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Flavour changing phenomenology of supersymmetric Z' model

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We carry out a study of new physical effects in flavor changing processes in the framework of the low-energy approximation of the supersymmetric extension of the SM with an additional gauge group $U(1)'$. The most important feature of the model is the non-universal charges of fermion on the additional group, and the presence of several new physical parameters in addition to the known mixing matrices of quarks and leptons in the SM. We analyse the dependence of the CP-asymmetries on the model parameters together with constraints to addititional model parameters from anomalous magnetic moment, lepton decays.

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1. Introduction

The Standard Model (SM) has explained many observables with a great consistency. However there are exists several observables that can not be explained within the SM. Among such processes, transitions involving $b \to s$ have attracted considerable interest, primarily due to the presence of anomalies (see, e.g., [1] and references therein) hinting at potential manifestations of new physics (NP). Noteworthy examples include anomalies observed in $B_s \to \mu^+\mu^-$, $B \to K^* l^+ l^-$, and also $B \to K^{(*)} \nu\bar{\nu}$ processes. We analysed these anomalies in our Refs.[2–4].

However there are exist CP-averaged (S_i) and CP-asymmetry (A_i) angular observable in $B \to K^{(*)}\mu^+\mu^-$. In this work we consider just triple-product ($A_{7,8,9}$) CP-asymmetries and their averaged ($S_{7,8,9}$) partners. These observables are measured by the LHCb collaboration, however, with large errors [5] and are consistent with zero. Observation of non-zero CP asymmetries in $b \to sll$ decays would be a clear signature of new physics. In the absence of a non-zero signal, precise measurements of the CP asymmetries $A_{7,8,9}$ can provide important bounds on beyond the SM (BSM) sources of CP violation in the form of imaginary parts of the Wilson coefficients.

Moreover a long-standing discrepancy between the SM prediction and the measured value of the anomalous magnetic moment of the muon and electron $a_{\mu(e)} \equiv (g-2)_{\mu(e)}$ hints at new physics beyond the SM. Recent experimental evidence has shown that the muon magnetic moment deviates from the SM predictions to 5.1σ [6], and 2.4σ [7] for electron

$$(\Delta a_\mu)_{exp} = (249 \pm 48) \cdot 10^{-11}, \tag{1}$$

$$(\Delta a_e)_{exp} = -8.7 \cdot 10^{-13}. \tag{2}$$

There are uncertainties in experimental measurements and theoretical calculations. If comparable progress in reducing uncertainties in both SM predictions and measurements can be achieved, we will have an unambiguous answer to the question whether $\Delta a_{\mu(e)}$ is evidence for BSM physics. Thus from a theoretical point of view, it is very important to investigate BSM scenarios which can account for the $(g-2)_{\mu(e)}$ anomalies. With this motivation, we discuss supersymmetric Z' extension of the SM (see [2, 4] for more detail).

In addition, charged lepton flavor-violating (LFV) processes, such as $\mu \to e\gamma$, $\mu \to 3e$, $\tau \to \mu\gamma$, $\tau \to 3\mu$, restrict both lepton-flavor-diagonal couplings of the Z', and flavor off-diagonal couplings to electrons and muons. The experimental limit on the branching ratios determined from the [8, 9]

$$\mathcal{B}(\mu \to e\gamma) < 4.2 \cdot 10^{-13}, \tag{3}$$

$$\mathcal{B}(\tau \to \mu\gamma) < 4.4 \cdot 10^{-8}, \tag{4}$$

$$\mathcal{B}(\mu \to 3e) < 1 \cdot 10^{-13}, \tag{5}$$

$$\mathcal{B}(\tau \to 3\mu) < 2.7 \cdot 10^{-8}. \tag{6}$$

The paper is organized as follows. In the context of our model, described in Sec. 2, we consider the dependendence of CP-asymmetries from additional model parameters and several correlations between S_i and A_i in Sec.3. Moreover we analyse constraints on model parameters from anomalous magnetic moment of electron and muon, $\mu \to e\gamma$, $\tau \to \mu\gamma$, $\mu \to eee$, $\tau \to \mu\mu\mu$ decays in Sec.4. We conclude in Sec. 5.

2. Description of supersymmetric Z' model (Uν_RMSSM)

In this Section we briefly describe our model and set up the notation. We consider a $U(1)'$ extension of the MSSM [2]. We have chiral multiplets of the MSSM, moreover introduce a singlet superfield S, which allows one to break $U(1)'$ spontaneously and generate mass for the corresponding Z' boson. We also added three right-handed chiral superfields $\nu^c_{1,2,3}$ to account for the massive neutrinos.

It is known that models with charge assignments $(B-L)_i$ and (L_i-L_j), where B_i (L_i) are baryon (lepton) numbers of the i generations are free from anomalies. Due to this, Ref. [10] was considered the model based on $Q' = a(B-L)_3+b(L_\mu-L_\tau)$ with $a = 3/2$ and $b = -2$. However in this case is not possible to reproduce PMNS matrix. Therefore we consider $Q' = a(B-L)_3+b(L_2-L_3)+c(L_1-L_2)$ and made the substitutions $L_3 \to H_d$, $\nu^c_3 \to S$. Among possible solutions we have chosen the one with $a = 3$, $b = -2$, $c = -1$: $Q' = 3B_3 - L_1 - L_2 - H_d + S + E^c_3$, where B_3 (L_i) assigns 1/3 (1) to the top-quark (i generation lepton) superfields (both LH and RH[1]). The corresponding charges can be found in Table 1.

field	Q'	field	Q'	field	Q'
$Q_{1,2}$	0	$U^c_{1,2}$	0	$D^c_{1,2}$	0
Q_3	+1	U^c_3	−1	D^c_3	−1
$L_{1,2}$	−1	$E^c_{1,2}$	+1	$\nu^c_{1,2}$	+1
L_3	0	E^c_3	+1	ν^c_3	0
H_d	−1	H_u	0	S	+1

Table 1: The anomaly-free $U(1)'$ charges considered in the paper.

The Z' boson couples to both left and right-handed leptons as [2]

$$\begin{aligned}
\Delta\mathcal{L}_{Z'} &= g_E J^\alpha Z'_\alpha = \\
&= g_E Z'_\alpha \sum_{q,q'=1,3}\Big[V_{R,3q}V^*_{R,3q'}(\overline{\mathcal{D}}_{qR}\gamma_\alpha\mathcal{D}_{q'R}) + V_{L,3q}V^*_{L,3q'}(\overline{\mathcal{D}}_{qL}\gamma_\alpha\mathcal{D}_{q'L})\Big] \\
&- \sum_{l=1,3}\Big[\overline{\mathcal{E}}_l\gamma_\alpha\mathcal{E}_{l'} + \overline{\mathcal{N}}_l\gamma_\alpha\mathcal{N}_{l'} - V^*_{L,3l}V_{L,3l'}(\overline{\mathcal{E}}_{lL}\gamma_\alpha\mathcal{E}_{l'L})\Big] \\
&+ \sum_{\nu\nu'=1,3}\Big[V^*_{L,3\nu}V_{L,3\nu'}(\overline{\mathcal{N}}_{\nu L}\gamma_\alpha\mathcal{N}_{\nu' L}) + V^*_{R,3\nu}V_{R,3\nu'}(\overline{\mathcal{N}}_{\nu R}\gamma_\alpha\mathcal{N}_{\nu' R})\Big]. \qquad (7)
\end{aligned}$$

Here g_E is the $U(1)'$ gauge coupling, and the fermionic current is given in terms of up ($\mathcal{U}_q$) and down ($\mathcal{D}_q$) quarks, charged ($\mathcal{E}_l$) and neutral ($\mathcal{N}_\nu$) leptons (see Ref. [2]).

In Eq. (7) the mixing-matrices elements for quarks $V_{L(R),3q}$ are defined as

$$\begin{aligned}
V_{L,3q} &= \{-s^d_{13}e^{-i\phi_{13}}, -c^d_{13}s^d_{23}e^{-i\phi_{23}}, c^d_{13}c^d_{23}\}, \\
V_{R,3q} &= \frac{\{-m_b m_s s^d_{13}e^{-i\phi_{13}}, -m_b m_d c^d_{13}s^d_{23}e^{-i\phi_{23}}, m_s m_d c^d_{13}c^d_{23}\}}{\sqrt{m_d^2(m_b^2 s_{23}^2 + m_s^2 c_{23}^2)c_{13}^2 + m_b^2 m_s^2 s_{13}^2}}, \qquad (8)
\end{aligned}$$

[1]We use LH charge-conjugated superfields to account for the RH particles.
[2]We neglect the mixing with the SM Z boson.

while for leptons one can write

$$V_{L,3l} = \left\{-s^e_{13}e^{i\chi_{13}}, -c^e_{13}s^e_{23}e^{i\chi_{23}}, c^e_{13}c^e_{23}\right\}, \qquad V_{R,3l} = 1, \tag{9}$$

$$V_{L,3\nu} = \left\{\tilde{U}_{l1}, \tilde{U}_{l2}, \tilde{U}_{l3}\right\}, \quad V_{R,3\nu} = \frac{\left\{m_{\nu_1}\tilde{U}_{l1}, m_{\nu_2}\tilde{U}_{l2}, m_{\nu_3}\tilde{U}_{l3}\right\}}{\sqrt{m^2_{\nu_3}|\tilde{U}_{l3}|^2 + m^2_{\nu_2}|\tilde{U}_{l2}|^2 + m^2_{\nu_1}|\tilde{U}_{l1}|^2}}. \tag{10}$$

For convenience, we introduce the following shorthand notation

$$\tilde{U}_{li} \equiv c^e_{13}(U_{\tau i}c^e_{23} - U_{\mu i}s^e_{23}e^{-i\chi_{23}}) - U_{ei}s^e_{13}e^{-i\chi_{13}}, \qquad i = \{1,2,3\}, \tag{11}$$

with U_{l_ij} being the matrix elements of PMNS matrix.

The mixing matrices (8)-(10) incorporate new model parameters as angles and phases, where $c^{d,e}_{13,23}$, $s^{d,e}_{13,23}$ – mixing angles between 1 and 3, 2 and 3 generation of quarks and leptons, respectively, with $c_\alpha \equiv \cos\alpha$, $s_\alpha \equiv \sin\alpha$, and $\phi_{13,23}$, $\chi_{13,23}$ – new CP-violating phases of quarks and leptons.

We can introduce the following notation

$$\begin{aligned} g^{qq'}_L &\equiv V_{L,3q}V^*_{L,3q'}, \qquad & g^{qq'}_R &\equiv V_{R,3q}V^*_{R,3q'}, \\ g^{ll'}_L &\equiv V_{L,3l}V^*_{L,3l'} - \delta_{ll'}, \qquad & g^{ll'}_R &\equiv 1, \\ g^{\nu\nu'}_L &\equiv V_{L,3\nu}V^*_{L,3\nu'} - \delta_{\nu\nu'}, \qquad & g^{\nu\nu'}_R &\equiv V_{R,3\nu}V^*_{R,3\nu'} - \delta_{\nu\nu'}, \end{aligned} \tag{12}$$

where $g^{ll'}_{L(R)}$ are the left-handed (right-handed) couplings of the Z' boson to leptons, $g^{\nu\nu'}_{L(R)}$ to neutrinos and $g^{qq'}_{L(R)}$ to quarks.

3. Restrictions for Uν_RMSSM parameters

Let us briefly consider CP-averaged and CP-asymmetry angular observables. The differential distribution of $\bar{B}^0 \to \bar{K}^{*0}(\to K^-\pi^+)l^+l^-$ decay can be parametrized in terms of invariant mass of the lepton pair ($q^2 = (p_B - p_{K^*})^2 = (p_{l^+} + p_{l^-})^2$, where p_B, p_{K^*}, and $p_{l^\pm}$ the four-momenta of $\bar{B}$, K^* mesons, and charged leptons, respectively) and three angles [11]: 1) the angle θ_K of K^- in the rest frame of $\bar{K}^*$ with respect to the direction of flight of the latter in the $\bar{B}$ rest system; 2) the angle θ_l of l^- in the dilepton rest frame with respect to the direction of the lepton pair in the $\bar{B}$ rest frame; 3) the angle ϕ between $K^-\pi^+$ decay plane and the plane defined by the dilepton momenta.

The full angular decay distribution of $\bar{B}^0 \to \bar{K}^{*0}(\to K^-\pi^+)l^+l^-$ [12] can be written in the form

$$\frac{d^4\Gamma}{dq^2 d\cos\theta_l d\cos\theta_K d\phi} = \frac{9}{32\pi}J(q^2,\theta_l,\theta_K,\phi), \tag{13}$$

where

$$\begin{aligned} J(q^2,\theta_l,\theta_K,\phi) = \\ J_{1s}\sin^2\theta_K + J_{1c}\cos^2\theta_K + (J_{2s}\sin^2\theta_K + J_{2c}\cos^2\theta_K)\cos 2\theta_l + \\ J_3\sin^2\theta_K\sin^2\theta_l\cos 2\phi + J_4\sin 2\theta_K\sin 2\theta_l\cos\phi + \\ J_5\sin 2\theta_K\sin\theta_l\cos\phi + (J_{6s}\sin^2\theta_K + J_{6c}\cos^2\theta_K)\cos\theta_l + \\ J_7\sin 2\theta_K\sin\theta_l\sin\phi + J_8\sin 2\theta_K\sin 2\theta_l\sin\phi + \\ J_9\sin^2\theta_K\sin^2\theta_l\sin 2\phi. \end{aligned} \tag{14}$$

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The expressions of these twelve angular coefficients $J_i(a)$ can be found, e.g., in Ref. [13]. These coefficients depend on the q^2 variable, on Wilson coefficients and various hadronic form factors. The expression for the CP conjugate decay mode $B^0 \to K^{*0}(\to K^+\pi^-)l^-l^+$ can be obtained by substituting θ_l by $(\pi - \theta_l)$ and ϕ by $-\phi$, which leads to

$$J^{(a)}_{1,2,3,4,7} \to \bar{J}^{(a)}_{1,2,3,4,7}, \qquad J^{(a)}_{5,6,8,9} \to -\bar{J}^{(a)}_{5,6,8,9}. \tag{15}$$

Here $\bar{J}^{(a)}_i$ equal to $J^{(a)}_i$, where all weak phases are conjugated.

The expressions for CP-averaged angular observables can be presented in the form [12]

$$S^{(a)}_i(q^2) = \frac{J^{(a)}_i(q^2) + \bar{J}^{(a)}_i(q^2)}{d(\Gamma + \bar{\Gamma})/dq^2}, \tag{16}$$

as well as the CP asymmetries

$$A^{(a)}_i(q^2) = \frac{J^{(a)}_i(q^2) - \bar{J}^{(a)}_i(q^2)}{d(\Gamma + \bar{\Gamma})/dq^2}. \tag{17}$$

Next we analyse the low-energy limit of our Uν_RMSSM model. In Ref.[4] we performed two type of fits of model parameters using more than 300 observables. We found two Benchmark points, where Fit$_1$:

$$\begin{aligned} \alpha_{13} = (2.0 \pm 4)\cdot 10^{-3}, \qquad \alpha_{23} = -0.207 \pm 0.022, \qquad \beta_{13} = 0.61 \pm 0.10, \\ \beta_{23} = 0 \pm 0.5, \qquad M_{Z'}/g_E = 16.1 \pm 0.6 \text{ TeV}, \\ \phi_{13} = \phi_{23} = \chi_{13} = \chi_{23} = 0, \end{aligned} \tag{18}$$

and Fit$_2$:

$$\begin{aligned} \alpha_{13} = (8 \pm 2)\cdot 10^{-3}, \qquad \alpha_{23} = 0.34 \pm 0.08, \qquad \beta_{13} = 0.76 \pm 0.17, \\ \beta_{23} = 0.0 \pm 0.3, \qquad M_{Z'}/g_E = 18.4 \pm 1.7 \text{ TeV}, \qquad \phi_{13} = \text{unconstrained}, \\ \phi_{23} = 2.49 \pm 0.24, \qquad \chi_{13} = \chi_{23} = 0. \end{aligned} \tag{19}$$

Let us finish this section by an illustration how triple-produce CP-asymmetries (A_7, A_8, A_9) together with their CP-even counterparts (S_7, S_8, S_9) change when NP parameters are varied around Fit$_2$ BMP (see Fig.1).

The choice of the angular coefficients is motived by the fact that the former are more sensitive to new weak phases as compared to the other observables. Furthermore, A_7 is very sensitive to the phase of C_{10}. We therefore expect that, if NP reveals itself through CP-violating effects in $B^0 \to K^*\mu^+\mu^-$, it will most likely be in $A_7 - A_9$.

In Fig.2 we illustrate the dependencies between S_i and A_i. As can be seen from picture there are exist several correlations. The observation of these correlations is very important for further testing of the model for consistency. This means that if the upcoming measurements are within our predicted regions, it can indicate that our model stands out among the others.

4. Other restrictions

Here we study lepton decays, as well as lepton anomalous magnetic moments.

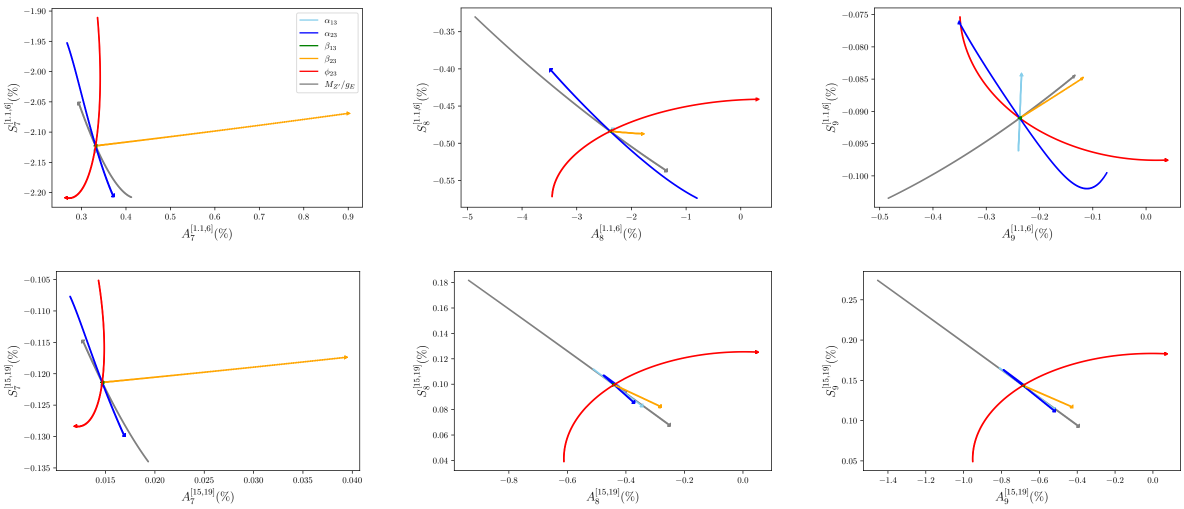

Figure 1: Dependencies on different model parameters of $(S_i - A_i)$ for central- and high-q^2 regions in the first and second rows, respectively.

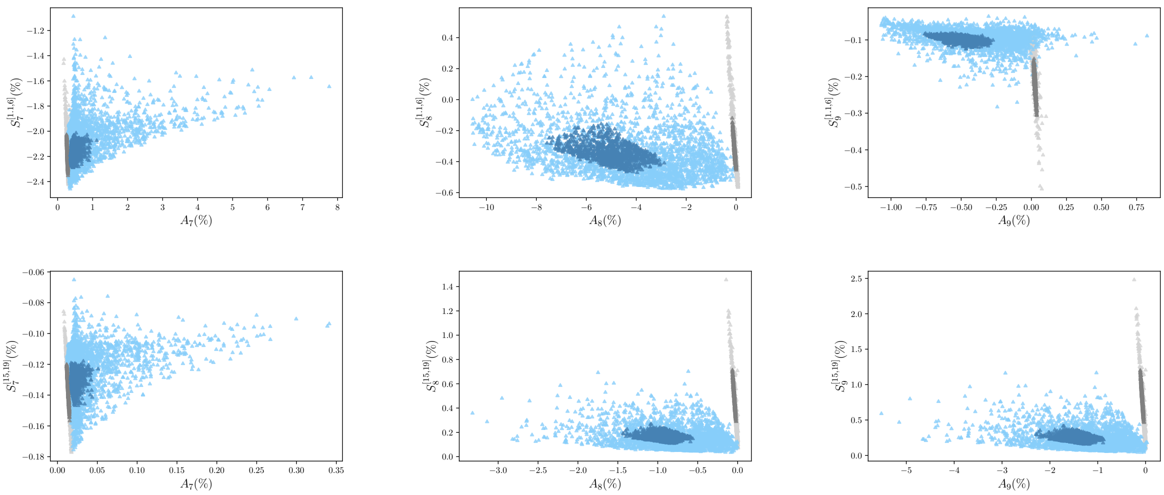

Figure 2: Correlations between A_i and S_i ($B^0 \to K^*\mu^+\mu^-$) for central-q^2 (first row), and for high-q^2 (second row). Grey and light grey regions correspond to $1, 3\sigma$ variation of model parameters around their central values for Fit$_1$; blue and light blue for Fit$_2$. The black star – is our BMP corresponding to Fit$_2$.

4.1 Anomalous magnetic moment

The one-loop beyond the SM corrections involving Z' boson to Δa_μ is given by [14]

$$\begin{aligned}\Delta a_\mu &= \frac{m_\mu g_E^2}{48\pi^2 M_{Z'}^2 (m_F^2 - m_{Z'}^2)^4}\Big[-m_\mu (C_A^2 + C_V^2)\Big(-14 m_F^6 M_{Z'}^2 - 38 m_F^2 M_{Z'}^6 \\ &+3 m_F^4 M_{Z'}^4 (13 - 6\log(\frac{m_F^2}{M_{Z'}^2})) + 5 m_F^8 + 8 m_Z^8\Big) \\ &-3 m_F (C_A^2 - C_V^2)(m_F^2 - M_{Z'}^2)\Big(3 m_F^2 M_{Z'}^4 (1 - 2\log(\frac{m_F^2}{M_{Z'}^2})) + m_F^6 - 4 M_{Z'}^6\Big)\Big] \\ &= \frac{m_\mu}{4\pi^2 M_{Z'}^2}(-m_\mu (C_A^2 + C_V^2) F(x) - m_F (C_A^2 - C_V^2) G(x)), \end{aligned} \tag{20}$$

where

$$\begin{aligned} F(x) &= \frac{5x^4 - 14x^3 + 39x^2 - 38x - 18x^2 \log x + 8}{12(1-x)^4}, \\ G(x) &= \frac{x^3 + 3x - 6x\log x - 4}{2(1-x)^3}, \end{aligned} \tag{21}$$

here $x = m_F^2/m_{Z'}^2$

Rewrite the equation (20) through g_L, g_R: $C_A = |g_R - g_L|/2$, $C_V = |g_R + g_L|/2$:

$$\Delta a_\mu = -\frac{m_\mu g_E^2}{8\pi^2 M_{Z'}^2}(m_\mu(|g_L|^2 + |g_R|^2) F(x) + m_F |g_L||g_R| G(x)) \tag{22}$$

The prediction for this observable in our model therefore takes the form

$$\begin{aligned}\Delta a_\mu^{Z'} &= -\frac{m_\mu^2 g_E^2}{8\pi^2 M_{Z'}^2}[(|g_{e_L}^{\mu e}|^2 + |g_R^{\mu e}|^2) F(x_e) + (|g_{e_L}^{\mu\mu}|^2 + |g_R^{\mu\mu}|^2) F(x_\mu) \\ &+ (|g_{e_L}^{\mu\tau}|^2 + |g_R^{\mu\tau}|^2) F(x_\tau) + Re(g_{e_L}^{\mu e} g_R^{*\mu e})\frac{m_e}{m_\mu} G(x_e) + Re(g_L^{\mu\mu} g_R^{*\mu\mu})\frac{m_\mu}{m_\mu} G(x_\mu) \\ &+ Re(g_L^{\mu\tau} g_R^{*\mu\tau})\frac{m_\tau}{m_\mu} G(x_\tau)]. \end{aligned} \tag{23}$$

Analogous formulas for Δa_e are obtained by formally $\mu \to e$:

$$\begin{aligned}\Delta a_e^{Z'} &= -\frac{m_e^2 g_E^2}{8\pi^2 M_{Z'}^2}[(|g_L^{ee}|^2 + |g_R^{ee}|^2) F(x_e) + (|g_L^{e\mu}|^2 + |g_R^{e\mu}|^2) F(x_\mu) \\ &+ (|g_L^{e\tau}|^2 + |g_R^{e\tau}|^2) F(x_\tau) + Re(g_L^{ee} g_R^{*ee}) G(x_e)\frac{m_e}{m_e} \\ &+ Re(g_L^{e\mu} g_R^{*e\mu}) G(x_\mu)\frac{m_\mu}{m_e} + Re(g_L^{e\tau} g_R^{*e\tau}) G(x_\tau)\frac{m_\tau}{m_e}]. \end{aligned} \tag{24}$$

The leading contribution to $\Delta a_{e,\mu}$ can be estimated as

$$\Delta a_{e(\mu)} = -\frac{m_e^2(m_\mu^2)}{8\pi^2}\frac{g_E^2}{M_{Z'}^2} C_{LR}, \tag{25}$$

where the dimensionless function C_{LR} is defined as

$$\begin{aligned} C_{LR} &= F(x_i) + G(x_i) = (|g_L^{le}|^2 + |g_R^{le}|^2)F(x_e) + (|g_L^{l\mu}|^2 + |g_R^{l\mu}|^2)F(x_\mu) \\ &+ (|g_L^{l\tau}|^2 + |g_R^{l\tau}|^2)F(x_\tau) + Re(g_L^{le} g_R^{*le})\frac{m_e}{m_l}G(x_e) \\ &+ Re(g_L^{l\mu} g_R^{*l\mu})\frac{m_\mu}{m_l}G(x_\mu) + Re(g_L^{l\tau} g_R^{*l\tau})\frac{m_\tau}{m_l}G(x_\tau). \end{aligned} \tag{26}$$

Approximation expressions if CP phases $\chi_{13} \to 0, \chi_{23} \to 0$

$$\Delta a_\mu = -\frac{m_\mu^2}{8\pi^2}\frac{g_E^2}{M_{Z'}^2}\left(-\frac{2}{3}(\cos^2\beta_{13}\cos 2\beta_{23} + \sin^2\beta_{13})\right), \tag{27}$$

$$\Delta a_e = -\frac{m_e^2}{8\pi^2}\frac{g_E^2}{M_{Z'}^2}\frac{1}{12}\left(2\left(\sin^2 2\beta_{13}\sin^2\beta_{23} + 12\sin^2\beta_{13} - 8\right) - \cos^4\beta_{13}(\cos 4\beta_{23} - 9)\right). \tag{28}$$

For our Fit$_1$ (only if we substitute the angles and evaluate the $M_{Z'}/g_E$ ratio), we have:

$$\Delta a_\mu = -\frac{m_\mu^2}{8\pi^2}\frac{g_E^2}{M_{Z'}^2}C_{LR} \sim 2.49 \cdot 10^{-9} \times \left(\frac{200\text{GeV}}{M_{Z'}/g_E[\text{GeV}]}\right)^2, \tag{29}$$

$$\Delta a_e = -\frac{m_e^2}{8\pi^2}\frac{g_E^2}{M_{Z'}^2}C_{LR} \sim -8.7 \cdot 10^{-13} \times \left(\frac{100\text{GeV}}{M_{Z'}/g_E[\text{GeV}]}\right)^2. \tag{30}$$

4.2 $l_i \to l_j\gamma$

The Lagrangian of $l_i \to l_j\gamma$ for general gauge and Yukawa interactions is given by [15]

$$\mathcal{L}_{l_i \to l_j\gamma} = \bar{\mathcal{E}}_i\left[Z'_\mu\gamma^\mu(g_L^{\mathcal{E}_i\mathcal{E}_j}P_L + g_R^{\mathcal{E}_i\mathcal{E}_j}P_R)\right]\mathcal{E}_j + h.c., \tag{31}$$

where $\mathcal{E}_j$ are external charged leptons, $\mathcal{E}_i$ internal fermions.

The branching fraction is then given by

$$\mathcal{B}(\mu \to e\gamma) = \frac{e^2 g_E^4}{16\pi\Gamma_{l_i}}\left(m_{l_i} - \frac{m_{l_j}^2}{m_{l_i}}\right)^3 [|\sigma_L|^2 + |\sigma_R|^2], \tag{32}$$

where $\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3} = 3 \cdot 10^{-19}\text{GeV}$

$$\begin{aligned} \sigma_L &= [F(x_\mu)g_L^{e\mu}g_L^{\mu\mu} + F(x_e)g_L^{ee}g_L^{\mu e} + F(x_\tau)g_L^{e\tau}g_L^{\mu\tau} + \\ &+ G(x_\mu)g_L^{e\mu}g_R^{\mu\mu}\frac{m_\mu}{m_\mu} + G(x_e)g_L^{ee}g_R^{\mu e}\frac{m_e}{m_\mu} + G(x_\tau)g_L^{e\tau}g_R^{\mu\tau}\frac{m_\tau}{m_\mu}, \\ \sigma_R &= \sigma_L(L \to R), \end{aligned} \tag{33}$$

and

$$\begin{aligned} F(x) &= \frac{5x^4 - 14x^3 + 39x^2 - 38x - 18x^2\log x + 8}{12(1-x)^4}, \\ G(x) &= \frac{x^3 + 3x - 6x\log x - 4}{2(1-x)^3}, \qquad x = m_F^2/m_{Z'}^2. \end{aligned} \tag{34}$$

Analytic expression for the branching fraction of $\mu \to e\gamma$:

$$\begin{aligned}\mathcal{B}(\mu \to e\gamma) &= \frac{\alpha}{1024\pi^4}\frac{m_\mu^5 g_E^4}{M_{Z'}^4\Gamma_\mu}[|[F(x_\mu)g_L^{e\mu}g_L^{\mu\mu} + F(x_e)g_L^{ee}g_L^{\mu e} + F(x_\tau)g_L^{e\tau}g_L^{\mu\tau} + \\ &+ G(x_\mu)g_L^{e\mu}g_R^{\mu\mu}\frac{m_\mu}{m_\mu} + G(x_e)g_L^{ee}g_R^{\mu e}\frac{m_e}{m_\mu} + G(x_\tau)g_L^{e\tau}g_R^{\mu\tau}\frac{m_\tau}{m_\mu}|^2 + |\sigma_L(L \to R)|^2] \\ &\lesssim \frac{\alpha}{1024\pi^4}\frac{m_\mu^5 g_E^4}{M_{Z'}^4\Gamma_\mu}\sin 2\beta_{13}\sin\beta_{23}\left(\frac{m_e}{m_\mu} + \frac{2}{3}\right) \\ &\sim 4.2\cdot 10^{-13}\times(\sin 2\beta_{13}\sin\beta_{23})\cdot\left(\frac{1}{M_{Z'}/g_E}\right)^4 \\ &\sim_{Fit_1} 4.2\cdot 10^{-13}\times\left(\frac{3\cdot 10^4\text{GeV}}{M_{Z'}/g_E[\text{GeV}]}\right)^4 \end{aligned} \tag{35}$$

and $\tau \to \mu\gamma$ is given by

$$\begin{aligned}\mathcal{B}(\tau \to \mu\gamma) &= \frac{\alpha}{1024\pi^4}\frac{m_\tau^5 g_E^4}{M_{Z'}^4\Gamma_\tau}[|[F(x_\mu)g_L^{\mu\mu}g_L^{\mu\tau} + F(x_e)g_L^{e\mu}g_L^{\tau e} + F(x_\tau)g_L^{\mu\tau}g_L^{\tau\tau} + \\ &+ G(x_\mu)g_L^{\mu\mu}g_R^{\mu\tau}\frac{m_\mu}{m_\mu} + G(x_e)g_L^{e\mu}g_R^{e\tau}\frac{m_e}{m_\mu} + G(x_\tau)g_L^{\mu\tau}g_R^{\tau\tau}\frac{m_\tau}{m_\mu}|^2 + |\sigma_L(L \to R)|^2] \\ &\lesssim \frac{\alpha}{1024\pi^4}\frac{m_\tau^5 g_E^4}{M_{Z'}^4\Gamma_\tau}(-\cos^2\beta_{13}\sin 2\beta_{23})\left(\frac{2}{3} + \frac{m_\mu}{m_\tau}\right)\left(\frac{1}{M_{Z'}/g_E}\right)^4 \\ &\sim 4.4\cdot 10^{-8}\times(-\cos^2\beta_{13}\sin 2\beta_{23})\cdot\left(\frac{1}{M_{Z'}/g_E}\right)^4 \\ &\sim_{Fit_1} 4.4\cdot 10^{-8}\times\cdot\left(\frac{1\cdot 10^4\text{GeV}}{M_{Z'}/g_E[\text{GeV}]}\right)^4 \end{aligned} \tag{36}$$

4.3 $l_i \to l_j l_k l_l$

The Lagrangian for $l_i \to l_j l_j l_k$ (e.g., $\mu^- \to e^- e^+ e^-$, $\tau^- \to \mu^-\mu^+\mu^-$) are given by

$$\begin{aligned}\mathcal{L} &= C_{LL}^{ikjj}(\bar{l}_i\gamma^\mu P_L l_k)(\bar{l}_j\gamma_\mu P_L l_j) + C_{LL}^{ijjk}(\bar{l}_i\gamma^\mu P_L l_j)(\bar{l}_j\gamma_\mu P_L l_k) \\ &+ C_{LR}^{ikjj}(\bar{l}_i\gamma^\mu P_L l_k)(\bar{l}_j\gamma_\mu P_R l_j) + C_{LR}^{ijjk}(\bar{l}_i\gamma^\mu P_L l_j)(\bar{l}_j\gamma_\mu P_R l_k) + (L \to R) + h.c. \end{aligned} \tag{37}$$

The branching fraction is given by [16]

$$\mathcal{B}(l_i \to l_j l_j l_k) = \frac{m_{l_i}^5}{1536\pi^3\Gamma_{l_i}}\left[|C_{LL}^{ikjj} + C_{LL}^{ijjk}|^2 + |C_{LR}^{ijjk}|^2 + |C_{LR}^{ikjj}|^2 + (L \to R)\right], \tag{38}$$

where masses of daughter leptons are neglected.

In this model, the Wilson coefficients are given by

$$C_{XY}^{ijkl} = \frac{1}{M_Z'^2}[g_X]^{ij}[g_Y]^{kl}, \tag{39}$$

where $X, Y = L, R$

These LFV three-body decays are dominated by Z' boson exchange. The Z' contributions to $\mu^- \to e^- e^+ e-$ and $\tau^- \to \mu^- \mu^+ \mu^-$ are estimated as

$$\mathcal{B}(\mu^- \to e^- e^+ e^-) \sim \frac{m_\mu^5}{1536\pi^3 \Gamma_\mu} \left(\frac{1}{M'_Z/g_E} \right)^4 \sin^2 2\beta_{13} \sin^2 \beta_{23} (\cos^4 \beta_{13} + 1) \\ \sim_{Fit_1} 1 \cdot 10^{-12} \times \left(\frac{500\text{GeV}}{M_{Z'}/g_E [\text{GeV}]} \right)^4 , \tag{40}$$

$$\mathcal{B}(\tau^- \to \mu^- \mu^+ \mu^-) \sim \frac{m_\tau^5}{1536\pi^3 \Gamma_\tau} \left(\frac{1}{M'_Z/g_E} \right)^4 \times \\ \times \cos^4 \beta_{13} \sin^2 2\beta_{23} ((\cos^2 \beta_{13} \sin^2 \beta_{23} - 1)^2 + 1) \\ \sim_{Fit_1} 2.7 \cdot 10^{-8} \times \left(\frac{600\text{GeV}}{M_{Z'}/g_E [\text{GeV}]} \right)^4 . \tag{41}$$

5. Conclusion

Thus, in this paper we have analyzed the dependencies of the model parameters on the CP-even and CP-odd observables and their correlations with each other. In addition, we have found constraints on the model parameters due to lepton decays and anomalous magnetic moment of the muon and electron. As can be seen from expressions (29),(30),(35),(36),(40),(41) our Uν_RMSSM can not accomodate for these observables if Z' is on the TeV scale.

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