PROCEEDINGS OF SCIENCE

Hybrid baryons in a constituent model

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In addition to conventional hadrons, such as baryons and mesons, quantum chromodynamics predict the existence of other hadronic states based on the principle of colour confinement. Among these, hybrid states are particularly intriguing. They arise from excitation in the gluonic field or, in a constituent approach, from the inclusion of a constituent gluon within the system. In recent years, both theoretical and experimental efforts have been dedicated to the study of hybrid mesons. Their identification seems, at first sight, easier since some P^{PC} quantum numbers are forbidden in a $q\bar{q}$ configuration but allowed for $q\bar{q}g$. On the other hand, hybrid baryons do not have such "smoking gun" signature since all quantum numbers J^P can be populated by conventional qqq configuration. On the theory side, hybrid baryons have been studied within the framework of the MIT bag model, flux tube model, QCD sum rules, large-N QCD and lattice QCD. Although these models predict the existence of hybrid baryons, their predictions for the masses and structures differ considerably from each other. On the experimental side, significant efforts are underway at the Jefferson Laboratory to identify these particles. In this presentation, we propose a constituent model for describing hybrid baryons with heavy quarks. First, the flavour-spin-colour wavefunctions of the core of quarks are computed based on the Pauli exclusion principle. Then, the spin of the core of quarks is coupled to the helicity of the gluon by using the two-body helicity formalism of Jacob and Wick, leading to a series of helicity states with fixed J^P quantum numbers. Eventually, the spectrum of the system is computed by the help of the method of the envelope theory, which was already used in the past for studying conventional baryons and hybrid mesons, with conclusive results.

10th International Conference on Quarks and Nuclear Physics (QNP2024)
8-12 July, 2024
Barcelona, Spain

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PoS(QNP2024)024

1. Introduction

Experimental candidates for exotic hadrons are increasingly viable. Beyond mesons, baryons, and multiquark states, QCD predicts the existence of hybrids, where gluonic field excitations contribute as valence components. Hybrid baryons, comprising three quarks and a gluon, are theoretically plausible and have been studied using models like the bag model, large-N approaches, flux-tube frameworks, lattice QCD, and QCD sum rules. However, these models diverge significantly in their predictions for masses and structures.

Although there is no experimental evidence for hybrid baryons yet, searches are ongoing at facilities such as Jefferson Lab's CEBAF Large Acceptance Spectrometer (CLAS). Identifying these states is challenging since their quantum numbers overlap with those of ordinary baryons, leading to potential mixings with excited three-quark states. However, hybrid baryons can be distinguished by their unique Q^2-dependent couplings and decay products, which reflect their additional gluonic component.

Given these challenges and the diversity of theoretical predictions, a robust model for hybrid baryons is essential. This study develops a semi-relativistic potential approach, previously successful for ordinary hadrons [1], and adapts it for hybrids.

Focusing on heavy hybrid baryons ($cccg$ and $bbbg$), this work assumes a two-body structure: a colour-octet quark core interacting with a gluon through a QCD-inspired potential. This choice simplifies the modeling because of the large mass asymmetry between the core and the gluon. Key aspects of the model, including the gluon's helicity, are detailed in Sec. 2. The masses, sizes, and quantum numbers of the quark core are computed in Sect. 3, followed by the description of the core-gluon interactions in Sec. 4. The results for the lowest $cccg$ and $bbbg$ hybrid baryons are presented in Sect. 5, with conclusions and outlook in Sec. 6.

More details for the computations and results can be found in the original paper [2].

2. Potential models with helicity

The gluon, as a massless particle, is characterised by its helicity, with only two projections (± 1) of intrinsic angular momentum, rather than a spin with three projections. Previous studies [3, 4] have shown that semi-relativistic potential models for two-gluon glueballs are feasible when helicity is properly accounted for. Extending this approach to hybrid baryons imposes similar constraints on modelling.

While the helicity formalism is well-established for two-body systems [5], it becomes significantly more complex for three or more particles [6]. To simplify the treatment, a hybrid baryon is approximated as a two-body system: a point-like gluon interacting with a colour-octet quark core in its ground state. The core is assumed to be inert, with no excitations allowed—a simplification discussed further in Sec. 6.

This model resembles the quark-diquark framework, widely used for baryons, where two quarks form a cluster that interacts with the third quark [7, 8]. Such structures are particularly favoured in systems with heavy quarks, as their large masses lead to tightly bound cores [7, 8]. By focusing on heavy hybrid baryons with three identical quarks ($cccg$ and $bbbg$), this work maximises the likelihood of forming a stable, tightly bound core minimally affected by gluon dynamics.

3. Quark core description

This work focuses on the mass differences between heavy hybrid baryons and the ground states of ordinary baryons with the same quark content ($qqq = ccc$ or bbb). Accurate solutions for the semi-relativistic three-body problem are obtained using an oscillator basis expansion [9–11], with natural units ($\hbar = c = 1$).

3.1 Ordinary baryons

For baryons composed of three identical quarks, the flavour and spatial wave functions are completely symmetric. The colourless state and the antisymmetry of the total wave function implies a fully symmetric spin state ($S_B = 3/2$). Thus, the reference baryons are characterised by $J_B^{P_B} = 3/2^+$.

The Hamiltonian for baryons is derived from the Cornell potential for mesons [12]

$$V_{q\bar{q}} = A\,r - \frac{\kappa}{r}, \tag{1}$$

where A is the string tension and κ relates to the strong coupling constant. For the three-body case, the Hamiltonian becomes

$$H_B = \sum_{i=1}^{3} \sqrt{\boldsymbol{p}_i^2 + m_q^2} + \frac{1}{2}\sum_{i<j}^{3} \left(f A r_{ij} - \frac{\kappa}{r_{ij}} \right), \tag{2}$$

where $f = 1.086$ approximates the flux-tube Y-junction [13], and the potential components are derived from quark-quark colour factors. Ground-state baryon masses for ccc and bbb systems are listed in Table 1.

3.2 Octet Quark Core

The colour-octet quark core (qqq) in its ground state has a symmetric flavour-space wave function but a mixed symmetry in spin ($S_C = 1/2$). The total spin-colour state is antisymmetric

$$\frac{1}{\sqrt{2}} \left(\chi^{\mathrm{MS}} \phi^{\mathrm{MA}} - \chi^{\mathrm{MA}} \phi^{\mathrm{MS}} \right), \tag{3}$$

resulting in $J_C^{P_C} = 1/2^+$.

In the flux-tube model, the gluon connects to the quark core through a colour-neutralising octet flux-tube. The quark core Hamiltonian is

$$H_C = \sum_{i=1}^{3} \sqrt{\boldsymbol{p}_i^2 + m_q^2} + \frac{1}{2}\sum_{i<j}^{3} \left(f A r_{ij} - \frac{1}{4}\frac{\kappa}{r_{ij}} \right), \tag{4}$$

where the factor $1/4$ accounts for octet colour correlations.

The spatial density of the core is approximated as

$$\rho(\boldsymbol{r}) = \frac{\lambda^3}{\pi^{3/2}} e^{-\lambda^2 r^2}, \tag{5}$$

with λ determined numerically. Table 1 summarises the calculated masses, gaps ($\Delta = m_C - m_B$), and size parameters (λ). The reduced Coulomb attraction increases the octet core mass relative to the baryon ground state, contributing to the hybrid baryon's mass.

State	m_B	m_C	Δ	λ
ccc	4.822	5.119	0.297	0.825
bbb	14.401	14.894	0.493	1.261

Table 1: Masses of baryons (m_B), octet quark cores (m_C), mass gaps (Δ), and size parameters (λ), in GeV.

4. Gluon-gluon and quark core-gluon interactions

The Hamiltonian for two-gluon glueballs is [3]

$$H_{gg} = 2\sqrt{\boldsymbol{p}^2 + m_g^2} + \frac{9}{4}\sigma r - 3\frac{\alpha_S}{r}, \tag{6}$$

where m_g is the gluon mass, σ the string tension, and α_S the strong coupling constant.

The interaction between the gluon and the quark core is assumed identical to V_{gg}, as the internal structure of the core is irrelevant at this level. However, since the core is extended, the point-like interaction must be convoluted with the core's density [8]

$$\tilde{V}(\boldsymbol{R}) = \int \rho(\boldsymbol{r}) V(|\boldsymbol{R} + \boldsymbol{r}|) d\boldsymbol{r}. \tag{7}$$

Using Eq.(5) for $\rho(r)$, the Hamiltonian for hybrid baryons is

$$H_{HB} = \sqrt{\boldsymbol{p}^2 + m_g^2} + \sqrt{\boldsymbol{p}^2 + m_C^2} + \frac{9}{4}\sigma\left[\frac{e^{-\lambda^2 r^2}}{\sqrt{\pi}\lambda} + \left(r + \frac{1}{2\lambda^2 r}\right)\mathrm{erf}(\lambda r)\right] - 3\alpha_S\frac{\mathrm{erf}(\lambda r)}{r}, \tag{8}$$

where m_C and λ are the quark core mass and size parameter, respectively.

5. Masses of heavy hybrid baryons

Eigenvalues and eigenstates of Hamiltonian (8) are computed using the Lagrange-mesh method [2, 15, 16]. We compute the mass gaps $m_{HB} - m_B$ relative to the ground-state baryon masses m_B from Table 1. We expect that results with spin are not physically relevant but include them for comparison with the helicity case, as done in [3] for glueballs.

For a gluon with helicity, the quark core-gluon states of hybrid baryons are constructed as $\left|H_\pm; J^P : \pm J_C 1\right\rangle$, where the gluon's helicity impacts the spin-orbit wave function. These states can be expanded in terms of $\left|^{2S+1}J_J\right\rangle$ states for computation, as shown in [2, 3]. The two lowest J^P states for a spin $J_C = 1/2$ are

$$|1/2^-; 1\rangle = \sqrt{\frac{2}{3}}|^2 0_{1/2}\rangle - \sqrt{\frac{1}{3}}|^4 2_{1/2}\rangle, \tag{9}$$

$$|1/2^+; 1\rangle = \sqrt{\frac{2}{3}}|^2 1_{1/2}\rangle - \sqrt{\frac{1}{3}}|^4 1_{1/2}\rangle. \tag{10}$$

For $3/2^+$, there are two states each with a corresponding matrix $w_{\alpha\beta}$, which gives effective orbital angular momenta l_{eff}. The states with the lowest quantum numbers are shown below.

The key findings from Table 2 are

J^P	n_r	l_{eff}	$cccg$	$bbbg$
$1/2^\pm$	0	1	1.842	1.784
$3/2^\pm$	0	1	1.842	1.784
$3/2^\pm$	0	2	2.350	2.336
$1/2^\pm$	1	1	2.552	2.469
$3/2^\pm$	1	1	2.552	2.469
$3/2^\pm$	1	2	2.938	2.880

Table 2: Mass gap $m_{HB} - m_B$ in GeV for the lowest $J = 1/2$ and $J = 3/2$ $cccg$ and $bbbg$ hybrid baryons for a gluon with helicity.

- The mass hierarchy is similar for $cccg$ and $bbbg$ states.
- States with $l_{\text{eff}} = 1$ are degenerate.
- The lowest states have $J^P = 1/2^\pm$ $3/2^\pm$and , with a mass gap of approximately 1.8 GeV above the ground-state baryon mass.
- No states with $l_{\text{eff}} = 0$ are present, consistent with the results for gg systems [3].

While these results apply to heavy hybrid baryons, the extension to lighter systems, targeted by future experiments, requires further work. In lattice QCD studies, only positive parity spectra are considered, with mass gaps around 1.5 GeV for hybrid-Δ. These results suggest similarities but also highlight the need to extend this model to lighter systems for more reliable comparisons. For heavy hybrids, the quantum numbers align with those of ordinary baryons, implying that mixing between the two configurations is minimal, as suggested in large-N_c approaches.

6. Concluding remarks and outlook

In this work, heavy hybrid baryons are modeled as two-body systems consisting of a quark core and a gluon interacting via a simple QCD-inspired potential, similar to the quark-diquark approximation for baryons. The model includes a flux-tube confinement potential, Casimir scaling for triplet and octet color sources, and a one-gluon exchange interaction. The key feature of the model is the correct treatment of the gluon's helicity. Masses for $cccg$ and $bbbg$ hybrid baryons were computed, with results that align reasonably well with light hybrid baryons from lattice QCD calculations. This serves as a proof of concept, with potential extensions to lighter hybrid baryons, which are more relevant for experimental studies.

However, three improvements are needed µ

- A more comprehensive potential model, incorporating relativized potentials and spin contributions as in [1], could provide better spectra for both ordinary and exotic hadrons. Recent developments, such as the inclusion of screening effects for linear confinement [17], may also be tested using Casimir scaling.

- The compact three-quark cluster formation is unlikely in light hybrid baryons due to strong mass asymmetry. This challenge may be addressed by allowing the quark core to exist in a superposition of states, with the gluon dynamics controlling the mixing.
- While the mixing is expected to be suppressed for heavy quarks, it remains a significant issue for light quarks. A potential model could treat this problem similarly to quark-diquark mixing.

For further insight into hadron phenomenology, combining potential models with the large-N_c limit of QCD [18, 19] offers a promising approach, particularly in the light baryon sector. This method could be used to study light hybrid baryons by tracking their properties from large N_c down to $N_c = 3$. Challenges include the mixed symmetry of the quark core's color wave function, which can be addressed in the Veneziano limit [20]. Additionally, the use of envelope theory [21, 22] could simplify the many-body quantum problem, enabling efficient calculations for baryon spectroscopy.

Acknowledgments

L.C. would like to thank the Fonds de la Recherche Scientifique - FNRS for the financial support. This work was also supported under Grant Number 4.45.10.08. L.C. would thank Cyrille Chevalier for providing accurate masses for the quark-cores.

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PoS(QNP2024)024