PROCEEDINGS OF SCIENCE

Nucleons and vector mesons in holographic QCD

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We review the holographic QCD model for nucleons and vector mesons proposed in [1]. The model can be thought of a consistent embedding of soft wall models in Einstein-dilaton gravity and it leads to hadronic correlators compatible with QCD in the large N_c limit. We compare our results for the hadronic masses and decay constants against previous models and available experimental data.

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1. Confinement in Einstein-dilaton holography

We start with the action of 5d Einstein-dilaton gravity in the Einstein frame:

$$S_E = \sigma \int d^5x \sqrt{-g} \left[R - \frac{4}{3} g^{mn} \partial_m \Phi \partial_n \Phi + \ell^{-2} V(\Phi) \right] , \tag{1}$$

where ℓ is the radius of 5d anti-de Sitter (AdS) space. In holographic QCD we consider the following ansatz for the metric and dilaton

$$ds^2 = \frac{1}{\zeta(z)^2} \left[-dt^2 + d\vec{x}^2 + dz^2 \right] \quad , \quad \Phi = \Phi(z) . \tag{2}$$

The inverse scale factor $\zeta(z)$ is usually expressed in terms of the warp factor $A(z)$ through the relation $A(z) = -\ln \zeta(z)$. Varying the action (1) and using the ansatz in (2) we obtain the following independent Einstein-dilaton field equations

$$\zeta'' - \frac{4}{9} \zeta \Phi'^2 = 0 \quad , \quad \ell^{-2} V - \zeta^5 (\zeta^{-3})'' = 0 , \tag{3}$$

where $' = d/dz$. At small z we impose the condition that the 5d metric becomes AdS and that the dilaton field vanishes, i.e. $\zeta(z \to 0) = z/\ell$ and $\Phi(z \to 0) = 0$. At large z (far from the bundary) we impose the condition $\Phi(z \to \infty) = kz^2$ which guarantees linear confinement [2] and linear Regge trajectories for vector mesons [3]. In this work we consider two analytical solutions that satisfy the conditions above. These are given by

$$\Phi_I(z) = kz^2 \quad , \quad \zeta_I(z) = \Gamma(5/4) \left(\frac{3}{k} \right)^{1/4} \frac{\sqrt{z}}{\ell} I_{1/4} \left(\frac{2}{3} kz^2 \right)$$
$$\Phi_{II}(z) = \frac{1}{2} \sqrt{k} z \sqrt{9 + 4kz^2} + \frac{9}{4} \sinh^{-1} \left(\frac{2}{3} \sqrt{k} z \right) \quad , \quad \zeta_{II}(z) = \frac{z}{\ell} \exp \left(\frac{2}{3} kz^2 \right) . \tag{4}$$

In the next sections we will describe the dynamics of the 5d gauge and Dirac fields dual to the 4d vectorial current and nucleon operators. The 5d actions will be described in the string frame where the warp factor takes the form

$$A_s(z) = A(z) + \frac{2}{3} \Phi(z) = -\ln \zeta(z) + \frac{2}{3} \Phi(z) . \tag{5}$$

2. Vector mesons in confining holographic QCD

We are interested in the physics of vector mesons in large N_c QCD with $N_f = 2$ flavors. This is described by the 4d flavour current operators $J^{\mu,a}(x) = \bar{q}(x) \gamma^\mu T^a q(x)$ and for simplicity we assume $SU(2)$ isospin symmetry ($m_u = m_d$). Inspired by previous works [4, 5] we map these currents to 5d non-Abelian fields $V_m^a(z,x)$ with a 5d action given by

$$S_V = - \int d^4x \, dz \frac{1}{4g_5^2} \sqrt{-g_s} \, e^{-\Phi} {v_{mn}^a}^2 , \tag{6}$$

where $v_{mn}^a = \partial_m V_n^a - \partial_n V_m^a$ [1] and $g_{mn}^s = e^{2A_s(z)} \eta_{\hat{m}\hat{n}}$ is the 5d metric in the string frame. The 5d coupling is fixed as $g_5^2 = 12\pi^2/N_c$ in order to reproduce the perturbative QCD result for

[1]Cubic or higher order on V_m^a are relevant only for interactions and will be neglected in this work.

the current correlator in the regime of small distances. The 5d gauge field is decomposed as $V^a_{\hat{m}} = (V^a_z, V^{\perp}_{\hat{\mu},a} + \partial_{\hat{\mu}}\xi^a)$. We use gauge symmetry to fix $V^a_z = 0$ and from the field equations one finds that $\xi = 0$. The only remaining field equation from (6) is

$$\Big[\partial_z + A_s' - \Phi'\Big]\partial_z V_{\perp}^{\hat{\mu},a} + \Box V_{\perp}^{\hat{\mu},a} = 0\,. \tag{7}$$

At small z the 5d gauge field admits the asymptotic solution

$$V_{\hat{\mu},a}(x,z) = V^{(0)}_{\hat{\mu},a}(x) + \cdots + V^{(2)}_{\hat{\mu},a}(x) z^2 + \dots\,, \tag{8}$$

where $V^{(0)}_{\hat{\mu},c}(x)$ and $V^{(2)}_{\hat{\mu},c}(x)$ are the source and VEV coefficients. It is also convenient to introduce the bulk to boundary propagator $K^{cd}_{\hat{\mu}\hat{\nu}}(z,x;y)$ via the relation

$$V^a_{\hat{\mu}}(z,x) = \int d^4y\, K^{ab}_{\hat{\mu}\hat{\nu}}(z,x;y) V_b^{\hat{\nu},0}(y), \tag{9}$$

As shown in [1], the holographic dictionary for the 4d current correlator takes the form

$$G^{cd}_{\hat{\mu}\hat{\nu}}(x-y) = \langle J_{\hat{\mu},c}(x) J_{\hat{\nu},d}(y)\rangle = \frac{1}{g_5^2}\Big[e^{A_s-\Phi}\partial_z K^{cd}_{\hat{\mu}\hat{\nu}}(z,x;y)\Big]_{z=\epsilon}\,. \tag{10}$$

In momentum space the bulk to boundary propagator can be written as

$$\tilde{K}^{ab}_{\hat{\mu}\hat{\nu}}(z,q) = \left(\eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\delta^{ab}\, V(z,q)\,. \tag{11}$$

From (7) we find that $V(z,q)$ satisfies the differential equation

$$\Big[\Big(\partial_z + A_s' - \Phi'\Big)\partial_z - q^2\Big] V(z,q) = 0\,, \tag{12}$$

which can be written in the Sturm-Liouville form

$$\Big[\mathcal{L} + \lambda\, r(z)\Big] y(z) = 0 \quad , \quad \mathcal{L} = \partial_z\Big(p(z)\partial_z\Big) - s(z)\,, \tag{13}$$

with

$$p(z) = e^{A_s-\Phi}\ ,\ s(z) = 0\ ,\ \lambda = -q^2\ ,\ r(z) = e^{A_s-\Phi}\,. \tag{14}$$

The corresponding Green's function satisfies the differential equation

$$\Big[\mathcal{L} + \lambda\, r(z)\Big] G(z;z') = \delta(z-z')\,. \tag{15}$$

Following Sturm-Liouville theory, the Green's function admits the spectral decomposition

$$G(z;z') = -\sum_n \frac{v^n(z) v^n(z')}{q^2 + m^2_{v^n}}\,, \tag{16}$$

where the Sturm-Liouville modes satisfty the differential equation

$$\Big[\partial_z\left(e^{A_s-\Phi}\partial_z\right) + m^2_{v^n} e^{A_s-\Phi}\Big] v^n(z) = 0 \tag{17}$$

and are normalized as $\int dz\, e^{A_s-\Phi} v^m(z) v^n(z) = \delta^{mn}$. As described in [1], the bulk to boundary propagator is related to the Green's function by $V(z', q) = -\left[e^{A_s-\Phi}\partial_z G(z; z')\right]_{z=\epsilon}$. Then using the holographic dictionary (10) one obtain the following decomposition for the current correlator

$$G^{ab}_{\hat{\mu}\hat{\nu}}(q) = \left(\eta_{\hat{\mu}\hat{\nu}} - \frac{q_{\hat{\mu}} q_{\hat{\nu}}}{q^2}\right)\delta^{ab}\sum_n \frac{F^2_{v^n}}{q^2 + m^2_{v^n}} \quad , \quad F_{v^n} = \frac{1}{g_5}\left[e^{A_s-\Phi}\partial_z v_n(z)\right]_{z=\epsilon} , \tag{18}$$

where F_{v^n} are the vector meson decay constants. The decomposition in (18) is consistent with large N_c QCD, see for example [6, 7].

3. Nucleons in confining holographic QCD

In QCD nucleons are usually described using interpolating fields. For the proton we use the Ioffe operator $O(x) = \epsilon_{abc}\left(u^T_a(x) C\gamma_\mu u_b(x)\right)\gamma_5\gamma^\mu d_c(x)$ [8, 9]. Inspired by previous works [10–12], we map this 4d operator to a 5d Dirac field with the following 5d action [2]

$$S_F = G_F \int d^5x\sqrt{-g_s}\left(\frac{i}{2}\bar{\psi}\not{D}\psi + \text{c.c.} - i\tilde{m}\bar{\psi}\psi\right) + \Delta S\,, \tag{19}$$

where $\not{D} = \Gamma^n D_n$ with $\Gamma^n = e^n_{\hat{a}}\Gamma^{\hat{a}}$ and $D_n = \partial_n + \frac{1}{4}\omega^{\hat{a}\hat{b}}_n \Gamma_{\hat{a}\hat{b}}$. The surface term ΔS is required by the variational principle. The quantities $e^n_{\hat{a}}$ and $\omega^{\hat{a}\hat{b}}_n$ are the veilbein and spin connection respectively while $\Gamma^{\hat{a}}$ are the gamma matrices in 5d flat space.

In holographic QCD the veilbein takes the form $e^n_{\hat{a}} = e^{-A_s(z)}\delta^n_{\hat{a}}$ and the non-vanishing components of the spin connection are $\omega^{\hat{z}\hat{\nu}}_\mu = -\omega^{\hat{\nu}\hat{z}}_\mu = -A'_s\delta^{\hat{\nu}}_\mu$. As shown in [1], the 5d coupling in (19) can be fixed to $G_F = 2\pi^{-4}$ to reproduce the perturbative QCD result for the nucleon correlator at small distances. The 5d Dirac field admits the decomposition $\psi = \psi_R + \psi_L$ where $\psi_{R/L} = \frac{1}{2}\left(\mathbb{1} \pm \Gamma^{\hat{z}}\right)\psi = P_{R/L}\psi$. The Dirac field equation arising from (19) decomposes as

$$\not{\partial}\psi_L = -\left(\partial_z + 2A'_s - e^{A_s}\tilde{m}\right)\psi_R \quad , \quad \not{\partial}\psi_R = \left(\partial_z + 2A'_s + e^{A_s}\tilde{m}\right)\psi_L\,, \tag{20}$$

where $\not{\partial} = \Gamma^{\hat{\mu}}\partial_\mu$. At small z one finds the asymptotic solutions

$$\begin{aligned}\psi_L(x,z) &= \alpha_L(x) z^{2-m} + \cdots + \beta_L(x) z^{3+m} + \ldots\,,\\ \psi_R(x,z) &= \alpha_R(x) z^{3-m} + \cdots + \beta_R(x) z^{2+m} + \ldots\,,\end{aligned} \tag{21}$$

where $\alpha_{R/L}$ and $\beta_{R/L}$ are the source and VEV coefficients respectively. In this work we will be interested only in the operator $O_R(x) = P_R O(x)$ so the only independent source is $\alpha_L(x)$. Again, it is convienent to define a bulk to boundary propagator $F_L(z,x;y)$ using the relation

$$\psi_L(z,x) = \int d^4y\, F_L(z,x;y)\,\alpha_L(y)\,, \tag{22}$$

and the holographic dictionary for the nucleon correlator takes the form [1]

$$\Gamma_R(x-y) = \langle O_R(x)\bar{O}_R(y)\rangle = iG_F P_R\frac{\not{\partial}_{x-y}}{\partial^2}\left(z^{2-m}e^{4A_s}(\partial_z + 2A'_s + e^{A_s}\tilde{m})F_L(z,x;y)\right)_{z=\epsilon} . \tag{23}$$

[2] The dilaton coupling was absorbed in a redefinition of the Dirac field $\psi \to e^{\Phi/2}\psi$.

The bulk to boundary propagator in momentum space satisfies the differential equation

$$\left[\left(\partial_z + 4A_s'\right)\partial_z + 2A_s'' + 4A_s'^2 + \partial_z(e^{A_s}\tilde{m}) - e^{2A_s}\tilde{m}^2 + Q^2\right] F_L(q,z) = 0\,. \tag{24}$$

Again, we use the Sturm-Liouville theory and follow the same steps as in the previous section to obtain a spectral decompostion for the nucleon correlator [1]

$$\Gamma_R^{\rm ren}(q) = -P_R \not{q} \sum_n \frac{\lambda_{N^n}^2}{q^2 + m_{N^n}^2} \quad , \quad \lambda_{N^n} = \sqrt{G_F}\left[z^{-2-m} f_{R,n}(z)\right]_{z=\epsilon}\,, \tag{25}$$

where λ_{N^n} are the nucleon "decay constants" which can be interpreted as probability amplitudes associated with the creation of nucleon states from the vacuum. The Sturm-Liouville modes satisfy the coupled differential equations

$$\left(\partial_z + 2A_s' \mp e^{A_s}\tilde{m}\right) f_{R/L}^n = \mp m_{N^n} f_{L/R}^n\,, \tag{26}$$

and are normalized as $\int dz\, e^{4A_s} f_L^m(z) f_L^n(z) = \delta^{mn}$. The spectral decomposition in (24) is consistent with large N_c QCD, see for example [6, 13]. Finally, from the analysis of the mode equations in (26) one finds that linear Regge trajectories are guaranteed considering a 5d mass term of the form $\tilde{m} = e^{-A_s}\left(\frac{1}{2}\Phi' - mA_s'\right)$ [1].

4. Results

In [1], we calculated the masses of vector mesons and nucleons, as well as the corresponding decay constants, comparing them with previous models and available experimental data. Here we present our main results for the vector meson and nucleon masses in table 1 and 2 respectively.

Ratio	Model I	Model II	Soft wall	Hard wall	Experimental
m_{ρ^1}/m_{ρ^0}	1.591	1.34	1.414	2.295	1.652 ± 0.048
m_{ρ^2}/m_{ρ^0}	2.015	1.611	1.732	3.598	1.888 ± 0.032
m_{ρ^3}/m_{ρ^0}	2.365	1.843	2	4.903	2.216 ± 0.026
m_{ρ^4}/m_{ρ^0}	2.67	2.049	2.236	6.209	2.443 ± 0.072
m_{ρ^5}/m_{ρ^0}	2.944	2.236	2.45	7.514	2.727 ± 0.265

Table 1: Ratio of vector meson masses m_{ρ^n}/m_{ρ^0} for the first excited states $n = 1, .., 5$ in the Einstein-dilaton models I and II, the soft wall model [3], the hard wall model [4], compared against experimental results. The experimental result for m_{ρ^1} was taken from [14] and for the other states were obtained from PDG [15].

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Ratio	Model I	Model II	Soft wall	Hard wall	Experimental [15]
m_{N^0}/m_{ρ^0}	0.896	0.952	1.732	2.136	1.209 ± 0.002
m_{N^1}/m_{ρ^0}	1.593	1.314	2	3.5	1.856 ± 0.039
m_{N^2}/m_{ρ^0}	2.04	1.595	2.236	4.832	2.204 ± 0.039
m_{N^3}/m_{ρ^0}	2.399	1.833	2.449	6.153	2.423 ± 0.065
m_{N^4}/m_{ρ^0}	2.708	2.043	2.646	7.468	2.706 ± 0.065

Table 2: Nucleon masses divided by the mass of the ρ_0 meson in the case $\Delta = 9/2$ ($m = 5/2$) in the Einstein-dilaton models, the soft wall model [12] and the hard wall model [10, 12], compared against the experimental results from PDG [15].

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