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Possible scenario of dynamical chiral symmetry breaking in the instanton liquid

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Based on simulations of the interacting instanton liquid model (IILM) with three-flavor quarks, we compute the free energy density of the QCD vacuum as a function of the quark condensate. We then evaluate the second derivative of the free energy density with respect to the quark condensate at the origin. This evaluation allows us to investigate whether chiral symmetry breaking in the IILM occurs in an anomaly-driven way. Such a breaking pattern of chiral symmetry has been proposed in a previous study to connect the QCD vacuum structure with meson properties, such as the mass of the sigma meson. We also perform the quenched simulations, in which no dynamical quarks interact with instantons. Comparing these results with the full calculations provides a better understanding of the pattern of chiral symmetry breaking in the IILM. We find that in the full IILM, chiral symmetry is dynamically broken in anomaly-driven way, whereas in the quenched IILM, it is broken through the ordinary mechanism. Based on these results, we suggest that chiral symmetry breaking in real QCD could also occur in an anomaly-driven way. Consequently, in phenomena where chiral symmetry breaking plays a crucial role, the anomaly effect may also have significant influence.

10th International Conference on Quarks and Nuclear Physics (QNP2024)
8-12 July, 2024
Barcelona, Spain

*Speaker

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1. Introduction

Spontaneous chiral symmetry breaking is a fundamental phenomenon in strong interaction, playing a crucial role in hadron physics. It governs key aspects such as the generation of hadron masses and the emergence of massless Goldstone bosons. Traditionally, chiral symmetry breaking is understood as follows: the attractive interaction mediated by gluon exchanges between quarks leads to the formation of a quark-antiquark condensate in the vacuum, and it breaks chiral symmetry spontaneously. This mechanism is widely described by the Nambu-Jona-Lasinio (NJL) model, an effective low-energy theory of quantum chromodynamics (QCD).

A recent study has revisited the relationship between the pattern of chiral symmetry breaking and the axial anomaly. One well-known example is the NJL model, which has been extensively studied ans is known to exhibit chiral symmetry breaking due to the attractive interaction between quarks, as described above. Quantitatively, this symmetry breaking occurs when the coupling g_S of the inter-quark interaction exceeds a critical value g_S^{crit}. The previous study demonstrated that even when $g_S < g_S^{\text{crit}}$, dynamical symmetry breaking can still occur if there is a sufficiently strong contribution from the axial anomaly, introduced via the Kobayashi–Maskawa–'t Hooft (KMT) term. In such cases, this anomaly-driven mechanism has been shown to influence physical quantities, such as causing the sigma meson mass to become lighter than 800 MeV [1].

Following to the previous study, we define patterns of chiral symmetry breaking, distinguishing between the conventional one known so far and the one proposed in the previous study due to the contribution of the axial anomaly. We refer to the former breaking pattern as "ordinary breaking" and the latter as "anomaly-driven breaking". Instead of classifying them based on the magnitude of the coupling constants, we adopt a more general approach. We define them by the sign of the second derivative of the vacuum energy density with respect to the quark condensate at the origin, where a negative (positive) value corresponds to ordinary (anomaly-driven) breaking [2].

We examine whether the anomaly-driven breaking can occur in other model rather than those used in the previous study. We focus on the fact that the anomaly effect is understood as the instanton induced interaction. Since such effect might be naturally incorporated in the instanton liquid model, which is a phenomenological framework describing the QCD vacuum with instanton degree of freedom, we expect that chiral symmetry could be broken in the anomaly-driven way. The details of this study are presented in our paper [2].

2. Partition function of the interacting instanton liquid model

The partition function of the interacting instanton liquid model is given by [3]

$$Z_{\text{IILM}} = \sum_{N_+,N_-=0}^{\infty} \frac{1}{N_+!N_-!} \int \prod_{i=1}^{N_++N_-} d\Omega_i f(\rho_i) e^{-S_{\text{int}}} \prod_{f=1}^{N_f} \det(i\gamma^\mu D_\mu + m_f), \tag{1}$$

where N_+ and N_- are number of instantons and antiinstantons inside the four volume V_4, the measure of the path integral $d\Omega$ consists of size, color orientation and position of the i-th instantons $d\Omega_i = d\rho_i dU_i d^4 z_i$, $f(\rho_i)$ represents the classical instanton amplitude, S_{int} is the instanton-instanton interaction and D_μ is covariant derivative for quark with current mass m_f for each flavor f.

3. Numerical calculation method

3.1 Vacuum energy density

The vacuum energy density ϵ of the vacuum is identified as the free energy density F at zero temperature ($T = 0$), $\epsilon = F$. For simplicity, we will refer to it as "free energy" below. From the standard thermodynamics relation, the free energy at $T = 0$ is expressed as $F = -(1/V_4) \ln Z$ with the partition function of the system Z. It is clear that to obtain the vacuum energy density ϵ at $T = 0$, it is sufficient to calculate the equivalent free energy F, and to calculate F, we only need to calculate Z. The partition function Z can be calculated as follows. One writes the partition function as $Z(\alpha) = \int d\Omega \exp[-S_{\rm eff}(\alpha)]$ such that the desired partition function Z is reproduced as $Z(\alpha = 1)$. Here, an effective action $S_{\rm eff}(\alpha)$ has the form of $S_{\rm eff}(\alpha) = S^0_{\rm eff} + \alpha S_1$ to interpolate between a known solvable action $S^0_{\rm eff} \equiv S_{\rm eff}(0)$ and the full one $S_{\rm eff}(1)$. With this decomposition, one obtains the partition function $Z(\alpha = 1)$ as follows:

$$\ln[Z(\alpha = 1)] = \ln[Z(\alpha = 0)] - \int_0^1 d\alpha \, \langle 0|S_1|0\rangle_\alpha \, , \tag{2}$$

where an expectation value $\langle 0|O|0\rangle_\alpha$ is defined by

$$\langle 0|O|0\rangle_\alpha = \frac{1}{Z(\alpha)} \int d\Omega O(\Omega) e^{-S_{\rm eff}(\alpha)}. \tag{3}$$

The explicit expression for the decomposition in our case is presented in Ref. [2], and it follows the one used in the previous study [3].

3.2 Quark condensate

In our calculation using the IILM, the quark condensate for a single flavor f with mass m_f is evaluated as an expectation value of the traced quark propagator at the same point:

$$\langle \bar{q}_f q_f \rangle = \sum_{A,\alpha} \langle q^\dagger_f(x)^A_\alpha q_f(x)^A_\alpha \rangle = -\lim_{y\to x} \frac{1}{Z} \int D\Omega {\rm Tr}[S(x,y;m_f)] e^{-S_{\rm int}} \det(\gamma^\mu D_\mu + m_f). \tag{4}$$

Here, we shortly write the measure in the path integral as $D\Omega$ given in the partition function. $A = 1, \ldots, N_c$ and $\alpha = 1, \cdots, 4$ represent the color and the Dirac indices, respectively, and the trace operation is taken over for the both indices. The quark propagator $S(x, y; m)$ is approximated as a sum of contributions from the free and the zero-mode propagators $S(x, y; m) \approx S_0(x, y) + S^{\rm ZM}(x, y; m)$ [4]. In our results, we only consider the zero-mode part of the propagator. Accordingly, the quark condensate is also considered only in terms of its zero-mode part. This is done to exclude the contribution of the free part, which diverges at the same point.

3.3 Estimate of the curvature

We are interested in the curvature of the free energy density at the origin with respect to the quark condensate. Since the free energy and the quark condensate can be obtained as a function of the instanton density, combining these results, we have the free energy as a function of the quark condensate. Consequently, we can estimate the curvature at the origin of the free energy from these

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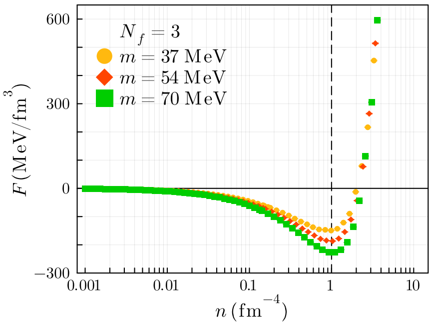

Figure 1: Free energy as a function of the instanton density. This figure is based on the work [2].

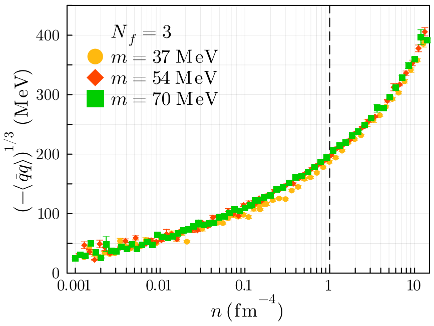

Figure 2: Quark condensate as a function of the instanton density. This figure is based on the work [2].

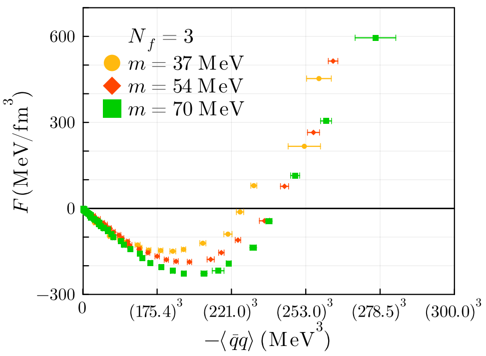

Figure 3: Free energy versus quark condensate. This figure is based on the work [2].

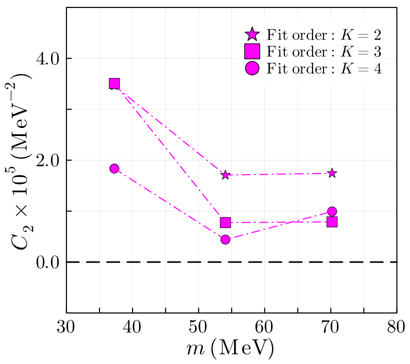

Figure 4: Coefficient C_2 for different current quark masses. This figure is based on the work [2].

data. To obtain the curvature, we fit the data to a polynomial in the quark condensate of degree K: $F(x) = \sum_{j=0}^{K} C_j x^j$, where we write the quark condensate as x, and C_j represents the coefficients resulting from the fitting. After fitting the data, we have the curvature at the origin of the free energy as the coefficient C_2. In the actual fitting, we performed fits for cases where the polynomial degree K are 2, 3 and 4, and evaluated the systematic errors of the coefficients C_j.

4. Numerical results

We simulate the partition function with fixed number of instantons, $N = N_+ + N_-$, using standard Monte-Carlo techniques. By varying the four-volume of the simulation while keeping N fixed, we control the instanton density. We perform two types of simulations. The first is a full simulation, which includes both inter-instanton and instanton-quark interactions. The second is a quenched simulation, where the partition function omits the quark determinant, thereby excluding the instanton-quark interaction from the system. For each simulation type, we perform the simulations with different quark masses m_f. In the full calculations, we use the quark masses of $m_f = 37.0, 54.0$ and 70.0 MeV in the flavor SU(3) limit. In contrast, for the quenched calculations, we use smaller quark masses of $m_f = 2.8$, 14 and 28 MeV.

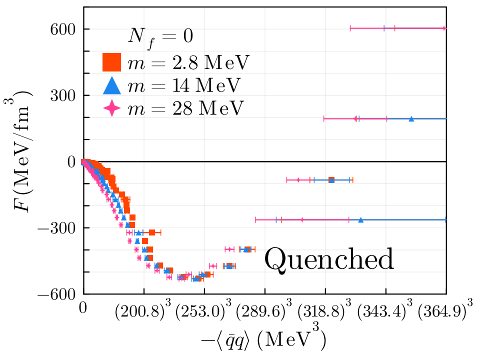

Figure 5: Free energy versus quark condensate. This figure is based on the work [2].

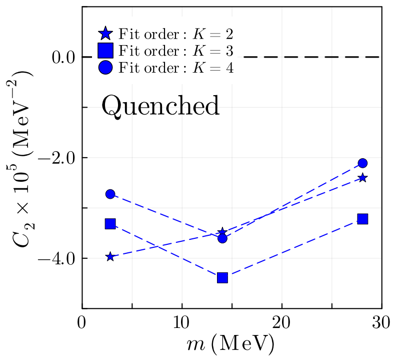

Figure 6: Coefficient C_2 for different current quark masses. This figure is based on the work [2].

4.1 Full calculation

Figure 1 shows the free energy as a function of the instanton density in the full calculation for three different quark masses. At lower instanton density, instantons feel attractive interaction and so the free energy decreases with increasing the density. While for higher density, the free energy rapidly grows. That shows repulsive interaction between instantons. Figure 2 shows the quark condensate in magnitude as a function of the instanton density in the full calculations. The magnitude of the quark condensate increases with growing the instanton density, which reproduces well the previous work [3]. Figure 3 shows the free energy versus the quark condensate in the full calculation. We find that chiral symmetry broken at the vacuum of the system where minimizing the free energy. Figure 4 shows the curvature versus the current quark masses in the full calculations. The curvature is evaluated as in the manner that we explained in Sect. 3.3. One can see that the value of C_2 is positive for the range we considered and for degree k of polynomial in our full calculation of the IILM. This suggests that the IILM shows the anomaly-driven breaking of chiral symmetry.

4.2 Quenched calculation

We perform the same analysis for the quenched calculations. Figure 5 shows the quenched calculation results for the free energy versus the quark condensate. From this, we again observe that chiral symmetry is broken at the vacuum of the system. We can see that the decrease in the free energy as a function of the quark condensate is more gradual compared to the case of the full calculation results. Figure 6 shows the curvature C_2 versus the current quark mass in the quenched calculations. We find the negative value of the curvature for a wide range of quark mass, and then we conclude that the quenched IILM shows the ordinary symmetry breaking pattern.

5. Discussion

The realization of the anomaly-driven breaking in the full IILM can be understood as follows. We have defined the anomaly-driven breaking of chiral symmetry as a situation where chiral symmetry is broken at the vacuum and the curvature of the vacuum energy density is positive at the origin. As investigated in previous work [1], such a situation occurs in the three flavor NJL model when the attractive interaction between quarks is not strong enough, but the KMT term has sufficient strength. The KMT term, which induces the anomaly effect, includes a six-quark interaction, and this can be understood as part of the 't Hooft vertex for three flavor [5]. This 't Hooft vertex is incorporated into the quark determinant in the IILM. Therefore, the full calculation that takes the quark determinant into account should include the 't Hooft vertex. Consequently, it can be naturally interpreted that the IILM exhibits anomaly-driven chiral symmetry breaking.

The results of the quenched calculations can be interpreted as supporting this interpretation. In the quenched calculations, the quark determinant is set to 1, which suppresses the effects of the instanton-quark interaction. As a result, the interaction corresponding to the six-quark KMT term is no longer included, and thus, anomaly-driven chiral symmetry breaking does not occur. This can be interpreted as manifesting in the negative curvature $C_2 < 0$ for the quenched calculation.

6. Outlook

We are interested in two directions for the further development of our work. The first is the investigation of chiral symmetry breaking patterns in intermediate flavor regimes, such as $N_f = 2$. Six-quark interactions like the KMT term appear when considering three quark flavors, while they do not manifest in cases with $N_f = 0, 1$, or 2. So, we are interested in understanding what chiral symmetry breaking patterns realize in such intermediate flavor scenarios. The second is to examine whether the consequence of the anomaly-driven breaking can also be reproduced in the IILM. The previous work has shown that the sigma meson as the chiral partner of the pion becomes lighter than about 800 MeV. This is an important exploration for connecting the discussion based on the sign of the curvature of the free energy with actual physical quantities.

7. Acknowledgements

This work by Y.S. is supported by JST SPRING, Grant No. JPMJSP2106. The work of D.J. was supported in part by Grants-in-Aid for Sicentific Reserach from JSPS (Grants No. JP21K03530, No. JP22H04917, and No. JP23K03427).

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