittings of higher charmonium hybrid states, of the bottomonium ones, and the error associated to them.

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