

Recent spectroscopy results from BESIII

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XYZ states are a series of unexpected new states observed above the open charm threshold and cannot be explained as conventional $c\bar{c}$ mesons. XYZ states are also referred to as charmonium-like states since these exotic states contain a charm-anti-charm quark pair but cannot be explained as conventional charmonium. In this proceeding, recent results in XYZ physics published by the BESIII Collaboration are reviewed.

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1. Introduction

The potential model has been effective in describing conventional charmonium states. However, in 2003, the Belle Collaboration reported a strange state referred to as $X(3872)$ [1]. Since then, numerous other exotic states (aka XYZ states) have been discovered above the $D\bar{D}$ threshold, posing challenges to the understanding of QCD [2]. Theoretically, in addition to conventional hadrons (mesons and baryons), QCD allows non-standard (exotic) hadrons, such as molecules, tetraquarks, hadro-quarkonia, hybrids, glueballs and pentaquarks [3]. The XYZ states naturally emerge as promising candidates for exotic hadrons.

The large data samples collected by the BESIII detector at the Beijing Electron Positron Collider (BEPCII) within the center-of-mass energy region of (3.00, 4.95) GeV provide a good opportunity to study XYZ states. In this proceeding, we review the recent results in XYZ physics at BESIII.

2. $X(3872)$

$X(3872)$ was observed in 2003 [1]. Twenty years later, its nature is still under discussion. Many theoretical interpretations have been proposed to explain the nature of $X(3872)$ such as a compact tetraquark, a molecule, a conventional charmonium $\chi_{c1}(2P)$, and a mixture of a molecule and a charmonium. Since $X(3872)$ is located almost exactly at the $D^{*0}\bar{D}^0$ threshold and is extremely narrow, the line shape is important to determine its nature. In 2020 LHCb released a result with a high-statistics $X(3872) \rightarrow \pi^+\pi^-J/\psi$ sample with both Breit-Wigner and Flatté models, however the two models can not be distinguished due to mass resolution [4]. Recently, a study of $X(3872) \rightarrow \pi^0 D^0 \bar{D}^0$ was released by Belle, in which a D^{*0} mass constraint was applied, thus the off-shell effect of the D^{*0} was not taken into account [5]. Based on the model described in Ref. [6], a simultaneous fit to $e^+e^- \rightarrow X(3872)$ with $X(3872) \rightarrow \pi^0 D^0 \bar{D}^0$ and $X(3872) \rightarrow \pi^+\pi^-J/\psi$ is performed by using the data sample with a total integrated luminosity of 9.0 fb^{-1} . The fit result is shown in Fig. 1. The coupled-channel effect and the width of the D^{*0} are taken into account. The lineshape mass parameter is obtained to be $M_X = (3871.63 \pm 0.13^{+0.06}_{-0.05}) \text{ MeV}$. This result improves the $X(3872)$ lineshape measurement and helps draw conclusions about its nature [7].

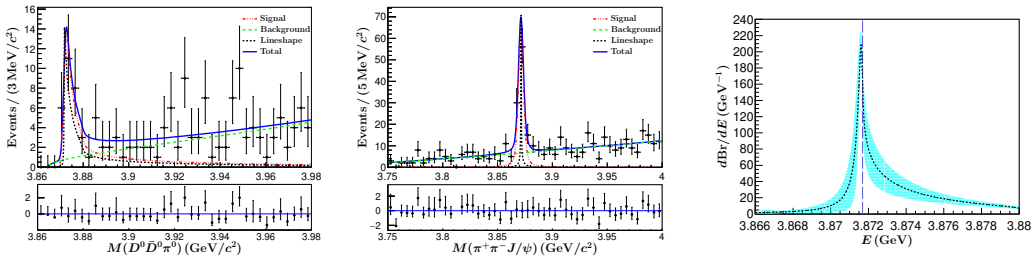


Figure 1: Distributions of $\pi^0 D^0 \bar{D}^0$ (left), $\pi^+\pi^-J/\psi$ (middle) invariant mass, and the $X(3872)$ lineshape at the best estimation (right).

In addition to exploring the $X(3872)$ via its decays, studying its production mechanisms is another way to investigate its internal structure. For the first time, BESIII group observe a new production process $e^+e^- \rightarrow \omega X(3872)$ with $X(3872)$ reconstructed via $X(3872) \rightarrow \pi^+\pi^-J/\psi$.

The study is performed based on the data samples at $\sqrt{s} = 4.661 - 4.951$ GeV with a total integrated luminosity of 4.7 fb^{-1} . The significance is determined to be 7.8σ , as shown in the left panel of Fig. 2. The \sqrt{s} -dependent Born cross section is measured, indicates that the observed $\omega X(3872)$ signals may be from decays of some non-trivial structures, i.e. a possible commonality between $X(3872)$ and Y states [8].

At the BESIII experiment, another possible production mode of $X(3872)$, although been suppressed, is via two-photon fusion. Based on 4 data samples collected at around the $X(3872)$ mass region with a total integrated luminosity of 322 pb^{-1} , a measurement of the cross section for the process $e^+e^- \rightarrow \pi^+\pi^-J/\psi$ is performed in search for the direct formation of $e^+e^- \rightarrow X(3872)$. The enhancement at $X(3872)$ is not significant. By assuming the total width of $X(3872)$ as $\Gamma_{\text{tot}}^{X(3872)} = 1.19 \pm 0.21 \text{ MeV}$, the upper limit is determined to be $\Gamma_{ee}^{X(3872)} \times \mathcal{B}(X(3872) \rightarrow \pi^+\pi^-J/\psi) < 7.5 \times 10^{-3} \text{ eV}$, which is an improvement of a factor of about 17 compared to the previous limit [9]. The result is within the prediction based on the vector meson dominance model [10].

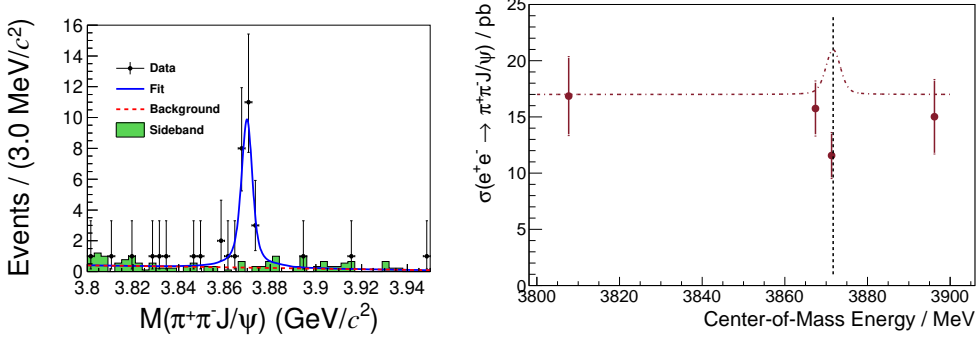


Figure 2: Fit to the $M(\pi^+\pi^-J/\psi)$ distribution in the $e^+e^- \rightarrow \omega X(3872)$ research (left) and the cross section of $e^+e^- \rightarrow \pi^+\pi^-J/\psi$ at around $X(3872)$ (right).

3. Y states

Y states are a set of vector charmonium-like states with $J^{PC} = 1^{--}$. By now, the number of observed Y states has exceeded the allowance of the conventional charmonium model. The first observed Y state is referred to as $Y(4260)$, reported by the BaBar Collaboration in the $\pi^+\pi^-J/\psi$ mass spectrum [11]. Using large data samples, in 2017 BESIII performed a cross section scan of $e^+e^- \rightarrow \pi^+\pi^-J/\psi$. The precise measurement found that $Y(4260)$ contains two structures [12]. With the total data samples collected in the charmonium mass region with a total integrated luminosity of 23fb^{-1} BESIII updates this measurement, improving the precision of $\sigma(e^+e^- \rightarrow \pi^+\pi^-J/\psi)$ [13] (Fig. 3).

By replacing the pions in the $e^+e^- \rightarrow \pi^+\pi^-J/\psi$ process with kaons, BESIII measures the cross section of $e^+e^- \rightarrow KKJ/\psi$, K is either charged or neutral kaons. In the charged channel of $e^+e^- \rightarrow K^+K^-J/\psi$, a structure with mass and width consistent with $Y(4230)$ is found. In the higher region, two new vector states referred to as $Y(4500)$ and $Y(4710)$ are found for the first time, as shown in the left panel of Fig. 4. The significance reach 8σ and 5.7σ for $Y(4500)$ and $Y(4710)$,

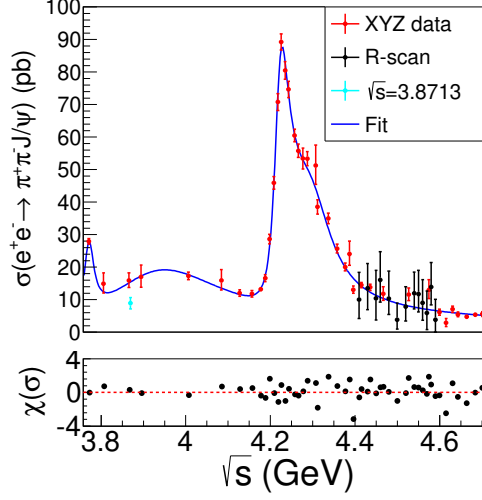


Figure 3: \sqrt{s} -dependent cross section of $e^+e^- \rightarrow \pi^+\pi^-J/\psi$.

respectively. The mass and width of the three structures are determined to be $M(Y(4230)) = (4225.3 \pm 2.3 \pm 21.5) \text{ MeV}/c^2$, $\Gamma(Y(4230)) = (72.9 \pm 6.1 \pm 30.8) \text{ MeV}$, $M(Y(4500)) = (4484.7 \pm 13.3 \pm 24.1) \text{ MeV}/c^2$, $\Gamma(Y(4500)) = (111.1 \pm 30.1 \pm 15.2) \text{ MeV}$, and $M(Y(4710)) = (4708^{+17}_{-15} \pm 21) \text{ MeV}/c^2$, $\Gamma(Y(4710)) = (126^{+27}_{-23} \pm 30) \text{ MeV}$. The hidden-charm tetraquark with strangeness Z_{c_s} is also studied and will be discussed below [14, 15]. While in the $K_S^0 K_S^0 J/\psi$ process, there is only (weak) evidence for $Y(4500)$ (1.5σ) and $Y(4710)$ (4.2σ) (Fig. 4, right panel). The average Born cross section ratio $\frac{\sigma^{\text{Born}}(K_S^0 K_S^0 J/\psi)}{\sigma^{\text{Born}}(K^+ K^- J/\psi)}$ over $\sqrt{s} = 4.128 - 4.600 \text{ GeV}$ is determined to be $0.388^{+0.035}_{-0.028} \pm 0.016$ ($0.426^{+0.038}_{-0.031} \pm 0.018$ when taking the three-body phase space into consideration). The ratio indicates an isospin violation effect in $e^+e^- \rightarrow KKJ/\psi$ with 3.1σ (1.9σ) significance [16]

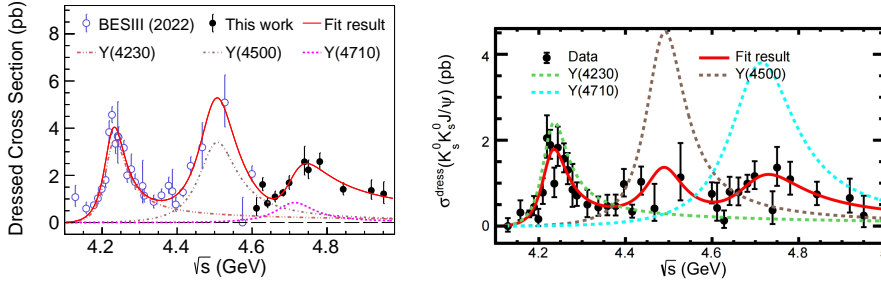


Figure 4: \sqrt{s} -dependent cross section of $e^+e^- \rightarrow K^+K^-J/\psi$ (left) and $e^+e^- \rightarrow K_S^0 K_S^0 J/\psi$ (right).

$\psi_2(3823)$ is a D-wave charmonium state with $J^{PC} = 2^{--}$, a new probe to study the vector states will be provided by selecting $\psi_2(3823)$. Recently BESIII release a measurement of $e^+e^- \rightarrow \pi^+\pi^-\psi_2(3823)$ based on the data samples with an integrated luminosity of 11.3 fb^{-1} . $\psi_2(3823)$ is constructed via partial reconstruction (missing one photon) and full reconstruction methods. $M^{\text{recoil}}(\pi^+\pi^-)$ distributions for the two reconstruction methods are shown in the first line of Fig. 5. The \sqrt{s} -dependent cross section is found nontrivial, and is described by using two coherent Breit-

Wigner resonances. The mass and width of the structures are determined to be $M(R1) = 4406.9 \pm 17.2 \pm 4.5 \text{ MeV}/c^2$, $\Gamma(R1) = 128.1 \pm 37.2 \pm 2.3 \text{ MeV}$, and $M(R2) = 4647.9 \pm 8.6 \pm 0.8 \text{ MeV}/c^2$, $\Gamma(R2) = 33.1 \pm 18.6 \pm 4.1 \text{ MeV}$, which are agree with $Y(4360)$ and $\psi(4660)$. However, a single resonance with $M(R) = 4417.5 \pm 26.2 \pm 3.5 \text{ MeV}/c^2$, $\Gamma(R) = 245 \pm 48 \pm 13 \text{ MeV}$ is also possible to interpret data, as shown in Fig. 5. For the first time the vector states decaying to D-wave charmonium state is observed. If take the model with two coherent Breit-Wigner resonances into consideration, a decay of $\psi(4660) \rightarrow \pi^+\pi^-\psi_2(3823)$ is reported with $\frac{\Gamma(\psi(4660) \rightarrow \pi^+\pi^-\psi_2(3823))}{\Gamma(\psi(4660) \rightarrow \pi^+\pi^-\psi(2S))} \sim 20\%$. The result disfavor the $f_0(980)\psi(2S)$ hadron molecule interpretation [18] for the $\psi(4660)$ nature, while also challenge the extended baryonium picture [19] and diquark-antidiquark tetraquark explanation [20]. This measurement also provides the most precise result of the mass and width of $\psi_2(3823)$, which are $M(\psi_2(3823)) = 3823.12 \pm 0.43 \pm 0.13 \text{ MeV}/c^2$ and $\Gamma(\psi_2(3823)) < 2.9 \text{ MeV}$ at the 90% C.L. [17]. The neutral channel $e^+e^- \rightarrow \pi^0\pi^0\psi_2(3823)$ is searched and reported by BESIII as well, the $M(\gamma\gamma J/\psi)$ distribution is shown in the last panel of Fig. 5 The ratio of cross section $\frac{\sigma(e^+e^- \rightarrow \pi^0\pi^0\psi_2(3823))}{\sigma(e^+e^- \rightarrow \pi^+\pi^-\psi_2(3823))}$ is determined to be $0.57 \pm 0.14 \pm 0.05$, which is consistent with the expectation from isospin symmetry. The observation of $e^+e^- \rightarrow \pi^0\pi^0\psi_2(3823)$ confirm the $\pi\pi$ system in this process comes from $f_0(500)$ instead of ρ^0 [21].

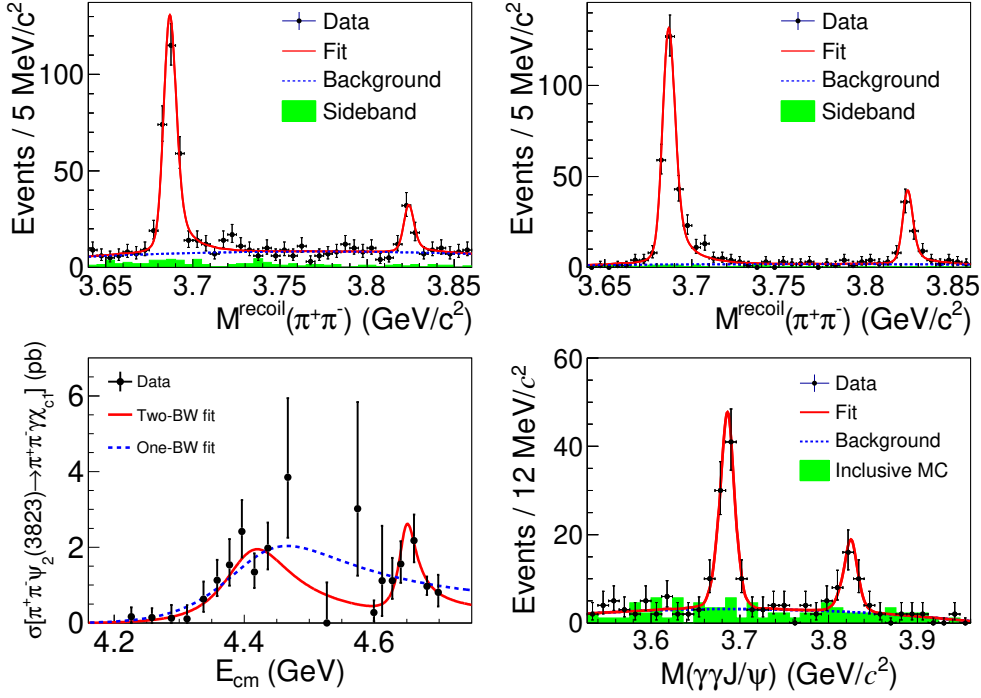


Figure 5: The first line shows $M^{\text{recoil}}(\pi^+\pi^-)$ distribution in the $e^+e^- \rightarrow \pi^+\pi^-\psi_2(3823)$ study based on partial and full reconstruction methods. The left panel in the bottom shows \sqrt{s} -dependent cross section of $e^+e^- \rightarrow \pi^+\pi^-\psi_2(3823)$. The last panel shows $M(\gamma\gamma J/\psi)$ distribution in $e^+e^- \rightarrow \pi^0\pi^0\psi_2(3823)$ study.

Using data samples with an integrated luminosity of 6.4 fb^{-1} BESIII searches the process $e^+e^- \rightarrow \gamma\phi J/\psi$. The processes of $e^+e^- \rightarrow \phi\chi_{c1,c2}$ are observed. The \sqrt{s} -dependent cross section of $e^+e^- \rightarrow \phi\chi_{c1}$ is well described with a continuum amplitude. For the $e^+e^- \rightarrow \phi\chi_{c2}$ process, there is an enhancement at around 4.7 GeV on the cross section lineshape, as shown in Fig. 6. Other

possible X -state such as $X(4140)$, $X(4274)$ and $X(4500)$ reported by the LHCb Collaboration [22] in the $\phi J/\psi$ system is searched as well. However we do not find any significant signal, so the upper limits on the cross sections at the 90% C.L. are given [23].

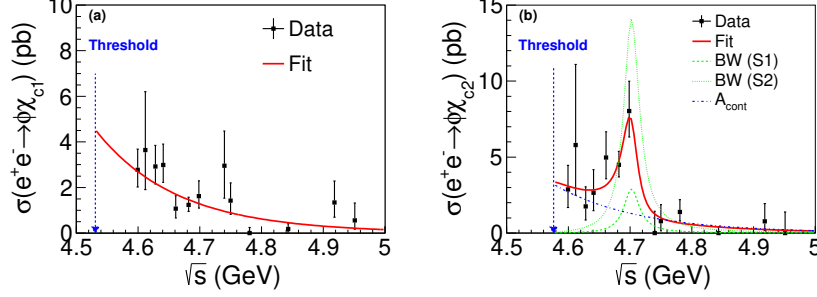


Figure 6: \sqrt{s} -dependent cross section of $e^+e^- \rightarrow \phi\chi_{c1}$ and $e^+e^- \rightarrow \phi\chi_{c2}$.

A follow-up study of $e^+e^- \rightarrow \eta J/\psi$ is released by BESIII by using the total data sets with an integrated luminosity of 22.42 fb^{-1} . The result agrees with the previous measurement. No structure is found on the cross section lineshape above 4.6 GeV, and the precision at the higher \sqrt{s} region improved. The reported $\sigma(e^+e^- \rightarrow \eta J/\psi)$ result is shown in Fig. 7. The lineshape of the measured cross sections is described with $\psi(4040)$, two additional resonances, and a non-resonant component. Mass and width of the two additional resonances are determined to be $M(R1) = 4219.7 \pm 2.5 \pm 4.5 \text{ MeV}/c^2$, $\Gamma(R1) = 80.7 \pm 4.4 \pm 1.4 \text{ MeV}$, and $M(R2) = 4386 \pm 13 \pm 17 \text{ MeV}/c^2$, $\Gamma(R2) = 177 \pm 32 \pm 13 \text{ MeV}$, consistent with the $Y(4230)$ and $Y(4360)$, respectively [24].

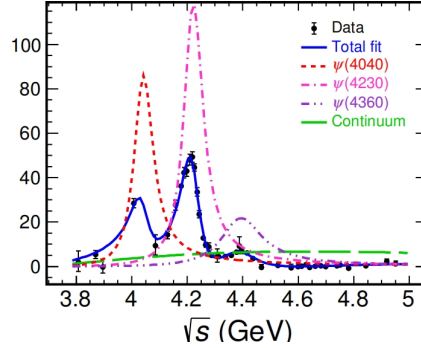


Figure 7: \sqrt{s} -dependent cross section of $e^+e^- \rightarrow \eta J/\psi$.

BESIII also contributes in the Y states measurement with open-charm final states. As shown in Fig. 8 (left), the \sqrt{s} -dependent cross section of $e^+e^- \rightarrow D^{*0}D^{*-}\pi^+$ is reported with the total integral luminosity of data sample is 17.9 fb^{-1} . Three resonances are used to describe the enhancements around 4.20, 4.47 and 4.67 GeV. The resonances have masses and widths of $M(Y(4230)) = (4209.6 \pm 4.7 \pm 5.9) \text{ MeV}/c^2$, $\Gamma(Y(4230)) = (81.6 \pm 17.8 \pm 9.0) \text{ MeV}$, $M(Y(4500)) = (4469.1 \pm 26.2 \pm 3.6) \text{ MeV}/c^2$, $\Gamma(Y(4500)) = (246.3 \pm 36.7 \pm 9.4) \text{ MeV}$, and $M(Y(4660)) = (4675.3 \pm 29.5 \pm 3.5) \text{ MeV}/c^2$, $\Gamma(Y(4660)) = (218.3 \pm 72.9 \pm 9.3) \text{ MeV}$. The second resonance is in agreement with the $Y(4500)$ firstly observed in $e^+e^- \rightarrow K^+K^-J/\psi$. Parameters of the first and third resonances agree with $Y(4230)$ and $\psi(4660)$, it is the first time $\psi(4660)$ is found in open-charm final states [25].

Cross section of the two-body open-charm final state $e^+e^- \rightarrow D_s^{*+}D_s^{*-}$ is measured based on a 17.9 fb^{-1} data sample. Three sets of fitting results with comparable goodness are found. In all cases an additional structure is found necessary to describe the lineshape at around 4.79 GeV, with statistical significance greater than 6.1σ . One of the fit result is shown in Fig. 8 (right). A unitary approach based on K-matrix formalism to fit the cross section of various exclusive channels is pointed out to be expected for the study of vector charmonium(-like) states [26].

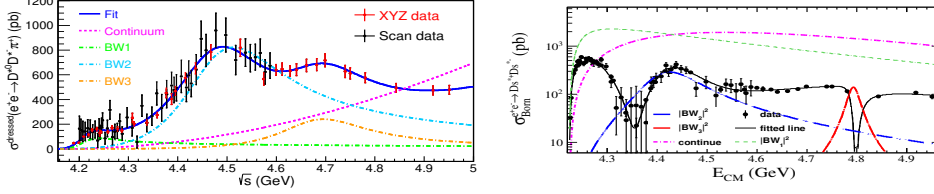


Figure 8: \sqrt{s} -dependent cross section of $e^+e^- \rightarrow D^{*0}D^{*-}\pi^+$ and $e^+e^- \rightarrow D_s^{*+}D_s^{*-}$.

4. Z_{cs} and Z'_{cs}

$Z_c^{(\prime)}$ states are states with non-zero isospin, with quark constituent of $c\bar{c}q\bar{q}'$ [27, 28]. Under SU(3) flavor symmetry, its flavour partner Z_{cs} ($c\bar{c}s\bar{q}$) is naturally expected. The charged Z_{cs} was found in 2021 by the BESIII and LHCb Collaborations [22, 29].

Recently BESIII release a research for Z_{cs}^0 via the processes $e^+e^- \rightarrow K_S^0 D_s^+ D^{*-}$ and $e^+e^- \rightarrow K_S^0 D_s^{*+} D^-$ and their charge conjugated processes with data samples at \sqrt{s} within (4.628, 4.699) GeV with an integrated luminosity of 3.8 fb^{-1} . Evidence of Z_{cs}^0 is found with significance as 4.6σ , as shown in Fig. 9 (the left panel). The mass and width are determined to be $M(Z_{cs}^0) = 3992.2 \pm 1.7 \pm 1.6 \text{ MeV}/c^2$ and $\Gamma(Z_{cs}^0) = (7.7_{-3.8}^{+4.1} \pm 4.3) \text{ MeV}$. The ratio of $\frac{\sigma(e^+e^- \rightarrow K_S^0 Z_{cs}^0)}{\sigma(e^+e^- \rightarrow K^- Z_{cs}^+)}$ is within isospin symmetry expectation. The measured mass $M(Z_{cs}^0)$ is a bit larger than $M(Z_{cs}^-) = 3982.5_{-2.6}^{+1.8} \pm 2.1 \text{ MeV}/c^2$ reported in Ref. [30]. The mass difference is within theoretical prediction [31].

Just similar as the observation of Z_{cs}^- via $e^+e^- \rightarrow K^+(D_s^- D^{*0} + D_s^{*-} D^0)$, the highly excited partner of Z_{cs}^- , Z'_{cs} , is searched by selecting $e^+e^- \rightarrow K^+ D_s^{*-} D^{*0} + c.c.$ There is only evidence with a significance of 2.1σ reported (or 3.9σ without systematic effect), $M^{\text{recoil}}(K^+)$ is shown in Fig. 9 (the middle panel). The integrated luminosity of corresponding data samples is 2.7 fb^{-1} . The upper limits of the product of $\sigma^{\text{Born}}(e^+e^- \rightarrow K^+ Z'_{cs} + c.c.) \cdot \mathcal{B}(Z'_{cs} \rightarrow D_s^{*-} D^{*0})$ are found to be at the level of $\mathcal{O}(1) \text{ pb}$ [32].

In the same analysis as the cross section measurement of $e^+e^- \rightarrow K^+ K^- J/\psi$ [15], Z_{cs} is searched in the $M(K^\pm J/\psi)$ spectrum as well. There is no significant structure observed, the statistical significance is determined to be 2.3σ with $M(Z_{cs})$, $\Gamma(Z_{cs})$ as well as the yields of all components are free, as shown in Fig. 9 (the right panel). The ratio of branching fraction is determined to be $\frac{\mathcal{B}(Z_{cs}(3985)^+ \rightarrow K^+ J/\psi)}{\mathcal{B}(Z_{cs}(3985)^+ \rightarrow (\bar{D}^0 D_s^{*+} + \bar{D}^{*0} D_s^+))} < 0.03$ at 90% C.L. The result disfavors the molecule assumption [33] while support that the $Z_{cs}(3985)^+$ and $Z_{cs}(4000)^+$ are two different states [34].

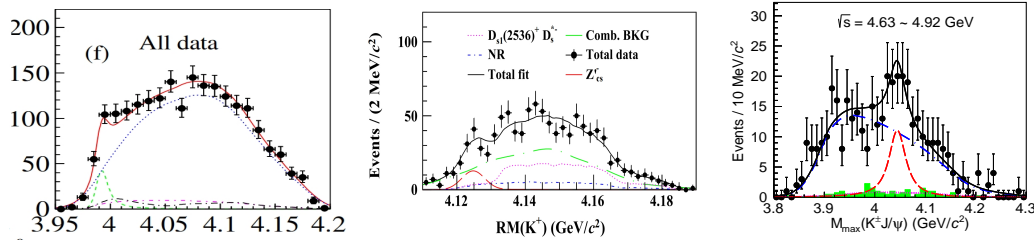


Figure 9: Z_{cs} distribution in the study of $e^+e^- \rightarrow (K_S^0 D_S^+ D^{*-} + K_S^0 D_S^{*+} D^-)$ (left), $e^+e^- \rightarrow K^+ D_s^{*-} D^{*0} + c.c.$ (middle), and $e^+e^- \rightarrow K^+ K^- J/\psi$ (right).

5. Summary

There are still puzzles, challenges and opportunities in the study of XYZ states. BEPCII collider will be upgraded in 2024, after that the luminosity in the XYZ mass region will increase by a factor of three, and the possible \sqrt{s} region will extend from 4.95 GeV to 5.6 GeV. Currently BESIII group is still quite active, more results will be released in the near future and contribute to the XYZ physics.

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