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Triple-leptoquark interactions for tree- and loop-level proton decays

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I present a study of the triple-leptoquark interactions within a context of matter stability, when these interactions cause proton to decay via both the tree- and one-loop level topologies. I show that the one-loop level processes are much more relevant when compared to the tree-level ones despite the usual loop-suppression factor. To support this claim, I analyse the triple-leptoquark interaction effects on the proton stability within one representative scenario, where the scenario in question simultaneously features a tree-level topology that yields three-body proton decay $p \to e^+e^+e^-$ and a one-loop level topology that induces two-body proton decays $p \to \pi^0 e^+$ and $p \to \pi^+ \bar{\nu}$. I also provide a comprehensive list of the leading-order proton decay channels for all cubic and quartic contractions that generate triple-leptoquark interactions of interest, where in the latter case one of the scalar multiplets is the Standard Model Higgs doublet.

Corfu Summer Institute 2023 "School and Workshops on Elementary Particle Physics and Gravity" (CORFU2023)
23 April - 6 May , and 27 August - 1 October, 2023
Corfu, Greece

[*] Speaker

PoS(CORFU2023)072

1. Introduction

In this work I present a study [1] of one particular class of proton decay processes that requires existence of either two or three scalar leptoquark multiplets in addition to the Standard Model (SM) particle content. The decay processes in question are based on two specific triple-leptoquark interaction topologies that are shown in Fig. 1, where q's and ℓ's denote generic quarks and leptons of the SM, while $\langle H \rangle$ stands for a vacuum expectation value of the SM Higgs boson doublet H. The scalar leptoquark states Δ^Q, $\Delta^{Q'}$, and $\Delta^{Q''}$ in Fig. 1 carry electric charges Q, Q', and Q'', respectively, and can originate, as I discuss later on, from either two or three different leptoquark multiplets. Note that κ is a generic cubic parameter of the Δ^Q-$\Delta^{Q'}$-$\Delta^{Q''}$ vertex in Fig. 1, whereas λ stands for a dimensionless quartic coupling of the Δ^Q-$\Delta^{Q'}$-$\Delta^{Q''}$-$\langle H \rangle$ vertex.

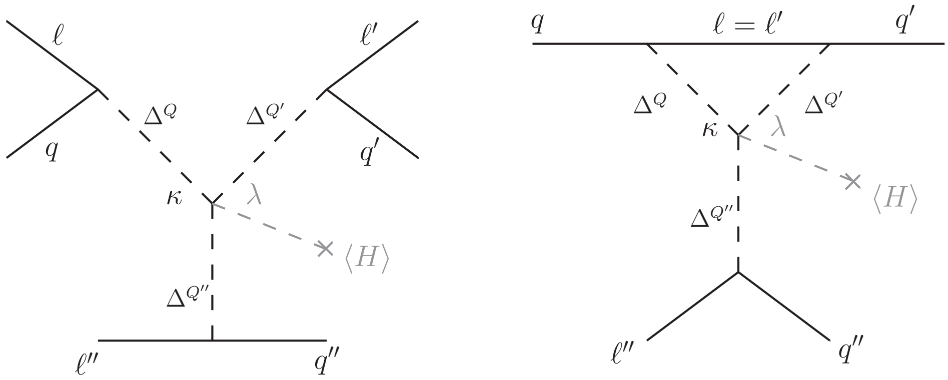

Figure 1: Two distinct proton decay topologies, with or without a Higgs vacuum expectation value insertion, generated by the triple-leptoquark interactions. q's and ℓ's denote generic quarks and leptons of the SM while Δ^Q, $\Delta^{Q'}$, and $\Delta^{Q''}$ are scalar leptoquark mass eigenstates with electric charges Q, Q', and Q'', respectively.

Both topologies of Fig. 1 have two different realisations. One is with and the other without the contraction with the Higgs boson doublet, where the diagrams that correspond to the former scenario include a vacuum expectation value insertion that is rendered in grey in Fig. 1.

It is possible to have a new physics scenario where the tree-level proton decay topology exists but the one-loop level one does not. This happens, for instance, if the leptoquarks Δ^Q and $\Delta^{Q'}$ couple to different leptons [2, 3], i.e., whenever $\ell \neq \ell'$ in Fig. 1. However, it is also possible to have a scenario where the tree-level proton decay topology is completely absent whereas the one-loop level one is not only present but also additionally enhanced due to propagation of, for example, the tau lepton in the loop. This study is especially applicable whenever $\ell = \ell'$ and, consequentially, $q = q'$ in Fig. 1.

Scalar leptoquark multiplets relevant for this study are specified in Table 1, where I also explicitly denote transformation properties of these multiplets under the SM gauge group $SU(3) \times SU(2) \times U(1)$. The notation that I use in Table 1 is self-explanatory and closely follows the notation of a contemporary review of the leptoquark phenomenology [4]. I suppress both the $SU(3)$ and $SU(2)$ indices in Table 1 for compactness and opt to show the flavor indices $i, j (= 1, 2, 3)$ instead. I furthermore use $\vec{\tau} = (\tau_1, \tau_2, \tau_3)$ to denote Pauli matrices and introduce $\vec{S}_3 = (S_3^1, S_3^2, S_3^3)$ for the

Leptoquark multiplets	Yukawa interactions
$R_2 = (\mathbf{3}, \mathbf{2}, 7/6)$	$-(y^L_{R_2})_{ij}\, \bar{u}_{R\,i} R_2 i\tau_2 L_j + (y^R_{R_2})_{ij}\, \bar{Q}_i R_2 e_{R\,j} + \text{h.c.}$
$\tilde{R}_2 = (\mathbf{3}, \mathbf{2}, 1/6)$	$-(y^L_{\tilde{R}_2})_{ij}\, \bar{d}_{R\,i} \tilde{R}_2 i\tau_2 L_j + \text{h.c.}$
$S_1 = (\bar{\mathbf{3}}, \mathbf{1}, 1/3)$	$(y^L_{S_1})_{ij}\, \bar{Q}^C_i i\tau_2 S_1 L_j + (y^R_{S_1})_{ij}\, \bar{u}^C_{R\,i} S_1 e_{R\,j} + \text{h.c.}$
$S_3 = (\bar{\mathbf{3}}, \mathbf{3}, 1/3)$	$(y^L_{S_3})_{ij}\, \bar{Q}^C_i i\tau_2 (\vec{\tau} \cdot \vec{S}_3) L_j + \text{h.c.}$
$\tilde{S}_1 = (\bar{\mathbf{3}}, \mathbf{1}, 4/3)$	$(y^R_{\tilde{S}_1})_{ij}\, \bar{d}^C_{R\,i} \tilde{S}_1 e_{R\,j} + \text{h.c.}$

Table 1: Scalar leptoquark multiplets and their interactions with the SM quark-lepton pairs.

$SU(2)$ components of the S_3 leptoquark multiplet. Throughout this work I consider only those scenarios where scalar leptoquark multiplets couple solely to the quark-lepton pairs.

The manuscript is organised as follows. In Sec. 2 I provide a comprehensive list of the leading-order proton decay channels for all non-trivial cubic and quartic contractions involving three scalar leptoquark multiplets that generate triple-leptoquark interactions of interest, where in the latter case one of the scalar multiplets is the Higgs boson doublet of the SM. In Sec. 3 I demonstrate that the one-loop level topology is much more relevant than the tree-level one when it comes to the proton decay signatures. There I also explicitly show how to extract limit on the energy scale associated with both of these topologies using the most accurate theoretical input and the latest experimental data on partial proton decay lifetimes. I briefly conclude in Sec. 4.

2. Classification

I consider extensions of the SM particle content with up to three different scalar leptoquark multiplets generically denoted with Δ, Δ', and Δ'' and study all possible cubic and quartic contractions of the generic forms Δ-Δ'-Δ'' and Δ-Δ'-Δ''-H, respectively, that yield triple-leptoquark interactions Δ^Q-$\Delta^{Q'}$-$\Delta^{Q''}$ and Δ^Q-$\Delta^{Q'}$-$\Delta^{Q''}$-$\langle H\rangle$. The aim is to specify the main tree- and one-loop level proton decay channels with topologies of Fig. 1 that can originate from these types of interactions and the associated Yukawa couplings of Table 1. My convention for the transformation properties of the Higgs boson doublet under the SM gauge group $SU(3) \times SU(2) \times U(1)$ is such that $H = (\mathbf{1}, \mathbf{2}, 1/2)$, where I denote its vacuum expectation value with $\langle H\rangle = (0 \;\; v/\sqrt{2})^T$.

If one only demands invariance of the cubic and quartic contractions under the $SU(2) \times U(1)$ part of the SM gauge group, one obtains the following potentially viable terms: $\tilde{R}_2$-$\tilde{R}_2$-S_1^* [5], $\tilde{R}_2$-$\tilde{R}_2$-S_3^* [5], R_2-$\tilde{R}_2$-$\tilde{S}_1^*$ [5], $\tilde{R}_2$-$\tilde{R}_2$-$\tilde{R}_2$-H^* [6], S_1-S_1-R_2^*-H [7], S_1-S_3-R_2^*-H [8], S_3-S_3-R_2^*-H [2], S_1-$\tilde{S}_1$-R_2^*-H^* [8], S_3-$\tilde{S}_1$-R_2^*-H^* [8], S_1-S_1-$\tilde{R}_2^*$-H^* [7], S_1-S_3-$\tilde{R}_2^*$-H^* [8], and S_3-S_3-$\tilde{R}_2^*$-H^* [8]. A thing to note is that it is always possible to replace S_1's with S_3's and vice versa in aforementioned contractions. If one furthermore demands invariance of these contractions under the $SU(3)$ gauge symmetry of the SM, one can demonstrate that the contractions $\tilde{R}_2$-$\tilde{R}_2$-$\tilde{R}_2$-H^*, S_1-S_1-R_2^*-H, $\tilde{R}_2$-$\tilde{R}_2$-S_3^*, and S_1-S_1-$\tilde{R}_2^*$-H^* all yield zero [8]. These contractions vanish due to a simple fact that they all come out to be symmetric under the exchange of two identical electric charge eigenstates which is in direct conflict with the antisymmetric nature of these contractions in the $SU(3)$ space. Of course,

	$SU(3)\times SU(2)\times U(1)$ level	$SU(3)\times U(1)_{\mathrm{em}}$ level	Ref.
(a)	$\kappa\tilde{R}_2^T i\tau_2\tilde{R}_2 S_1^*$	$-2\kappa\epsilon_{abc}\tilde{R}_{2a}^{-1/3}\,\tilde{R}_{2b}^{2/3}\,S_{1c}^{-1/3}$	[5]
(b)	$\kappa R_2^T i\tau_2\tilde{R}_2\tilde{S}_1^*$	$\kappa\epsilon_{abc}\left(R_{2a}^{5/3}\,\tilde{R}_{2b}^{-1/3}\,\tilde{S}_{1c}^{-4/3}-R_{2a}^{2/3}\,\tilde{R}_{2b}^{2/3}\,\tilde{S}_{1c}^{-4/3}\right)$	[5]
(c)	$\lambda H^\dagger i\tau_2(\vec{\tau}\cdot\vec{S}_3)^* i\tau_2 R_2 S_1^*$	$\lambda\frac{v}{\sqrt{2}}\epsilon_{abc}\left(-S_{3a}^{-1/3}\,R_{2b}^{2/3}\,S_{1c}^{-1/3}+\sqrt{2}S_{3a}^{-4/3}\,R_{2b}^{5/3}\,S_{1c}^{-1/3}\right)$	[8]
(d)	$\lambda H^\dagger i\tau_2(\vec{\tau}\cdot\vec{S}_3)^*(\vec{\tau}\cdot\vec{S}_3)^* i\tau_2 R_2$	$\lambda v\sqrt{2}\epsilon_{abc}\left(\sqrt{2}S_{3a}^{-1/3}\,S_{3b}^{-4/3}\,R_{2c}^{5/3}-S_{3a}^{-4/3}\,S_{3b}^{2/3}\,R_{2c}^{2/3}\right)$	[2]
(e)	$\lambda H^T i\tau_2 R_2 S_1^*\tilde{S}_1^*$	$-\lambda\frac{v}{\sqrt{2}}\epsilon_{abc}R_{2a}^{5/3}\,S_{1b}^{-1/3}\,\tilde{S}_{1c}^{-4/3}$	[8]
(f)	$\lambda H^T(\vec{\tau}\cdot\vec{S}_3)^* i\tau_2 R_2\tilde{S}_1^*$	$\lambda\frac{v}{\sqrt{2}}\epsilon_{abc}\left(\sqrt{2}S_{3a}^{2/3}\,R_{2b}^{2/3}\,\tilde{S}_{1c}^{-4/3}+S_{3a}^{-1/3}\,R_{2b}^{5/3}\,\tilde{S}_{1c}^{-4/3}\right)$	[8]
(g)	$\lambda H^T(\vec{\tau}\cdot\vec{S}_3)^* i\tau_2\tilde{R}_2 S_1^*$	$\lambda\frac{v}{\sqrt{2}}\epsilon_{abc}\left(\sqrt{2}S_{3a}^{2/3}\,\tilde{R}_{2b}^{-1/3}\,S_{1c}^{-1/3}+S_{3a}^{-1/3}\,\tilde{R}_{2b}^{2/3}\,S_{1c}^{-1/3}\right)$	[8]
(h)	$\lambda H^\dagger(\vec{\tau}\cdot\vec{S}_3)^*(\vec{\tau}\cdot\vec{S}_3)^* i\tau_2\tilde{R}_2$	$\lambda v\sqrt{2}\epsilon_{abc}\left(\sqrt{2}S_{3a}^{2/3}\,S_{3b}^{-1/3}\,\tilde{R}_{2c}^{-1/3}+S_{3a}^{-4/3}\,S_{3b}^{2/3}\,\tilde{R}_{2c}^{2/3}\right)$	[8]

Table 2: Cubic and quartic leptoquark multiplet contractions at the $SU(3)\times SU(2)\times U(1)$ level and the associated triple-leptoquark interactions at the $SU(3)\times U(1)_{\mathrm{em}}$ level.

it is always possible to have a new physics scenario where the scalars that transform in the same manner under the SM gauge group are not identical to each other. If that is the case, one would need to revisit those contractions that otherwise would have trivially vanished such as $\tilde{R}_2$-$\tilde{R}_2$-$\tilde{R}_2$-H^*.

I summarize all non-trivial cubic and quartic scalar contractions that yield triple-leptoquark interactions in Table 2 at both the $SU(3)\times SU(2)\times U(1)$ and $SU(3)\times U(1)_{\mathrm{em}}$ levels. There are two cubic and six quartic contractions, all in all, that generate triple-leptoquark interactions of interest.

Note that the superscript in the second column of Table 2 denotes electric charge Q of leptoquark Δ^Q in units of electric charge of positron, while a, b, and c are the leptoquark $SU(3)$ indices. I write $\Delta^{Q*}\equiv\Delta^{-Q}$ in the second column of Table 2 for simplicity, where I also define the electric charge eigenstates of S_3 leptoquark via $S_3^{1/3}=S_3^3$, $S_3^{4/3}=(S_3^1-iS_3^2)/\sqrt{2}$, and $S_3^{-2/3}=(S_3^1+iS_3^2)/\sqrt{2}$.

I can finally specify main proton decay mediating processes for both topologies of Fig. 1 using Yukawa couplings presented in Table 1 together with the cubic and quartic interaction terms given in Table 2. These results are shown in Table 3 under the assumption that the final states comprise exclusively the first generations of both quarks and charged leptons. I also write down in Table 3 relevant operators behind these processes.

3. Phenomenological analysis

I extract, in this Section, lower limits on the energy scales that are associated with the proton decay signatures due to the presence of triple-leptoquark interactions by using the latest experimental constraints on partial proton decay lifetimes for both the tree-level and one-loop level topologies of Fig. 1. I denote these energy scales simply with Λ, where Λ is a common scale for the masses of all those scalar leptoquarks that participate in proton decay processes under consideration.

Contractions	Operators	Proton decay (tree)	Proton decay (one-loop)
(a) $\tilde{R}_2$-$\tilde{R}_2$-S_1^*	$ddd\bar{e}\nu\bar{\nu}$	$p \to \pi^+\pi^+e^-\nu\bar{\nu}$	–
	$ddue\bar{e}\bar{\nu}$	$p \to \pi^+e^+e^-\nu$	$p \to \pi^+\nu$
(b) R_2-$\tilde{R}_2$-$\tilde{S}_1^*$	$ddde\bar{e}\bar{e}$	$p \to \pi^+\pi^+e^-e^+e^-$	–
	$ddue\bar{e}\bar{\nu}$	$p \to \pi^+e^+e^-\nu$	$p \to \pi^+\nu$
(c) S_1-S_3-R_2^*-H	$ddue\bar{e}\nu$	$p \to \pi^+e^+e^-\bar{\nu}$	$p \to \pi^+\bar{\nu}$
	$duue\nu\bar{\nu}$	$p \to e^+\nu\bar{\nu}$	$p \to \pi^0e^+$
	$duuee\bar{e}$	$p \to e^+e^+e^-$	$p \to \pi^0e^+$
	$uuue e\bar{\nu}$	$p \to \pi^-e^+e^+\nu$	–
(d) S_3-S_3-R_2^*-H	$ddue\bar{e}\nu$	$p \to \pi^+e^+e^-\bar{\nu}$	$p \to \pi^+\bar{\nu}$
	$duue\nu\bar{\nu}$	$p \to e^+\nu\bar{\nu}$	–
	$duuee\bar{e}$	$p \to e^+e^+e^-$	$p \to \pi^0e^+$
(e) S_1-$\tilde{S}_1$-R_2^*-H^*	$ddue\bar{e}\nu$	$p \to \pi^+e^+e^-\bar{\nu}$	$p \to \pi^+\bar{\nu}$
	$duuee\bar{e}$	$p \to e^+e^+e^-$	$p \to \pi^0e^+$
(f) S_3-$\tilde{S}_1$-R_2^*-H^*	$ddue\bar{e}\nu$	$p \to \pi^+e^+e^-\bar{\nu}$	$p \to \pi^+\bar{\nu}$
	$duue\nu\bar{\nu}$	$p \to e^+\nu\bar{\nu}$	$p \to \pi^0e^+$
	$duuee\bar{e}$	$p \to e^+e^+e^-$	$p \to \pi^0e^+$
(g) S_1-S_3-$\tilde{R}_2^*$-H^*	$ddu\nu\bar{\nu}\nu$	$p \to \pi^+\nu\bar{\nu}\bar{\nu}$	$p \to \pi^+\bar{\nu}$
	$ddue\bar{e}\nu$	$p \to \pi^+e^+e^-\bar{\nu}$	$p \to \pi^+\bar{\nu}$
	$duue\nu\bar{\nu}$	$p \to e^+\nu\bar{\nu}$	$p \to \pi^0e^+$
	$duuee\bar{e}$	$p \to e^+e^+e^-$	$p \to \pi^0e^+$
(h) S_3-S_3-$\tilde{R}_2^*$-H^*	$ddu\nu\bar{\nu}\nu$	$p \to \pi^+\nu\bar{\nu}\bar{\nu}$	$p \to \pi^+\bar{\nu}$
	$ddue\bar{e}\nu$	$p \to \pi^+e^+e^-\bar{\nu}$	–
	$duue\nu\bar{\nu}$	$p \to e^+\nu\bar{\nu}$	$p \to \pi^0e^+$

Table 3: List of all non-trivial Δ-Δ'-Δ'' and Δ-Δ'-Δ''-H contractions and associated proton decay channels at both the tree- and one-loop levels.

3.1 Tree- vs. one-loop level proton decays

Let us first estimate the expected proton decay widths for the tree- and one-loop level topologies from Fig. 1 to demonstrate that the latter dominates in all instances. I focus, for illustrative purposes, on the processes $p \to e^+e^+e^-$ and $p \to \pi^0 e^+$ that exist for most scenarios in Table 3.

I find the decay rate of the tree-level process, using naive dimensional analysis, to be

$$\Gamma(p \to e^+e^+e^-) \simeq \frac{m_p}{(16\pi)^3}\left(\frac{m_p^5 v}{\Lambda^6}\right)^2 |\lambda\, y_{ue}^2\, y_{de}|^2\,, \tag{1}$$

where y_{qe} denotes generic leptoquark couplings to electrons and valence quarks $q(= u, d)$, and Λ, again, stands for the common mass of leptoquarks that are taken to be mass-degenerate. The dependence on Λ in Eq. (1) arises from the leptoquark propagators depicted in the left panel of Fig. 1. The estimate in Eq. (1) is derived for the scenarios of the type Δ-Δ'-Δ''-H, but it also applies to the Δ-Δ'-Δ'' scenarios if one replaces κ with a product λv.

One can also perform an analogous estimate of the decay width of the corresponding one-loop level process $p \to \pi^0 e^+$ if one notes that the loop contribution in Fig. 1 can be schematically seen as an effective diquark coupling that reads

$$y_{ud} \simeq \frac{1}{16\pi^2}\frac{m_f v}{\Lambda^2}\lambda\, y_{ue} y_{de}^*\,, \tag{2}$$

where m_f can be either the valence quark mass m_q, or the mass of the lepton running in the loop, i.e., m_e in this case, depending on the specific scenario, as will be discussed below. (See also Ref. [1] for more details.) The loop-induced proton decay can then be expressed as follows

$$\Gamma(p \to \pi^0 e^+) \simeq \frac{m_p}{16\pi}\left(\frac{m_p^2}{\Lambda^2}\right)^2 |y_{ud}\, y_{ue}|^2\,. \tag{3}$$

By combining Eqs. (1) and (3), and taking the electron mass as a benchmark value for m_f, I find that

$$\frac{\Gamma(p \to e^+e^+e^-)}{\Gamma(p \to \pi^0 e^+)} \simeq \frac{1}{\pi^2}\left(\frac{m_p^3}{m_f\,\Lambda^2}\right)^2 \simeq 10^{-7}\left(\frac{m_e}{m_f}\right)^2\left(\frac{1\,\text{TeV}}{\Lambda}\right)^4, \tag{4}$$

where the dependence on the leptoquark couplings cancels out to the first approximation.

It is now transparent that proton decays faster through the one-loop level induced two-body process than through the tree-level three-body one. If one also takes into account that the experimental limit for the partial lifetime of $p \to \pi^0 e^+$ [9] is approximately of the same strength as the one for the three-body decay such as $p \to e^+e^+e^-$ [10], one can conclude with certainty that the loop-induced processes are more sensitive probes of the triple-leptoquark interactions than the tree-level ones. This estimate is based on the scenario where $m_f = m_e$ and it would be even further exacerbated if the chirality is flipped in the quark lines, leading to $m_f = m_q$, or if heavier leptons are running in the loop.

I finally opt to show how to accurately perform the extraction of a lower limit on the leptoquark masses that I denote with Λ within the framework of scenario (d) that is defined in Tables 2 and 3 for both the tree-level $p \to e^+e^+e^-$ decay, and the one-loop level decays $p \to \pi^0 e^+$ and $p \to \pi^+\bar{\nu}$. To deduce Λ I will eventually set all of the dimensionless couplings to one and focus exclusively on the leptoquark couplings to the first generation of fermions in Secs. 3.2 and 3.3.

3.2 Tree-level leptoquark mediation of $p \to e^- e^+ e^+$

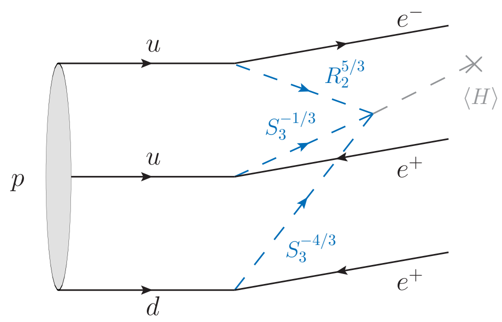

Figure 2: Tree-level diagram contributing to the process $p \to e^+ e^+ e^-$ in scenario (d) defined in Table 2.

Let us first consider decay amplitude for $p \to e^+ e^+ e^-$ and determine the corresponding decay rate for scenario (d) from Table 2, i.e., the S_3-S_3-R_2^*-H contraction. The relevant proton decay process is depicted in Fig. 2. I, again, focus on the case where leptoquarks couple only to fermions of the first generation. The starting point of this analysis is the $d = 9$ effective Lagrangian obtained after integrating out R_2 and S_3 scalar leptoquarks that reads

$$\mathcal{L}_{\text{eff}}^{(d=9)} \supset \sum_{X=L,R} \epsilon_{abc}\, C_X\, (\bar{u}_a^C P_L e)(\bar{d}_b^C P_L e)(\bar{e} P_X u_c) + \text{h.c.}\,, \tag{5}$$

where a, b, c are color indices and the effective coefficients read

$$C_L = \frac{2\sqrt{2}\lambda v}{m_{S_3}^4 m_{R_2}^2} (V^* y_{S_3}^L) y_{S_3}^L (V y_{R_2}^R)^*\,, \tag{6}$$

$$C_R = -\frac{2\sqrt{2}\lambda v}{m_{S_3}^4 m_{R_2}^2} (V^* y_{S_3}^L) y_{S_3}^L (y_{R_2}^L)^*\,. \tag{7}$$

Note that all flavor indices i and j of Table 1 are set to 1 and thus not written out. Also, V in Eqs. (6) and (7) is the CKM matrix that is taken to reside in the up-type quark sector. I keep it in this calculation for bookkeeping purposes even though it can be treated as an identity matrix in what follows.

To estimate the decay width for the process $p \to e^+ e^+ e^-$, I use the Fierz relations in Eq. (5) that produce the following scalar matrix elements

$$\begin{aligned} \epsilon_{abc} \langle 0 | (\bar{u}_a^C P_R d_b) P_L u_c | p \rangle &= \alpha_p P_R u_p\,, \\ \epsilon_{abc} \langle 0 | (\bar{u}_a^C P_L d_b) P_L u_c | p \rangle &= \beta_p P_L u_p\,, \end{aligned} \tag{8}$$

where u_p is the proton spinor, whereas $\alpha = -0.0144(3)(21)\,\text{GeV}^3$ and $\beta = +0.0144(3)(21)\,\text{GeV}^3$ are hadronic parameters that have been obtained by numerical simulations of QCD on the lattice [11].

I can finally write in full generality that

$$\Gamma(p \to e^+ e^+ e^-) = \frac{m_p^5}{6(16\pi)^3} \left(\beta_p^2 |C_L|^2 + \alpha_p^2 |C_R|^2 \right)\,. \tag{9}$$

If I furthermore take $y^L_{S_3} = y^L_{R_2} = y^R_{R_2} = \lambda = 1$ with $m_{S_3} = m_{R_2} = \Lambda$ and require that the calculated rate via Eq. (9) does not saturate the experimental limit, i.e., $\tau(p \to e^+e^+e^-) > 3.4\times10^{34}$ years [10], I find the following constraint

$$p \to e^+e^+e^- : \qquad \Lambda \geq 1.6 \times 10^2 \text{ TeV} . \tag{10}$$

Again, the lower bound on Λ in Eq. (10) corresponds to the energy scale at which the experimental limit for $p \to e^+e^+e^-$ is saturated for order one dimensionless couplings and mass-degenerate leptoquarks in scenario (d) from Table 2. Similar scale estimate has been performed before in Refs. [2, 3]. I also note that the process $p \to e^+e^+e^-$ has been discussed before in Ref. [12] in the supersymmetric context.

3.3 Loop-level leptoquark mediation of $p \to \pi^0 e^+$ and $p \to \pi^+\bar{\nu}$

I now turn attention to the two-body proton decays $p \to \pi^0 e^+$ and $p \to \pi^+\bar{\nu}$ that are induced at the one-loop level through the diagrams in the right panel of Fig. 1. I, again, consider scenario (d) of Table 2 with the assumption that the leptoquarks couple only to the first generation SM fermions.

I start by discussing the main features of the one-loop level decay topology depicted in Fig. 1. This contribution can be understood as a loop-induced diquark coupling of the leptoquark state $\Delta^{Q''}$, which then contributes to the two-body proton decay modes in the usual way, i.e., via $d = 6$ operators. However, the $SU(3)$ structure of the one-loop level topology in Fig. 1 imposes important restrictions on the possible external quark states. Since the triple-leptoquark vertex is fully antisymmetric in the $SU(3)$ indices, the one-loop level contributions vanish if the quarks q and q' in Fig. 1 are identical. In other words, the one-loop level contributions are only present if q and q' carry different flavors.

In the following, I discuss the phenomenology of scenario (d), deferring the details of the one-loop computation to Ref. [1].

$\underline{p \to \pi^0 e^+}$**:** The most interesting probe of triple-leptoquark interactions is the decay $p \to \pi^0 e^+$ due to the stringent experimental limit $\tau(p \to \pi^0 e^+)^{\text{exp}} > 2.4 \times 10^{34}$ years [9]. In the scenario I consider, there is only one diagram that contributes to this process at one loop, as depicted in Fig. 3. I assume that the leptoquark states are degenerate in mass, with $m_{R_2} = m_{S_3} \equiv \Lambda$. After integrating out the leptoquarks, I obtain an effective Lagrangian

$$\mathcal{L}^{(d=6)}_{\text{eff}} \supset C^{udeu}_{LL} \left(\overline{u}^C P_L d\right)\left(\overline{e}^C P_L u\right) + C^{udeu}_{LR} \left(\overline{u}^C P_L d\right)\left(\overline{e}^C P_R u\right) + \text{h.c.} , \tag{11}$$

where the Wilson coefficients, at the scale Λ, read

$$C^{udeu}_{LL} = \frac{\sqrt{2}\lambda}{8\pi^2}\frac{v m_e}{\Lambda^4}(V^* y^L_{S_3})\left[y^L_{S_3}(V y^R_{R_2})^* + \frac{m_d}{4m_e} y^L_{S_3}(y^L_{R_2})^*\right] , \tag{12}$$

$$C^{udeu}_{LR} = \frac{\lambda}{32\pi^2}\frac{v m_u}{\Lambda^4}(V^* y^L_{S_3})^2 (y^L_{R_2})^* . \tag{13}$$

I, again, omit flavor indices since they are all set to one. The hadronic matrix elements needed to compute $p \to \pi^0 e^+$ can be parameterized in full generality as [11, 13]

$$\left\langle \pi^0 \middle| O^{\Gamma\Gamma'} \middle| p \right\rangle = \left[W^{\Gamma\Gamma'}_0(q^2) - \frac{i\not{q}}{m_p} W^{\Gamma\Gamma'}_1(q^2)\right] P_{\Gamma'} u_p , \tag{14}$$

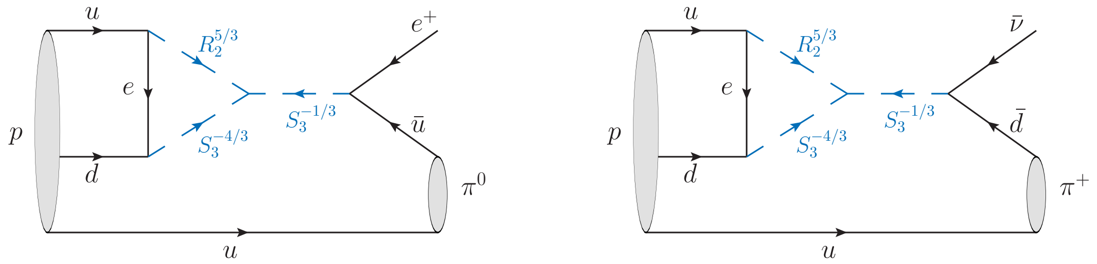

Figure 3: One-loop level diagrams contributing to $p \to \pi^0 e^+$ (left panel) and $p \to \pi^+ \bar{\nu}$ (right panel) in scenario (d) of Table 2.

where $O^{\Gamma\Gamma'} = \left(\overline{u}^C P_\Gamma d\right) P_{\Gamma'} u$, with $\Gamma, \Gamma' = R, L$. The proton spinor is, again, denoted by u_p, q stands for the momentum exchanged in this transition, and $W_0^{\Gamma\Gamma'}(q^2)$ and $W_1^{\Gamma\Gamma'}(q^2)$ are two independent hadronic form-factors. For the $p \to \pi^0 e^+$ transition, the latter form-factors can be neglected since their contributions are suppressed by m_e/m_p. The proton decay width can then be expressed in terms of the W_0 form factors as follows,

$$\Gamma(p \to \pi^0 e^+) = \frac{m_p}{32\pi}\left(1 - \frac{m_\pi^2}{m_p^2}\right)^2 \left[(W_0^{LL})^2 |C_{LL}^{udeu}|^2 + (W_0^{RL})^2 |C_{LR}^{udeu}|^2\right], \tag{15}$$

where the electron mass has been neglected. $W_0^{\Gamma\Gamma'} \equiv W_0^{\Gamma\Gamma'}(0)$ have been computed on the lattice and they are predicted, for this specific transition, to be $W_0^{LL} = 0.134(5)\,\mathrm{GeV}^2$ and $W_0^{LR} = -0.131(4)\,\mathrm{GeV}^2$ [11].

Using the expressions derived above, and assuming that $y_{S_3}^L = y_{R_2}^R = y_{R_2}^L = \lambda = 1$, I obtain that the scale Λ for $p \to \pi^0 e^+$ should satisfy

$$p \to \pi^0 e^+ : \quad \Lambda \geq 1.8 \times 10^4\,\mathrm{TeV}\,. \tag{16}$$

This limit is much more stringent than the limit derived from the tree-level proton decay $p \to e^+ e^+ e^-$ presented in Eq. (10). This is in agreement with the initial estimate from Sec. 3.1.

$\underline{p \to \pi^+ \bar{\nu}}$**:** Another interesting proton decay mode is $p \to \pi^+ \bar{\nu}$, where the experimental limit on this partial decay lifetime is currently at $\tau(p \to \pi^+ \bar{\nu})^{\mathrm{exp}} > 3.9 \times 10^{32}$ years [14]. This process can be induced by the right diagram in Fig. 3, which contributes to the effective Lagrangian

$$\mathcal{L}_{\mathrm{eff}}^{(d=6)} \supset C_{LL}^{ud\nu d}\left(\overline{u}^C P_L d\right)\left(\overline{\nu}^C P_L d\right) + C_{RL}^{ud\nu d}\left(\overline{u}^C P_R d\right)\left(\overline{\nu}^C P_L d\right) + \mathrm{h.c.}\,, \tag{17}$$

with the Wilson coefficients

$$C_{LL}^{ud\nu d} = -\frac{\sqrt{2}\lambda}{8\pi^2}\frac{v m_e}{\Lambda^4}(y_{S_3}^L)^2\left[(V y_{R_2}^R)^* + \frac{m_d}{4 m_e}(y_{R_2}^L)^*\right], \tag{18}$$

$$C_{RL}^{ud\nu d} = -\frac{\lambda}{32\pi^2}\frac{v m_u}{\Lambda^4}(V^* y_{S_3}^L)^2 (y_{R_2}^L)^*\,. \tag{19}$$

I, once again, assume degenerate leptoquark masses $m_{R_2} = m_{S_3} \equiv \Lambda$ and omit all flavor indices. By combining the isospin relation

$$\langle \pi^+ | \left(\overline{u}^C P_\Gamma d \right) P_{\Gamma'} d | p \rangle = \sqrt{2} \langle \pi^0 | \left(\overline{u}^C P_\Gamma d \right) P_{\Gamma'} u | p \rangle \,, \tag{20}$$

with Eq. (14), I find that

$$\Gamma(p \to \pi^+ \nu) = \frac{m_p}{16\pi} \left(1 - \frac{m_\pi^2}{m_p^2} \right)^2 \left[(W_0^{LL})^2 |C_{LL}^{udvd}|^2 + (W_0^{RL})^2 |C_{RL}^{udvd}|^2 \right] . \tag{21}$$

If I again assume that $y^L_{S_3} = y^L_{R_2} = y^R_{R_2} = \lambda = 1$, I find that the scale Λ should satisfy the following limit to be consistent with the current experimental input

$$p \to \pi^+ \bar{\nu} \; : \qquad \Lambda \geq 1.2 \times 10^4 \,\text{TeV} \,. \tag{22}$$

This constraint on Λ is slightly weaker than the one quoted in Eq. (16) for $p \to \pi^0 e^+$ due to a less stringent experimental limit but it is still much more relevant than the one derived from three-body proton decay $p \to e^+ e^+ e^-$ in Eq. (10).

4. Conclusions

I present a phenomenological impact of triple-leptoquark interactions on proton stability for two different decay topologies under the assumption that scalar leptoquarks of interest couple solely to the quark-lepton pairs. The first topology has the tree-level structure and it yields three-body proton decays at the leading order. The other topology is of the one-loop nature and it generates two-body proton decay processes instead. The tree-level topology has been analysed in the literature before in the context of baryon number violation while the one-loop level one has not been featured in any scientific study to date.

I demonstrate that it is the one-loop level topology that is producing more stringent bounds on the scalar leptoquark masses of the two, if and when they coexist, thus rendering the extraction of limits using the tree-level topology processes redundant. To quantitatively support this claim I extract a lower limit on the mass scale Λ that is associated with the leading order proton decay signatures for both topologies within one particular scenario using the latest theoretical and experimental input. I show that the limit on this scale for the one-loop level process $p \to \pi^0 e^+$ reads $\Lambda \geq 1.8 \times 10^4$ TeV when the charged lepton in the loop is an electron. The corresponding limit for the tree-level topology process $p \to e^+ e^+ e^-$ is $\Lambda \geq 1.6 \times 10^2$ TeV. To generate these limits I identify scale Λ with a common scale for the masses of all those leptoquarks that participate in a given baryon number violating process and set all of the dimensionless couplings to one under the assumption that leptoquarks couple solely to the first generation SM fermions.

I also specify the most prominent proton decay signatures due to the presence of all non-trivial cubic and quartic contractions involving three scalar leptoquark multiplets, where in the latter case one of the scalar multiplets is the SM Higgs doublet.

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