Comparison of molecular and compact states for the $T_{cc}(3875)$ and $X(3872)$

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The dilemma between molecular states and compact quark states is the subject of a continuous debate in hadron physics. We explore this important question from a novel and different perspective. In this talk, firstly we present our recent work on $T_{cc}(3875)$ state [Phys Lett B 846 (2023) 138200], in which the general formalism is developed in the single-channel calculation, we derive the molecular probability, the scattering length and effective range. We find that the binding energy by itself cannot give a proof of the nature of the state, which clarify the previous conclusion on this important issue. Secondly, we make an extension to the coupled-channel calculation for $X(3872)$ state [Phys Rev D 108 (2023) 114017]. The calculated results are discussed in different scenarios. Compared with the available experimental information, we find that the unavoidable molecular nature of these two states are dominated.

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1. Introduction

The dilemma between molecular states and compact (genuine) quark states is the subject of a continuous debate in hadron physics. Here we take $T_{cc}(3875)$ as an example. Its quark content is composed of $T_{cc}^+(3875)$ ($cc\bar{u}\bar{d}$), the mass $M_{T_{cc}} = M_{D^{*+}D^0} + \delta m_{\exp}$ and width $\Gamma = 48 \pm 2^{+0}_{-14}$ keV, with $M_{D^{*+}D^0} = 3875.09$ MeV, $\delta m_{\exp} = -360 \pm 40^{+4}_{-0}$ keV [1, 2]. What is the nature of $T_{cc}(3875)$ state? Is it a molecular state or a compact quark state or a mixture of both? We can see the discussion in Ref. [3] for detail about the debate for the nature of $T_{cc}(3875)$ state. Similary for $X(3872)$ state, which is another important and interesting state, the detail can be found in Ref. [4] .

In this talk, we explore this important question from a novel and different perspective. We develop the general formalism for evaluating the molecular probability, the scattering length and effective range, both in a single-channel case and also in the coupled-channel case. We focus on the calculations for the two interesting and important states of $T_{cc}(3875)$ and $X(3872)$ from PDG [5].

2. $T_{cc}(3875)$ state in the single-channel case

Now we develop a general Formalism in the single-channel case. We assume a hadronic state of bare mass m_R, which is not generated by the interaction of meson-meson components, for instance a compact quark state. For simplicity, we consider an $I = 0$ state in the single-channel case ($D^{*+}D^0$). The consequences are general and would apply to the lowest threshold of the $D^{*+}D^0$ component. We can write for the DD^* amplitude the diagram of Figure 1 and the DD^* amplitude of Eq. (1).
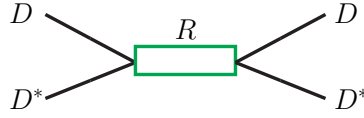

Figure 1: DD^* amplitude based on the genuine resonance R.

$$\tilde{t}_{DD^*,DD^*}(s) = \frac{\tilde{g}^2}{s - s_R} \tag{1}$$

This amplitude is not unitary. It is rendered unitary immediately by iterating the diagram of
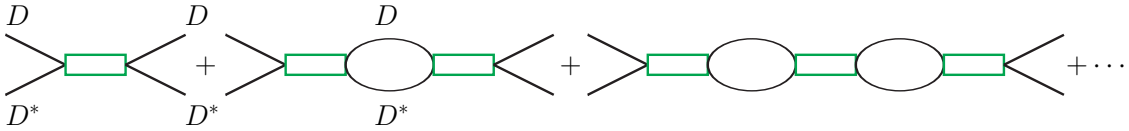

Figure 2: Iterated diagram of Figure 1 implementing unitarity of the DD^* amplitude.

Figure 1 as shown in Figure 2. What we are doing with the diagram of Figure 2 is to insert the DD^* selfenergy in the propagator of Eq. (1). We have then

$$t_{DD^*,DD^*}(s) = \frac{\tilde{g}^2}{s - s_R - \tilde{g}^2 G_{DD^*}(s)} \tag{2}$$

where $G_{DD^*}(s)$ is the DD^* selfenergy which we choose to regularize with a sharp cutoff.

$$G_{DD^*}(s) = \int_{|q|<q_{max}} \frac{d^3q}{(2\pi)^3} \frac{\omega_1 + \omega_2}{2\,\omega_1\,\omega_2} \frac{1}{s - (\omega_1 + \omega_2)^2 + i\epsilon} \tag{3}$$

where $\omega_i = \sqrt{q^2 + m_i^2}$. The selfenergy is negative, we take $s_R = m_R^2$ above the DD^* threshold. The condition that a pole appears at s_0 (the square of the mass of the physical state) below the threshold

$$s_0 - s_R - \tilde{g}^2 G_{DD^*}(s_0) = 0 \tag{4}$$

thus the value of $\tilde{g}^2$ can be obtained.

By using L'Hôpital rule, we obtain the molecular probability

$$P = -\frac{\tilde{g}^2 \frac{\partial G}{\partial s}}{1 - \tilde{g}^2 \frac{\partial G}{\partial s}}\Big|_{s=s_0} \tag{5}$$

We have three limits:

$$\begin{cases} 1)\ \tilde{g}^2 \to 0\,, P \to 0\,, \text{the genuine state survives} \\ 2)\ \tilde{g}^2 \to \infty\,, P \to 1\,, \text{the state becomes pure molecular} \\ 3)\ s_0 \to s_{th}\,, P \to 1\,, \text{the state becomes pure molecular} \end{cases} \tag{6}$$

The third case 3) is interesting, it is a consequence of unitarity and analyticity of the t and G functions. When the binding energy goes to zero, the state becomes fully molecular, the compact component has been fagocitated by the molecular component.

In Figure. 3 we show the results for the molecular probability P of Eq. (5) for different scales of $\Delta\sqrt{s_R}$, here $\sqrt{s_R} = \sqrt{s_{th}} + \Delta\sqrt{s_R}$. It is noticed that when $\sqrt{s_0} \to \sqrt{s_{th}}, P \to 1$. However, at $s_0^{exp} = \sqrt{s_{th}} - 0.36$ MeV, we notice different P for different $\Delta\sqrt{s_R}$, it means that the "scale" shows up clearly. Therefore, we come to the conclusion that the binding energy by itself cannot give a proof of the nature of the state.
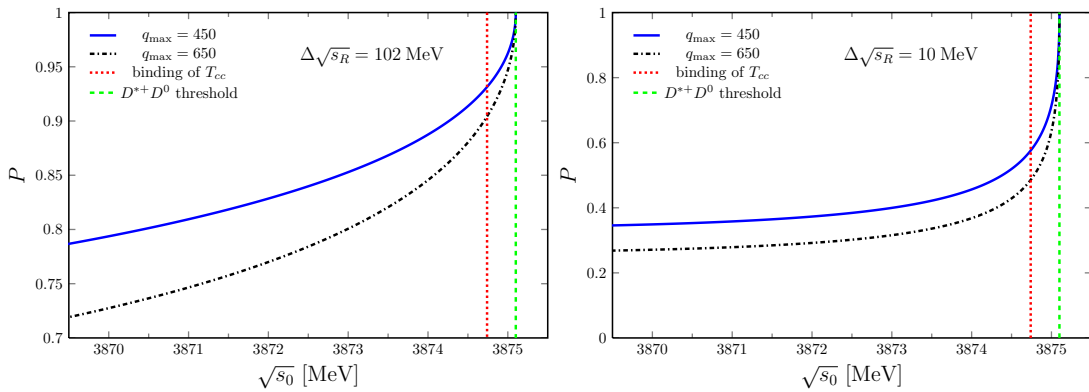

Figure 3: Molecular probability based on the genuine resonance as a function of $\sqrt{s_0}$ (the assumed value of the square of the energy of the bound state).

So what other magnitudes can really tell us about the nature of the state? We further derive the formalism for scattering length and effective range.

The unitarity of the t_{DD^*,DD^*} amplitude implies

$$\operatorname{Im} t^{-1} = \operatorname{Im}\left(\frac{s - s_R}{\tilde{g}^2} - G_{DD^*}(s)\right) = -\operatorname{Im} G_{DD^*}(s) = \frac{k}{8\pi\sqrt{s}} \tag{7}$$

with k the meson-meson on shell momentum.

The relationship with the Quantum Mechanics $f^{\rm QM}$ is

$$t = -8\pi\sqrt{s}\, f^{\rm QM} \simeq -8\pi\sqrt{s}\, \frac{1}{-\frac{1}{a} + \frac{1}{2} r_0\, k^2 - ik} \tag{8}$$

It is easy to induce

$$-\frac{1}{a} = \frac{s_{\rm th} - s_R}{\tilde{g}^2} - \operatorname{Re} G_{DD^*}(s_{\rm th}) \tag{9}$$

$$r_0 = 2\frac{\sqrt{s}}{\mu}\frac{\partial}{\partial s}\left\{(-8\pi\sqrt{s})\left(\frac{s - s_R}{\tilde{g}^2} - \operatorname{Re} G_{DD^*}(s)\right)\right\}\Big|_{s=s_{\rm th}} \tag{10}$$

It can be seen in Table 1 that as $\Delta\sqrt{s_R}$ becomes smaller (decreasing the P), a becomes smaller and

Table 1: The obtained scattering length and effective range for $q_{\rm max} = 450$ MeV.

$\Delta\sqrt{s_R}$ [MeV]	a [fm]	r_0 [fm]
0.1	0.87	-114.07
0.3	1.19	-79.33
1	2.10	-38.20
5	4.62	-9.26
10	5.74	-4.51
50	7.25	-0.47
70	7.39	-0.17
102	7.51	0.06

smaller and r_0 grows indefinitely. The lesson we draw from here is that the scattering length and effective range are very useful to determine the molecular probability of the state.

3. Extension to $X(3872)$ state in the coupled-channel case

Same as above in the single-channel case for $T_{\rm cc}(3875)$, we start with a bare mass m_R in the coupled-channel case for $X(3872)$, in which $D^{*0}\bar{D}^0$ and $D^{*+}D^-$ are considered. The scattering amplitude is

$$t_{D^*\bar{D}}(I=0) = \frac{\tilde{g}^2}{s - s_R} \tag{11}$$

By using L'Hopital's rule, we can derive the couplings

$$g_1^2 = \frac{\frac{1}{2}\tilde{g}^2}{1 - \frac{1}{2}\tilde{g}^2\frac{\partial}{\partial s}(G_1 + G_2)}\Bigg|_{s_0} ; \qquad g_2 = g_1 \tag{12}$$

and probabilities

$$P_1 = -g_1^2 \frac{\partial G_1}{\partial s}\Big|_{s_0} = -\frac{\frac{1}{2}\tilde{g}^2 \frac{\partial G_1}{\partial s}}{1 - \frac{1}{2}\tilde{g}^2 \frac{\partial}{\partial s}(G_1 + G_2)}\Big|_{s_0},$$

$$P_2 = -g_2^2 \frac{\partial G_2}{\partial s}\Big|_{s_0} = -\frac{\frac{1}{2}\tilde{g}^2 \frac{\partial G_2}{\partial s}}{1 - \frac{1}{2}\tilde{g}^2 \frac{\partial}{\partial s}(G_1 + G_2)}\Big|_{s_0}. \tag{13}$$

Similarly as Eq. (6), we also have three limits in the coupled-channel case. However, we are only interested in the case where $\sqrt{s_0}$ is closer to the first threshold of $D^{*0}\bar{D}^0$, and we obtain $P_1 \to 1, P_2 \to 0$ which means we have a completely molecular state dominated by the $D^{*0}\bar{D}^0$ component.

In this case, we further include the direct interaction V from the exchange of vector mesons in the local hidden gauge approach, thus Eq. (11) becomes

$$\frac{\tilde{g}^2}{s - s_R} \to \frac{\tilde{g}^2}{s - s_R} + \beta V, \tag{14}$$

where $\beta = 0$ for the compact scenario, and $\beta \neq 0$ for the hybrid scenario (including the direct interaction).
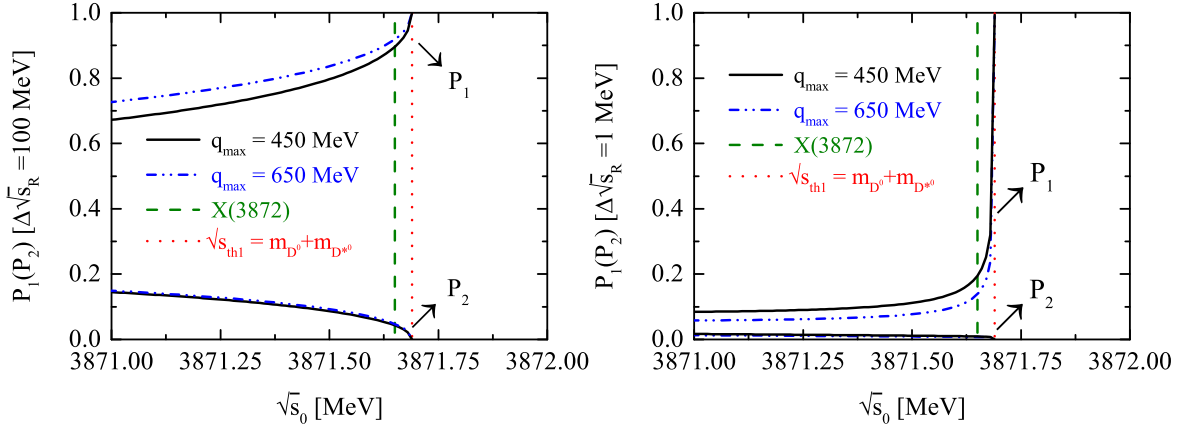

Figure 4: The same as Figure. 3 but in the coupled-channel case for $\beta = 0$.

The same as for the $T_{cc}(3875)$ state, we perform similar calculations for P_1 and P_2 for different scales $\Delta\sqrt{s}_R$ for $\beta = 0$, which are shown in Figure. 4. We notice that the "scale" shows up clearly again. It is clear that the binding energy by itself does not give us the molecular probability. In Figure. 5, we can see that the presence of a reasonable direct meson-meson interaction would increase drastically the molecular probability of the state.

In Table 2, we show the obtained scattering length and effective range for the $\beta \neq 0$ scenario. It is noticed that $\Delta\sqrt{s}_R = 1$ MeV can be acceptable with the current uncertainty of LHCb data [6], and $P_1 + P_2 \sim 0.95$ in this scenario, which is interesting and should not be discarded.

4. Conclusions

We develop the general formalisms in single-channel and coupled-channel calculations. As an application, we investigate the $T_{cc}(3875)$ and $X(3872)$ state. We find that the binding energy itself
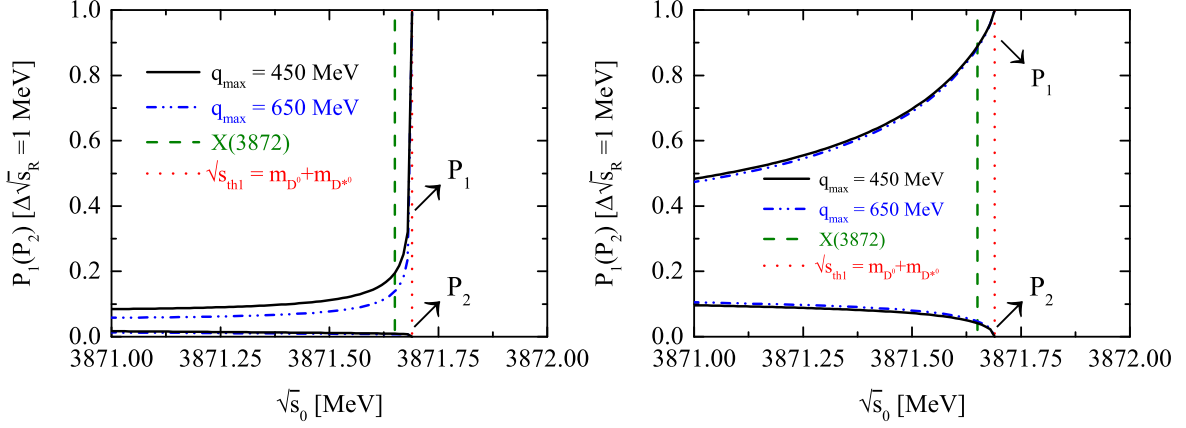
Figure 5: The same as Figure. 4 but for $\beta = 0$ (left) and $\beta \neq 0$ (right) .

Table 2: The same as Table 1 but in the coupled-channel case for $\beta \neq 0$ scenario.

$\Delta\sqrt{s_R}$	a_1[fm]	$r_{0,1}$[fm]	a_2[fm]	$r_{0,2}$[fm]
0.1	15.60	-24.97	$0.7068 - i\,1.116$	$1.17 - i1.56$
0.3	19.65	-7.13	$0.7060 - i1.118$	$1.16 - i1.56$
1	21.38	-2.30	$0.7024 - i1.125$	$1.14 - i1.56$
10	22.13	-0.63	$0.7818 - i0.780$	$-3.62 - i1.56$
100	22.21	-0.47	$0.7385 - i1.038$	$1.15 - i1.56$

does not determine the nature of a state, but the additional information of the scattering length and effective range can provide an answer. Taking $X(3872)$ as an example, the present data definitely rule out the possibility of a dominant nonmolecular component, and we obtain $P_1 + P_2 \sim 0.95$ which is dominantly molecular component.

Acknowledgments

References

[1] R. Aaij et al., (LHCb Collaboration), Nature Phys. **18** (2022) 751

[2] R. Aaij et al. (LHCb Collaboration), Nature Commun. **13** (2022) 3351

[3] L. R. Dai, J. Song, E. Oset, Phys. Lett. B **846** (2023) 138200

[4] J. Song, L. R. Dai, E. Oset, Phys. Rev. D **108** (2023) 114017

[5] S. Navas et al. (Particle Data Group), Phys. Rev. D **110**, 030001 (2024)

[6] R. Aaij et al. (LHCb Collaboration), Phys. Rev. D **102** (2020) 092005