PROCEEDINGS OF SCIENCE

Barrelet Zeros Extraction in Pion-Nucleon Scattering

Marta Campos Fornés and Vincent Mathieu

Departament de Física Quàntica i Astrofísica and Institut de Ciències del Cosmos, Universitat de Barcelona, E08028, Spain

E-mail: martacampos1999@gmail.com, vmathieu@ub.edu

A Monte-Carlo simulator has been designed to generate events of a proton-pion scattering employing a realistic model based on the unified Chew-Mandelstam SAID parametrization [PRC 86 1 (2012), 015202]. Using the generated data, a partial-wave analysis of the final state of the system is performed. The energy-dependent partial-wave amplitudes are derived analytically using the Barrelet Zeros of the moments. This study also discusses the existence of ambiguous solutions, phase uncertainty and the maximum angular momentum value considered in the data analysis.

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1. General outline of the analysis program

For each simulated proton-pion scattering event, two random values are generated, corresponding to the total energy of the system in the center-of-mass frame E_{cm} and $z = \cos\theta$, where θ is the scattering angle. Given that the simulation is performed in the center-of-mass frame, the two independent variables $s = E_{\text{cm}}^2$ and z are enough to fully describe a single event. The acceptance criterion for the Monte Carlo simulation is defined using the realistic SAID model [1].

The first step towards the data analysis of the system is the division of the full energy range obtained during the simulation into n evenly spaced intervals. Following this, the next step consists on the calculation of the moments of the system for each energy interval, which are defined as the coefficients necessary to express the differential cross-section of the scattering as a Fourier-Legendre series:

$$\frac{\mathrm{d}\sigma(s,z)}{\mathrm{d}\Omega} = \sum_{L=0}^{2\ell_{\max}} (2L+1) H_L(s) P_L(z) \tag{1}$$

Where $P_L(z)$ are the angle-dependent Legendre polynomials. From definition (1), the inverse relation can be extracted, which will allows one to calculate the moments:

$$H_L(s) = \frac{1}{2}\int_{-1}^{1} \frac{\mathrm{d}\sigma(s,z)}{\mathrm{d}z} P_L(z)\mathrm{d}z \quad \forall\, L \in \{0,...,2\ell_{\max}\} \tag{2}$$

The partial-wave analysis performed in this program is based on the extraction of roots for the polynomial $\sum (2\ell+1)\, a_\ell(s) P_\ell(z) = 0$, where z is the considered variable and s is fixed for each specific energy interval. These roots are known as the Barrelet Zeros [2] and are considered extremely valuable, as they allow one to establish a direct and simple analytic relation to the energy-dependent partial-wave amplitudes:

$$A(s,z) = \sum_{\ell=0}^{\ell_{\max}} (2\ell+1)\, a_\ell(s) P_\ell(z) = N(s) \prod_{i=1}^{\ell_{\max}} (z - z_i(s)) \tag{3}$$

Where $z_i(s)$ are the Barrelet Zeros and $N(s) \propto \left|a_{\ell_{\max}}(s)\right|$. Equivalently to the moments, the Barrelet Zeros are computed for each energy interval individually, which results in the extraction of a different set of partial-wave amplitude solutions for each energy value. The relation between the system's scattering amplitude and differential cross-section is key for the Barrelet Zeros calculation:

$$\frac{\mathrm{d}\sigma(s,z)}{\mathrm{d}\Omega} = \sum_{L=0}^{2\ell_{\max}} (2L+1)\, H_L(s) P_L(z) = \frac{1}{16\pi^2 s} |A(s,z)|^2 = \frac{|N(s)|^2}{16\pi^2 s} \prod_{i=1}^{\ell_{\max}} [z - z_i(s)] \cdot \left[z - z_i^*(s)\right] \tag{4}$$

Having obtained the values of the moments of the system $H_L(s)$ from equation (2), imposing the equality $\sum (2L+1)\, H_L(s) P_L(z) = 0$ allows one to extract the $\ell_{\max}$ pairs of complex conjugate roots $\{z_i(s_k), z_i^*(s_k)\}$ (with $i \in \{1,\ldots,\ell_{\max}\}$) for each energy bin k. This family of roots contains both the Barrelet Zeros and their complex conjugates, which can not distinguished just from the root extraction. Consequently, an ambiguity in the Barrelet Zeros is generated, which is further transmitted to the partial-wave amplitudes. These ambiguities will be further discussed in section 2.

Finally, the energy-dependent partial-wave amplitudes are computed using equation (3) and the extracted Barrelet Zeros. This concludes the general outline of the analysis program. In the next sections, some specifics of the program's operations will be further discussed.

2. Solution ambguities

During the described process, certain challenges appear in the data analysis: two different types of ambiguities need to be considered when displaying and discussing the extracted results.

The first type of ambiguity is related to the global phase of the system's scattering amplitude $A(s,z)$. Given that the only information obtained during the Monte Carlo simulation is $\frac{\mathrm{d}\sigma(s,z)}{\mathrm{d}\Omega}$ and $\frac{\mathrm{d}\sigma(s,z)}{\mathrm{d}\Omega} \propto |A(s,z)|^2$, the general phase of $A(s,z)$ will be unknown. To represent the results for the different partial-wave amplitudes, a general phase will have to be chosen. Since the focus is placed in the relative phase between different partial-wave amplitudes, the global phase election can be subject to the most convenient value for the resolution of the presented equations. Accordingly, a value for $L \in \{0,\ldots,\ell_{\max}\}$ can be chosen such that $a_L \in \mathbb{R}^+$. Initially, $a_{\ell_{\max}} \in \mathbb{R}$ was selected to facilitate calculations, leading to $a_{\ell_{\max}} \propto \sqrt{16\pi^2 s \cdot H_{2\ell_{\max}}}$. However, the general consensus is to choose $a_0 \in \mathbb{R}^+$. Therefore, before representing the obtained results for $a_L(s)$, $\forall L, s$, a general phase shift is applied such that $a_0 \in \mathbb{R}^+$.

The second type of ambiguity arises from the Barrelet Zeros extraction. As previously mentioned, a set of $\ell_{\max}$ pairs of complex conjugated roots $\{z_i, z_i^*\}$ is obtained from the polynomial $\sum (2L+1) H_L(s) P_L(z) = 0$. For each pair, it is impossible to distinguish which value corresponds to the original Barrelet Zero and which is its conjugate. All the different possible assignations generate an ambiguity in the computation of $a_L, \forall L \in \{0,\ldots,\ell_{\max}\}$. Specifically, the number of solutions obtained is $2^{\ell_{\max}}$. However, since half of these solutions are the complex conjugates of the other half, only $2^{\ell_{\max}-1}$ are considered as distinct solutions.

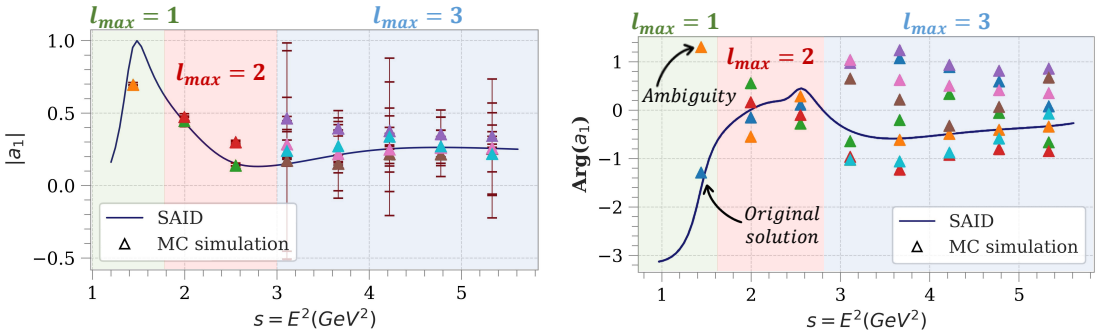
3. Variation of $\ell_{\max}$ in the data analysis

Theoretically, the sums presented in expansions (1) and (3) should go from 0 to ∞. However, when analyzing data in a realistic case, a maximum orbital angular momentum ($\ell_{\max}$) must be chosen. Components of higher orbital angular momentum than the fixed maximum are considered negligible. In an experimental context, the adequate $\ell_{\max}$ that needs to be chosen for the data analysis is unknown. Therefore, a previous study of the system must be performed and $\ell_{\max}$ must be determined from the highest non-zero moment $H_L(s)$ in each specific energy.

Throughout the development of this program, a parallelism with the experimental context was detected. It was observed that adapting $\ell_{\max}$ for the data analysis in each energy bin is beneficial for higher accuracy results. Thus, considering the adequate value for $\ell_{\max}$ in $A(s,z) = \sum_{\ell=0}^{\ell_{\max}} (2\ell+1)\, a_\ell(s) P_\ell(z)$ and $\frac{\mathrm{d}\sigma(s,z)}{\mathrm{d}z} = \sum_{L=0}^{2\ell_{\max}} (2L+1)\, H_L(s) P_L(z)$ for each energy value can positively impact the coherency of the obtained solutions.

This may sound confusing, as the real $\ell_{\max}$ is fixed for all energy bins by the number of partial-wave amplitudes used during the event simulation. However, the highest order partial-wave amplitudes are very close to 0 for small energies, which causes a problem in the Barrelet Zeros extraction. Since all solutions for the energy-dependent partial-wave amplitudes $a_\ell(s)$ are calculated using the extracted Barrelet Zeros, this possible root alteration poses a big issue.

Failing to follow this approach in the data analysis can also lead to significantly large error bars for $a_\ell(s)$ and introduce unnecessary ambiguities among the potential solutions. The divergence of the error bars stems from the same analytic limitation discussed in the previous paragraph. Regarding ambiguities, since the number of solutions obtained for each $a_\ell(s)$ is $2^{\ell_{\max}}$ for each energy value, using a lower maximum orbital momentum in the data analysis will result in fewer ambiguities among the extracted results.



(a) Absolute value of a_1 as a function of s.

(b) Phase of a_1 as a function of s.

Figure 1: These results are presented choosing a global phase such that $a_0 \in \mathbb{R}^+$. Error bars are not included in (b) for visibility purposes. The ambiguity in the extraction of the Barrelet Zeros (z_i, z_i^*) is responsible for the appearance of $2^{\ell_{\max}-1}$ different solutions for each value of $|a_1(s)|$ and $2^{\ell_{\max}}$ for each value of $Arg(a_1(s))$. Size of the simulated set of events used to compute a_1: 10^8.

As it can be observed in Figure 1, the adaptation of $\ell_{\max}$ in terms of the energy s does not have a significant impact on the solutions' deviation from their theoretical value. The choice of the appropriate $\ell_{\max}$ for each energy bin was performed in terms of the value of $H_{2\ell_{\max}}(s)$, for which a threshold relative to its maximum value was established.

4. Conclusions

Both the correct choice of ℓ_{max} for each energy interval in the data analysis and a large enough set of simulated events are essential factors to obtain coherent $a_\ell(s)$ results. A deep understanding of the different types of ambiguities that appear throughout the resolution process is also key for the development of a successful partial-wave analysis of the system.

5. Acknowledgments

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References

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