

Naturally small neutrino mass from asymptotic safety

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We consider the dynamical generation of an arbitrarily small neutrino Yukawa coupling in two scenarios embedded in trans-Planckian asymptotic safety, first the Standard Model with right-handed neutrinos, then the gauged $B - L$ model. We show that this mechanism is more in line with existing theoretical calculations in quantum gravity when it is applied to the $B - L$ model. Interestingly, this scenario can accommodate, in full naturalness and without extensions, the possibility of purely Dirac, pseudo-Dirac, and Majorana neutrinos with any see-saw scale. We investigate eventual distinctive signatures of these cases in the detection of gravitational waves from first-order phase transitions.

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1. Introduction

Asymptotic safety (AS) is the property of a quantum field theory to develop fixed points of the renormalization group (RG) flow of the action [1]. Following the development of functional renormalization group (FRG) techniques a few decades ago [2, 3], it was shown that AS could arise quite naturally in quantum gravity and provide the key ingredient for the non-perturbative renormalizability of the theory. Fixed points were identified initially for the rescaled Newton coupling and the cosmological constant in the Einstein-Hilbert truncation of the effective action [4–6], and later confirmed in the presence of gravitational operators of increasing mass dimension [7–15], and of matter-field operators [16–18].

The properties of asymptotically safe quantum gravity may also influence particle physics in four space-time dimensions, as not only the gravitational action but the full system of gravity and matter may feature ultraviolet (UV) fixed points in the energy regime where gravitational interactions become strong [19–30]. A trans-Planckian fixed point may thus induce some specific boundary conditions for some of the *a priori* free couplings of the matter Lagrangian, as long as they correspond to *irrelevant* directions in theory space. Early “successes” of AS applied to particle physics are a gravity-driven solution to the triviality problem in U(1) gauge theories [31–33]; a ballpark prediction for the value of the Higgs mass (more precisely, of the quartic coupling of the Higgs potential) obtained a few years ahead of its discovery [34]; and the retroactive “postdiction” of the top-mass value [35].

In these proceedings we report on two recent papers, Refs. [36, 37], in which we proposed a way of obtaining dynamically a naturally small Yukawa coupling in the framework of AS.

A large amount of atmospheric, reactor, and accelerator data have robustly shown that neutrinos have a mass, and that their mass is much smaller than the masses of the other fermions of the Standard

Model (SM). If (Dirac) neutrino masses were generated via the Higgs mechanism, they would require a minuscule Yukawa coupling, of the order of 10^{-13} , lower by several orders of magnitude than the other SM Yukawa couplings, which range between $\sim 10^{-5}$ and 1. To deal with such uncomfortably and potentially unnaturally small values, numerous new physics (NP) constructions have been developed in recent decades with the goal of dynamically generating the neutrino mass [38–49]. Unlike in those works, our mechanism does not have to be suppressed by a large Majorana scale, like in the see-saw mechanism, nor is it parameterized by the small spontaneous breaking of lepton-number symmetry, like in the inverse see-saw model. Rather, the trans-Planckian RG flow of the neutrino Yukawa coupling develops a Gaussian infrared (IR)-attractive fixed point. The neutrino can naturally be a Dirac particle, because its Yukawa coupling will be exponentially suppressed.

The simple ingredient beneath our construction is that the trans-Planckian renormalization group equations (RGEs) should accommodate a negative critical exponent for the Gaussian fixed point of the neutrino Yukawa coupling. Such a feature should ideally emerge from a first-principle calculation based on the FRG. It turns out, however, that at least in the SM with right-handed neutrinos (SMRHN) it may not be easy to obtain full consistency between the quantum-gravity calculation and a phenomenologically viable neutrino-mass generation, because of stringent indirect constraints on the gravitational parameters stemming in this construction from the measured value of the hypercharge gauge coupling. The problem can be avoided by substituting the dynamical effect of the hypercharge-coupling fixed point with an equivalent effect due to other couplings that are not well-measured yet, trading thus a constraint for a prediction. We considered in Ref. [37] perhaps the simplest and most natural extension of the SMRHN, the well-known gauged $B - L$ model [50, 51], which extends the SM gauge group with an abelian $U(1)_{B-L}$ symmetry. The gauge coupling g_{B-L} and kinetic mixing g_ϵ can generate a negative critical exponent for the neutrino Yukawa coupling in the same way as g_Y does in the SMRHN. Interestingly, by being endowed with a NP scalar field, the model provides a natural framework for the spontaneous generation of intermediate scales, either directly or via dimensional transmutation with the Coleman-Weinberg mechanism [52]. Assuming the latter applies, it is then interesting to compute potential gravitational-wave (GW) signatures from first-order phase transitions (FOPTs) [53–56] (see also Ref. [57] for a recent comprehensive review). We investigated them in Ref. [37], with the ultimate hope of associating some of their features to the dynamical generation of a Majorana or a Dirac neutrino mass in the context of AS.

2. General notions of asymptotic safety

The scale-dependence of all Lagrangian couplings is encoded in the RG flow. In AS, quantum gravity effects kick in at about the Planck scale, where the flow of the gravitational action develops dynamically a fixed point. Let us consider a (renormalizable) matter theory with gauge and Yukawa interactions. The RGEs receive modifications above the Planck scale that look like

$$\frac{dg_i}{dt} = \beta_i^{(\text{matter})} - f_g g_i \quad (1)$$

$$\frac{dy_j}{dt} = \beta_j^{(\text{matter})} - f_y y_j, \quad (2)$$

where we indicate the renormalization scale with $t = \ln \mu$, g_i and y_j (with $i, j = 1, 2, 3 \dots$) are the set of gauge and Yukawa couplings, respectively, and the original beta functions (without gravity)

are indicated schematically with $\beta_{i,j}^{(\text{matter})}$.

The trans-Planckian gravitational corrections f_g and f_y are universal, in the sense that they multiply linearly all matter couplings of the same kind, in agreement with the expectation that gravity should not distinguish the internal degrees of freedom of the matter theory. The f_g and f_y coefficients depend on the fixed points of the operators of the gravitational action, and can be computed using the techniques of the FRG. Their computation is subject to extremely large uncertainties, which relate to the choice of truncation in the gravity/matter action, to the selected renormalization scheme, to the gauge-fixing parameters, and other effects [6, 7, 12, 58–62]. Despite this, explicit forms of f_g and f_y exist in the literature. For example, it was found in Refs. [33, 35] that, for a matter theory with gauge and Yukawa couplings, an FRG calculation in the Einstein-Hilbert truncation of the gravity action, with Litim-type regulator and $\alpha = 0, \beta = 1$ gauge-fixing choice, yields

$$f_g = \tilde{G}^* \frac{1 - 4\tilde{\Lambda}^*}{4\pi (1 - 2\tilde{\Lambda}^*)^2}, \quad f_y = -\tilde{G}^* \frac{96 + \tilde{\Lambda}^* (-235 + 103\tilde{\Lambda}^* + 56\tilde{\Lambda}^{*2})}{12\pi (3 - 10\tilde{\Lambda}^* + 8\tilde{\Lambda}^{*2})^2}, \quad (3)$$

where $\tilde{\Lambda} = \Lambda/\mu^2$, $\tilde{G} = G\mu^2$ are the dimensionless cosmological and Newton constant, which parameterize the Einstein-Hilbert action, and we have indicated the trans-Planckian (interactive) fixed-point values with an asterisk. We will come back to these expressions later on.

Let us close this section by recalling that a trans-Planckian fixed point of the system of Eqs. (1) and (2) is a set $\{g_i^*, y_j^*\}$, corresponding to a zero of the beta functions: $\beta_{i(j)}^{(\text{matter})}(g_i^*, y_j^*) - f_{g(y)} g_i^*(y_j^*) = 0$. The RGEs of couplings $\{\alpha_k\} \equiv \{g_i, y_j\}$ are then linearized around the fixed point to derive the stability matrix M_{ij} , which is defined as

$$M_{ij} = \partial\beta_i / \partial\alpha_j |_{\{\alpha_k^*\}}. \quad (4)$$

Eigenvalues of the stability matrix define the opposite of critical exponents θ_i , which characterize the power-law evolution of the matter couplings in the vicinity of the fixed point. If θ_i is positive, the corresponding eigendirection is dubbed as *relevant* and UV-attractive. All RG trajectories along this direction will asymptotically reach the fixed point and, as a consequence, a deviation of a relevant coupling from the fixed point introduces a free parameter in the theory (this freedom can be used to adjust the coupling at the high scale so that it matches an eventual measurement at the low scale). If θ_i is negative, the corresponding eigendirection is dubbed as *irrelevant* and IR-attractive. There exists in this case only one trajectory that the coupling's flow can follow in its run to the low scale, thus potentially providing a clear prediction for its value at the experimentally accessible scale.

3. Small Yukawa couplings from UV fixed points

In a gauge-Yukawa theory embedded in trans-Planckian AS, it is possible to concoct a dynamical mechanism that makes some Yukawa couplings naturally small [36, 63]. If there exists an IR-attractive, Gaussian fixed point for those Yukawa couplings, their flow from a different, UV-attractive fixed point will asymptotically tend to zero as they approach the Planck scale from above.

We exemplify this pattern by considering a simple generic system of matter RGEs, comprising one (abelian) gauge coupling g_Y , and two Yukawa couplings, y_X and y_Z . In the deep trans-Planckian regime the system takes the form of Eqs. (1) and (2),

$$\frac{dg_Y}{dt} = \frac{b_Y}{16\pi^2} g_Y^3 - f_g g_Y \quad (5)$$

$$\frac{dy_X}{dt} = \frac{y_X}{16\pi^2} [\alpha_X y_X^2 + \alpha_Z y_Z^2 - \alpha_Y g_Y^2] - f_y y_X \quad (6)$$

$$\frac{dy_Z}{dt} = \frac{y_Z}{16\pi^2} [\alpha'_X y_X^2 + \alpha'_Z y_Z^2 - \alpha'_Y g_Y^2] - f_y y_Z, \quad (7)$$

where $b_Y, \alpha_X^{(r)}, \alpha_Y^{(r)}, \alpha_Z^{(r)} \geq 0$ are one-loop coefficients. The reader may think of Eqs. (5)-(7) as the system of hypercharge/top Yukawa/neutrino Yukawa coupling in the SMRHN, after all other couplings have been set to their (UV-attractive) Gaussian fixed point. The discussion however is generic and can be applied to any gauge-Yukawa system [36].

The dynamical flow of $y_Z(t)$ towards the trans-Planckian IR can be extracted by integrating Eq. (7). After replacing $f_y \rightarrow (\alpha_X y_X^{*2} - \alpha_Y g_Y^{*2})/16\pi^2$, $y_Z(t)$ is expressed in terms of y_X^* and g_Y^* , plus an arbitrary constant κ setting the boundary condition at the Planck scale. One gets

$$y_Z(t, \kappa) = \left[\frac{c_Y g_Y^{*2} - c_X y_X^{*2}}{e^{-(c_Y g_Y^{*2} - c_X y_X^{*2})(t/8\pi^2 - 2\kappa)} + \alpha'_Z} \right]^{1/2}, \quad (8)$$

where we have defined $c_Y = \alpha_Y - \alpha'_Y$ and $c_X = \alpha_X - \alpha'_X$. Imposing $\theta_Z < 0$ implies that $y_Z(t, \kappa)$ is monotonically increasing with t in the trans-Planckian regime, and that its value at the Planck scale is set by its “distance” from $16\pi^2\kappa$. As expected, y_Z can reach arbitrarily small values without fine tuning, being parameterized exclusively by the integration constant κ .

The mechanism just described applies to any gauge-Yukawa particle physics model embedded in asymptotically safe quantum gravity, as long as the corresponding RGEs take the form of Eqs. (5)-(7). In the SMRHN this mechanism can give rise to a Dirac neutrino mass without fine tuning after electroweak symmetry breaking. Moreover, the asymptotically safe SMRHN turns out to be consistent with all the existing data on mass-squared differences and mixing angles, if the normal ordering of neutrino masses is assumed [36]. The trans-Planckian flow of the gauge and Yukawa couplings of the SMRHN is shown in Fig. 1 (left).

Concerns arise when pondering the consistency of this mechanism with quantum-gravity calculations of f_g and f_y based on the FRG and this is particularly true in the SMRHN. Let us recall that by imposing $g_Y^* \neq 0$ along an irrelevant direction we imply that the RG flow of $g_Y(t)$, followed from the fixed point down to low energies, yields a specific prediction for the hypercharge gauge coupling. This requires in turn that only one value of f_g is allowed to emerge from the FRG calculation:

$$f_g \approx \frac{b_Y g_Y^{*2}(M_{\text{Pl}})}{16\pi^2}. \quad (9)$$

The numerical value of Eq. (9) ought to be computed very precisely, more precisely than f_y since the uncertainties on the experimental determination of g_Y are smaller than those on, *e.g.*, the \overline{MS} value of $y_t(M_t)$ and other Yukawa couplings. Even considering that FRG calculations are

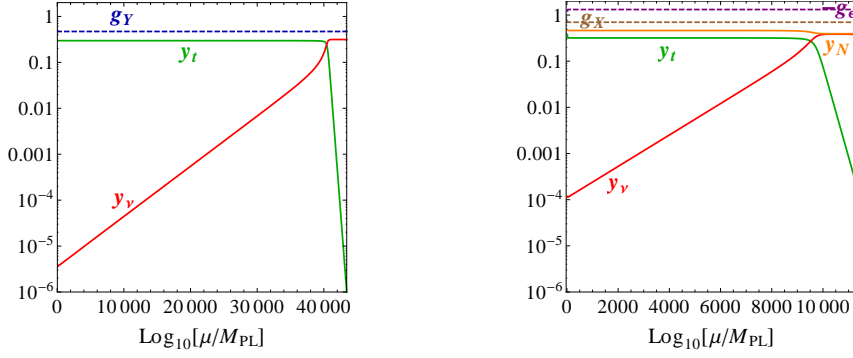


Figure 1: (left) An example of trans-Planckian trajectories for the RGE system composed of g_Y (blue, dashed), y_t (green, solid), and y_ν (red, solid) in the SMRHN with $f_g = 0.0097$, $f_y = 0.0005$. Neutrino Yukawa coupling y_ν can be made arbitrarily small at the Planck scale by adjusting the integration constant κ in Eq. (8). (right) The same in the gauged $B - L$ model. We set $f_g = 0.05$ and $f_y = -0.005$. Besides y_ν and y_t , we plot the trans-Planckian flow of g_X (brown, dashed), $-g_\epsilon$ (purple, dashed) and y_N (orange, solid). Note that g_Y (not shown) is here a relevant parameter.

marred by large theoretical uncertainties, it may seem exceedingly constraining that such a specific outcome ought to emerge from the deep UV construction.

A simple way out comes from generalizing the dynamical generation of small Yukawa couplings to models less dependent on precisely measured quantities. This allows us to modify the system of Eqs. (5)-(7) in two possible ways. One can either add some extra irrelevant gauge couplings to the system, one can add extra Yukawa couplings, or both. Both solutions are implemented straightforwardly if instead of the SMRHN one embeds the gauged $B - L$ model in trans-Planckian AS.

In the gauged $B - L$ model the SM symmetry is extended by an abelian gauge group $U(1)_{B-L}$, with gauge coupling g_{B-L} . The particle content is extended with a SM-singlet complex scalar field S , whose vev v_S spontaneously breaks $U(1)_{B-L}$. The abelian charges of the SM and NP fields can be found, *e.g.*, in Refs. [64, 65].

The Yukawa part of the Lagrangian includes

$$\mathcal{L}_M = -y_N^{ij} S \nu_{R,i} \nu_{R,j} + \text{H.c.}, \quad (10)$$

in terms of a new Yukawa coupling matrix in flavor space, y_N^{ij} . The vev v_S generates the Majorana mass upon spontaneous breaking of $U(1)_{B-L}$. In this scenario, $v_S \gg v_H$, so that one can work in the basis where the Majorana mass is diagonal. If the boundary conditions from AS require that they are irrelevant, all three diagonal couplings will be equal, $y_N^{ii} \equiv y_N$.

The RGEs of the gauged $B - L$ model are given at one loop in, *e.g.*, Ref. [37]. The RG flow of the neutrino Yukawa coupling admits a Gaussian irrelevant fixed point driven by the irrelevant fixed points of new gauge and Yukawa couplings, $g_X^* \neq 0$, $g_\epsilon^* \neq 0$, and $y_N^* \neq 0$. We show the trans-Planckian flow of the couplings in Fig. 1 (right). Note that the behavior of y_ν mimics exactly the SMRHN case, while g_Y can originate from a relevant fixed point and remain free independently of the value of f_g .

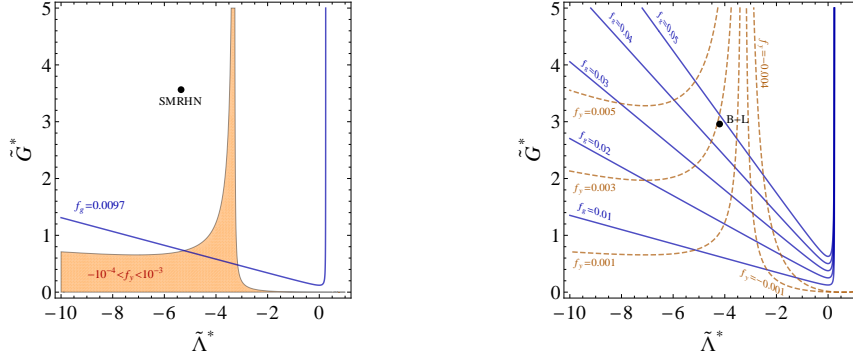


Figure 2: (left) The region of $(\tilde{\Lambda}^*, \tilde{G}^*)$ parameter space consistent with a small neutrino Yukawa coupling in the SMRHN. Blue solid line shows $f_g = 0.0097$, which is required for generating an irrelevant $g_Y^* \neq 0$. Orange region is consistent with the \overline{MS} value of the top mass. Black dot shows the outcome of a calculation with FRG techniques [33]. (right) The same in the $B-L$ model. All contours of f_g (solid blue) are consistent with a small neutrino Yukawa coupling. f_y contours (dashed brown) will be subject to constraints. Black dot shows the outcome of a calculation with FRG techniques [33].

3.1 Possible connections to the FRG

The freedom of adjusting the value of f_g arbitrarily without spoiling the dynamical generation of a small neutrino Yukawa coupling becomes valuable when confronting the phenomenologically viable parameter space with existing computations from the FRG. As was discussed in Sec. 2, calculations of f_g and f_y from first principles are marred by large theory uncertainties. We can nonetheless refer to some of the explicit existing cases in the literature and make the point that, even considering those uncertainties, the $B-L$ model likely provides a more flexible framework than the SMRHN to match a UV calculation with the low-scale phenomenology.

Let us return to the explicit f_g, f_y computations of Refs. [33, 35], which were recalled in Eq. (3). In Fig. 2 (left) we show in the $(\tilde{\Lambda}^*, \tilde{G}^*)$ plane the parameter space consistent with the generation of a small neutrino Yukawa coupling and phenomenological constraints in the SMRHN. The solid blue line corresponds to $f_g = 0.0097$, cf. Eq. (9), and the shaded (orange) region corresponds to the requirement of having the correct top mass. The black dot shows the outcome of a calculation with FRG techniques [33].

Conversely, one can recast the above discussion in the framework of the gauged $B-L$ model. Now, $f_g = 0.047$, $f_y = 0.0028$, and the fixed point is indicated as a black dot in Fig. 2 (right). We draw in Fig. 2 (right) as solid blue contours some sample values of f_g , which are *all* currently allowed by phenomenological constraints and give rise to different predictions for the $B-L$ gauge coupling and kinetic mixing. Brown dashed lines show the contours of selected values of f_y . It appears that the spectrum of possibilities for the eventual outcome of an FRG calculation opens up significantly in the $B-L$ model with respect to the SMRHN.

4. Boundary conditions of the $B-L$ model

We discuss in this section the trans-Planckian fixed points of the $B-L$ gauge-Yukawa RGE system. Increasing the value of $f_g > 0.0097$ allows one to find a relevant Gaussian solution $g_Y^* = 0$.

	f_g	f_y	g_X^*	g_ϵ^*	y_N^*	$g_X (10^{5,7,9} \text{ GeV})$	$g_\epsilon (10^{5,7,9} \text{ GeV})$	$y_N (10^{5,7,9} \text{ GeV})$
BP1	0.01	0.0005	0.10	-0.55	0.12	0.29, 0.29, 0.30	-0.26, -0.27, -0.28	0.16, 0.16, 0.16
BP2	0.05	-0.005	0.70	-1.32	0.47	0.40, 0.41, 0.44	-0.52, -0.56, -0.61	0.42, 0.44, 0.45
BP3	0.02	-0.0015	0.10	-0.75	0.0	0.12, 0.12, 0.12	-0.33, -0.35, -0.37	0.0
BP4	0.03	-0.004	0.10	0.75	0.0	0.09, 0.09, 0.09	0.23, 0.25, 0.28	0.0

Table 1: The values of f_g and f_y , trans-Planckian fixed points of the irrelevant couplings (indicated with an asterisk), and predicted values of those couplings at three low scales of reference for the four benchmark points selected in this study. All four points admit irrelevant $y_\nu^* = 0$.

However, no irrelevant fixed point with $y_\nu^* = 0$ can be identified when all three abelian gauge couplings correspond to relevant directions. Thus, the fixed points of the two abelian NP couplings g_X and g_ϵ must be irrelevant. It turns out that they are not independent, but instead belong to an ellipse, parameterized by f_g :

$$12g_X^{*2} + \frac{32}{3}g_X^*g_\epsilon^* + \frac{41}{6}g_\epsilon^{*2} - 16\pi^2 f_g = 0. \quad (11)$$

Furthermore, we impose several conditions on the Yukawa sector:

- y_t^* has to be real, *i.e.*, $17g_\epsilon^{*2} + 20g_\epsilon^*g_X^* + 8g_X^{*2} + 192\pi^2 f_y > 0$. This also guarantees that the top Yukawa coupling is irrelevant, $\theta_t < 0$.
- $y_\nu^* = 0$ has to be irrelevant, *i.e.*, $\theta_\nu < 0$.
- if $y_N^* \neq 0$, it has to be real, *i.e.*, $3g_X^{*2} + 8\pi^2 f_y > 0$. This also guarantees that y_N is irrelevant, $\theta_N < 0$.
- if $y_N^* = 0$, it has to be irrelevant, *i.e.*, $\theta_N = 3g_X^{*2} + 8\pi^2 f_y < 0$.

We present four benchmark points and their characteristic features in Table 1. The last three columns of Table 1 show the predicted values of irrelevant couplings g_X , g_ϵ , and y_N at three sub-Planckian scales of interest, 10^5 GeV, 10^7 GeV, and 10^9 GeV. Those are our chosen reference scales for the analysis of gravitational wave signatures in Sec. 5.

BP1 and BP2 feature $y_N^* \neq 0$, which is of order 1 in size. Equation (10) implies in this case that the Majorana mass scale is $M_N = \sqrt{2}y_N\nu_S$. It is a canonically relevant parameter of the theory and can thus be chosen anywhere, as long as it is in agreement with phenomenological constraints on the scalar potential. Note that the see-saw mechanism can be invoked here to give mass to the active neutrinos, $m_\nu \sim y_\nu^2\nu_H^2/(\sqrt{2}M_N)$. The theory is consistent with AS whatever the Majorana mass scale is, since the correct size of the neutrino Yukawa coupling can be generated dynamically in the trans-Planckian flow.

On the other hand, BP3 and BP4 in Table 1 feature $y_N^* = 0$ along irrelevant directions, similarly to y_ν^* . These cases allow for the interesting possibility that the sterile-neutrino Yukawa coupling sits tight at the irrelevant Gaussian fixed point $y_N^* = 0$. The Majorana mass is never generated, and its absence is protected along the entire RG flow by quantum scale invariance. The theory thus supports Dirac neutrinos with mass $m_\nu \sim y_\nu\nu_H/\sqrt{2}$, where the required minuscule Yukawa coupling is generated dynamically.

In alternative, the theory might originate from a UV-attractive y_N fixed point, and dynamically flow towards the IR-attractive one, thus generating – besides an arbitrarily small y_ν – also an arbitrarily small y_N . This case supports the existence of a Majorana mass, but the latter may be naturally decoupled from the size of v_S and the constraints on the scalar potential. Thus, BP3 and BP4 may additionally provide a natural framework for accommodating the phenomenologically interesting possibility of pseudo-Dirac neutrinos.

4.1 Scalar potential

The tree-level scalar potential of the gauged $B - L$ model is given by

$$V(H, S) = m_H^2 H^\dagger H + m_S^2 S^\dagger S + \lambda_1 (H^\dagger H)^2 + \lambda_2 (S^\dagger S)^2 + \lambda_3 (H^\dagger H) (S^\dagger S), \quad (12)$$

where H is the SM-like Higgs $SU(2)_L$ doublet, which is neutral under $U(1)_{B-L}$, and S is a complex scalar SM singlet, charged under $U(1)_{B-L}$ with $Q_S = 2$. The spontaneous breaking of $U(1)_{B-L}$ generates the mass of the abelian Z' gauge boson, which is approximately proportional to the vev along the S direction: $m_{Z'} \approx 2 g_X v_S$.

Since $v_S \gg v_H = 246$ GeV, as a consequence of stringent LHC lower bounds on the mass of new Z' particles, the two directions of the scalar potential effectively decouple. The vev v_S may arise from the presence of a large mass m_S^2 in Eq. (12). However, we rather decide to investigate the well-known possibility that the scalar potential of the $B - L$ model develop its vevs from dimensional transmutation [66–70], through the usual Coleman-Weinberg mechanism. In particular, we next discuss whether the possibility of developing a radiatively generated minimum is consistent with the benchmark points in Table 1 and with AS in general.

Let us define $\phi \equiv \sqrt{2} \text{Re}(S)$, and project the potential to the ϕ direction. The corresponding, RGE-improved Coleman-Weinberg potential in the $B - L$ model reads

$$V(\phi) = \frac{1}{2} m_S^2(t) \phi^2 + \frac{1}{4} \lambda_2(t) \phi^4 + \frac{1}{128 \pi^2} [20 \lambda_2^2(t) + 96 g_X^4(t) - 48 y_N^4(t)] \phi^4 \left(-\frac{25}{6} + \ln \frac{\phi^2}{\mu^2} \right), \quad (13)$$

where $t = \ln \mu$ is the renormalization scale. The potential can develop a minimum due to a large finite 1-loop contribution. If, at the scale μ , one finds

$$\lambda_2 \approx \frac{1}{55} \left(12 \pi^2 - 2 \sqrt{-3630 g_X^4 + 1815 y_N^4 + 36 \pi^4 + 330 \pi^2 m_S^2 / \mu^2} \right), \quad (14)$$

the minimum resides at $v_S \approx \mu$.

One may categorize the outcome of the eventual FRG calculation of f_λ – the UV parameter that induces the fixed points of the beta functions in the scalar sector, equivalent to f_g and f_y in the gauge-Yukawa system. We considered in Ref. [37] two broad classes inducing different qualitative behavior:

Case A. $f_\lambda \ll -2$. Under this condition we find the Gaussian fixed point $\tilde{m}_H^{2*} = 0$, $\tilde{m}_S^{2*} = 0$, fully irrelevant. The potential of Eq. (12) becomes thus scale-invariant and it remains so at all scales, protected by quantum scale symmetry.

In the context of the $B - L$ model with Coleman-Weinberg potential, conformal symmetry makes the model very predictive. However, following the flow of λ_2 from the Planck scale to the low energy one obtains large negative values (e.g., $\lambda_2 = -0.56$ at $v_S = 10^5$ GeV), which end up destabilizing the scalar potential from below in the S direction.

Case B. $f_\lambda \gg 0$. In this case the fixed points of the scalar potential are relevant. Masses and quartic couplings cannot be predicted from UV considerations, as any adopted value is eventually consistent with AS. For the purposes of this paper, which is phenomenological in spirit, we assume that this is what happens, which allows us to connect the asymptotically safe fixed point in the deep UV to any desired Planck-scale boundary condition for the quartic couplings.

5. Gravitational waves

It has long been known that a GW signal from the $B - L$ FOPT can be strong enough to allow detection in new-generation interferometers [71–76]. It is therefore enticing to investigate predictions for GWs associated with the boundary conditions from trans-Planckian AS discussed in Sec. 4.

In the presence of a hot plasma in the early Universe, the effective scalar potential receives thermal corrections [77, 78], which generate a thermal barrier between the false ($\phi = 0$) and true ($\phi = v_S$) $B - L$ vacua. Tunneling from the former to the latter can then proceed through bubble nucleation [79, 80], leading to the generation of GWs.

The GW physics is governed by several parameters that mainly depend on the shape of the effective thermal potential and on the bubbles' profile. These are the latent heat α , the nucleation speed β , and the reheating temperature T_{rh} . The present-day GW signal is then characterized by the peak amplitude $\Omega^{\text{peak}}(\alpha, \beta, T_{\text{rh}})$ and the peak frequency $f^{\text{peak}}(\alpha, \beta, T_{\text{rh}})$,

$$h^2 \Omega_{\text{GW}}(f) = h^2 \Omega^{\text{peak}} \times \mathcal{F}(f/f^{\text{peak}}), \quad (15)$$

where $h = H_0/(100 \text{ km/s/Mpc})$ is the present-day dimensionless Hubble parameter and \mathcal{F} is a function of the frequency f .

We calculate the GW spectra for the benchmark points listed in Table 1. The tree-level scalar mass m_S^2 plays an important role in the phenomenological prediction. In Case A of Sec. 4.1 we considered $f_\lambda \ll -2$, so that $m_S^2 = 0$ is protected by quantum scale symmetry. Scale invariance is typically associated with strong supercooling, and may give rise to large GW signals [81–85]. The presence of Yukawa couplings $y_N \neq 0$ is another key factor determining the properties of the FOPT. The height of the thermal barrier is directly proportional to y_N , forcing the nucleation and percolation temperature to be lower than with $y_N = 0$ and therefore enhancing the impact of supercooling. This effect is particularly prominent for BP2, where we find that $T_p < 0.1$ GeV, independently of the chosen value of v_S . At $T \approx 0.1$ GeV the QCD phase transition takes place, a case that we did not analyze in our study.

We observe the same behavior in BP1 at $v_S = 10^9$ GeV. Conversely, for BP1 at $v_S = 10^5, 10^7$ GeV, the non-zero value of y_N sets the nucleation/percolation temperature at several GeV. At the same time, $y_N \neq 0$ makes the Coleman-Weinberg minimum shallower, so that the probability of tunneling at a given temperature is reduced. As a result, the nucleation termination condition is

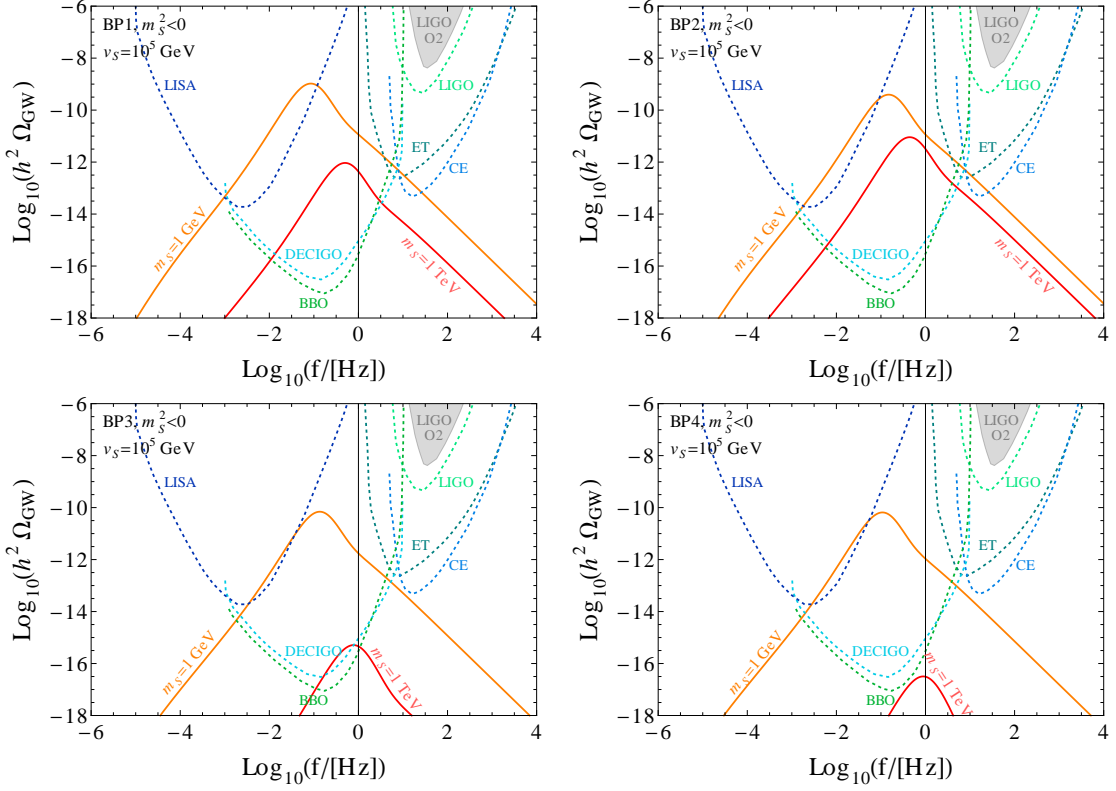


Figure 3: Gravitational-wave spectra of the benchmark points listed in Table 1, for two selected values of the scalar mass parameter $m_S \equiv \sqrt{|m_S^2|}$ (solid lines). The scalar vev is set at $v_S = 10^5$ GeV. Also shown are the sensitivity curves for various GW interferometers (dotted lines).

not satisfied. Those effects combined lead us to the conclusion that no GW signal can be observed for BP1 and BP2 with $m_S^2 = 0$.

BP3 and BP4 are characterized by relatively small values of the $B-L$ coupling. The Coleman-Weinberg minimum is thus rather shallow, indicating low decay rate of the false vacuum, $S_3(T)/T \gg 100$. As a result, there is no FOPT in BP3 and BP4 when $m_S^2 = 0$ for all chosen values of v_S .

On the other hand, we discussed in Sec. 4.1 that the purely conformal case, $m_S^2 = 0$, may be in strong tension with the requirement of a stable (bounded from below) scalar potential at the chosen scales v_S . A more natural possibility in the quantum gravity setup is thus that the mass terms in Eq. (12) remain relevant parameters, in agreement with their canonical scaling, a situation described in Case B of Sec. 4.1. No quantum scale symmetry can in those cases prevent the presence of tree-level masses in the Lagrangian. We consider here the situation where $m_S^2 < 0$ is small enough not to interfere with the generation of a thermal barrier but large enough to enhance the decay rate of the false vacuum, thus triggering the phase transition at nucleation (and percolation) temperature larger than it would be required in the conformal case.

We present in Fig. 3 the expected GW signal at $v_S = 10^5$ GeV for the four benchmark points in Table 1 given two selected values of the mass: $\sqrt{|m_S^2|} = 1$ GeV (yellow) and $\sqrt{|m_S^2|} = 1$ TeV (red). The signal is confronted with integrated sensitivity curves for the Big-Bang Observer (BBO) [86, 87], Cosmic Explorer (CE) [88], Deci-Hertz Interferometer Gravitational-Wave Observatory (DE-

CIGO) [89, 90], Einstein Telescope (ET) [91, 92], Laser Interferometer Gravitational-Wave Observatory (LIGO) [93–95] and Laser Interferometer Space Antenna (LISA) [96, 97], which are shown as dotted curves. Gray region in the upper part of the plot indicates the current exclusion bound by the LIGO-VIRGO O2 run [98, 99].

Benchmark point BP1 is shown in the upper left panel and BP2 in the upper right panel. Despite obtaining in the two cases quite different predictions from the fixed-point analysis, we observe similar GW amplitudes and frequencies. This is because the phase transitions are triggered by the mass term, so that they feature, given equivalent $m_S^2 < 0$, a similar, relatively fast nucleation speed β . Analogous behavior is observed for BP3 in the lower left panel and BP4 in the lower right panel, with the biggest difference with respect to BP1 and BP2 being that the signal amplitude and frequency show greater sensitivity to the m_S^2 spread. This is due to the much lesser depth of the minimum in BP3 and BP4, which enhances the impact of a finite m_S^2 on the bounce action. All in all, we observe strong similarities of signatures in the four cases. We are thus forced to conclude that different fixed points cannot be distinguished with a detection of GWs from FOPTs. Equally bleak are the prospects of distinguishing the Majorana vs. Dirac nature of the neutrino with this method, since the fixed points with $y_N^* \neq 0$ and those with $y_N^* = 0$ show very similar spectra, shaped in all cases by the m_S^2 relevant parameter.

6. Conclusions

In Refs. [36, 37] we considered the dynamical generation of an arbitrarily small neutrino Yukawa coupling based on the existence of Gaussian IR-attractive fixed points of the trans-Planckian RG flow. While in the first of those studies the low-energy theory that is completed in the UV with boundary conditions consistent with asymptotically safe quantum gravity was the SM with three right-handed neutrinos (SMRHN), in the second study we focused on the well-known gauged $B - L$ model.

The $B - L$ model offers several advantages with respect to the SMRHN. Some are well established – like requiring the existence of right-handed neutrino spinor fields based on the cancellation of gauge anomalies and the reliance on gauge rather than global or accidental symmetries – others apply more directly to the realm of trans-Planckian AS and the quantum-gravity nature of the UV completion. We have shown that the $B - L$ model may justify a richer phenomenology in the context of neutrino mass-generation, since it seems to be able to accommodate quite naturally each and every feature that neutrinos may eventually show experimentally: purely Dirac neutrinos, pseudo-Dirac neutrinos, and Majorana neutrinos. The see-saw scale too, being a canonically relevant parameter of the Lagrangian, can freely assume any desired value.

Among the several interesting signatures of the model, we investigated in detail the generation of gravitational waves from FOPTs. For our four different benchmark points we found that, while it will be easy to observe a clear signal in future interferometers, it will prove extremely more challenging to be able to discern different fixed points – their gauge and Yukawa coupling values and, more importantly, the Majorana vs. Dirac nature of the neutrino – from one another. This is because an explicit mass term in the effective scalar potential is necessary to trigger the $B - L$ phase transition. While this is a welcome feature for the theoretical consistency of the model – scalar masses are relevant parameters within the trans-Planckian UV completion so that conformal

symmetry is not a property enforced by RG running – it also makes the GW spectrum extremely sensitive to parameters that cannot, by their own nature, be predicted from UV considerations.

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