

X-ray and γ **-ray periodic analysis of the fast rotating, highly magnetized white dwarf EUVE J0317-85.5**

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We report possible X-ray and γ -ray pulsations from the fast rotating, highly magnetic white dwarf EUVE J0317-85.5. Pulsed modulations were found at periods $P=724.648\pm0.001s$ (-log(P_r)=5.60) at a significance of ~4.59 σ and P=361.685±0.001s (-log(P_r)=5.19) at ~4.18 σ in the *Fermi*-LAT γ -ray data set. These periods are close to, but significantly different from the spin period and half the spin period of the white dwarf, respectively. Although these γ -ray periods statistically exclude for instance, the optical periods (see Table [1\)](#page-6-0), the folded γ -ray light curve and folded optical light curves are in agreement which could possibly suggest that the optical and γ -rays are related. Possible pulsed X-ray emission was found at a period of P=362.82 \pm 0.04s (FAP=3.28 \times 10⁻⁵) in the ROSAT X-ray data and at a period P∼725.72s but at a low significance (FAP∼0.60). The P∼362.8s X-ray period is close to, but significantly different from the first harmonic of the spin period which was also observed in recent optical observations (see Table [1\)](#page-6-0). These results may suggest that this white dwarf display pulsar-like properties with emission of radiation from its polar cap regions. The pulsed γ -ray emission is most likely produced by electrons moving along the magnetic field lines emitting curvature radiation, while the X-rays are possibly produced by pair-produced particles flowing back towards and heating the polar cap regions of the white dwarf.

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1. Introduction

EUVE J0317-85.5 (hereafter J0317) was identified by the ROSAT Wide Field Camera (WFC) [\[1\]](#page-6-1) and Extreme Ultraviolet Explorer (EUVE, [\[2\]](#page-6-2)) surveys. [\[1\]](#page-6-1) identified this source as a highly magnetized white dwarf (WD) with a polar field strength of B=340 MG which showed optical variations at a period of $P=725.4\pm0.9$ s. These authors used a dipole model that is offset along the dipole axis by 0.2 R_{WD} in the direction of the southern magnetic pole to estimate the magnetic field strength. They also proposed that this optical modulation could be due to the rotation of the WD. [\[1\]](#page-6-1) found that J0317 is associated with another WD star, LB 9802, forming a resolved double DA (class of WD stars showing pressure-broadened hydrogen absorption lines in their spectra [\[3\]](#page-6-3)) system but could not find any evidence of accretion occurring in this system, meaning that J0317 is an isolated WD. Utilizing the variation in the wavelength-averaged circular polarization in the extreme ultraviolet data from the EUVE Deep Survey, [\[4\]](#page-6-4) determined a magnetic field strength of B=450 MG offset by 35% of the WD radius (viewed at an angle of 30°-60° to the dipole axis) and a spin period $P=725.5\pm0.8s$. [\[4\]](#page-6-4) also determined pulsations at a period of $P=362.9s$ which corresponds to half the spin period of the WD. [\[5\]](#page-6-5) further refined this spin period using optical circular spectropolarimetry and found a period of $P=725.727\pm0.001$ s. Given J0317's unusually high magnetic field and relatively fast spin period for an isolated WD, it is expected that this source behaves as a WD pulsar. A WD pulsar emits pulsed emission in a similar way as a neutron star (NS) pulsar, but the WD pulsar rotates at a much slower rate due to its large moment of inertia and its lower magnetic field $(10^3 - 10^9)$ G, (6) . The pulsed emission from pulsars are mainly due to electrons getting accelerated by huge electric potentials, ΔV_{max} , usually above the polar caps and can be estimated by the following equation [\[7\]](#page-6-7)

$$
\Delta V_{max} = \frac{B_p \Omega^2 R^3}{2c^2} \tag{1}
$$

Close to the polar cap regions, the electrons will rapidly radiate the perpendicular component of their acceleration due to the synchrotron mechanism in a timescale $\tau_s \sim 10^{-10}$ s [\[8\]](#page-7-0) leaving only the electron's parallel component of acceleration along the magnetic field lines to produce curvature photons with energy [\[9\]](#page-7-1) [\[10\]](#page-7-2) [\[11\]](#page-7-3)

$$
\epsilon_{CR} = \frac{3\hbar c \gamma_e^3}{2R_c} \tag{2}
$$

where R_c is the radius of curvature given by

$$
R_c \approx \sqrt{\frac{Rc}{\Omega}}\tag{3}
$$

and γ_e is the Lorentz factor of the electrons given by

$$
\gamma_e = \frac{e\Delta V}{m_e c^2} \tag{4}
$$

A recent study [\[8\]](#page-7-0) showed that curvature radiation in WD pulsar magnetospheres could produce γ -rays with energies $\epsilon_{\gamma} \leq 10$ GeV. The curvature photons then interact with the magnetic fields of the compact object and produce e^{\pm} pairs in the following process [\[12\]](#page-7-4) [\[13\]](#page-7-5)

$$
\gamma + B \to e^- + e^+ \tag{5}
$$

The electrons from the process could then be further re-accelerated along the open magnetic field lines to produce γ -rays which could result in a pair cascade process. The positrons, on the other hand, could be accelerated back towards the surface of the WD which could result in the emission of X-rays ([\[14\]](#page-7-6), [\[8\]](#page-7-0) and references therein).

2. Data Processing and Analysis

2.1 ROSAT X-ray Data

X-ray data of J03[1](#page-2-0)7 was obtained from the online archive HEASARC¹. J0317 was observed with the ROSAT X-ray $(0.1-2.4 \text{ keV})$ space telescope using the high resolution imager (HRI) [\[15\]](#page-7-7). Observations were made from 01/04/1996, 11:54:28.000 (UT) to 14/04/1996, 23:40:52.000 (UT). Additional observations were made from 29/08/1996, 17:49:13.00 (UT) to 21/10/1996, 11:27:42.00 (UT). The event files were barycentric corrected using the command rosbary that is part of HEAsoft^{[2](#page-2-1)} data analysis software provided by NASA. X-ray events were extracted from a region of interest (ROI) of 17" centered around J0317 (RA=03 $h17m16.1750s$, DEC=-85°32'25.45", [\[16\]](#page-7-8)). This ROI included LB 9802 because the ROSAT spacecraft could not resolve these two WDs. These events were then background subtracted using the same ROI in the same field where no source were detected to correctly model the background. Light curves binned at 100 seconds were then created using the events outlined above. These light curves were then used to search for possible periodic pulsations. The generalised Lomb-Scargle technique (GLS) [\[17\]](#page-7-9) was used to search for periodic behaviour in the X-ray data as it is more sensitive than the standard Lomb-Scargle technique ([\[18\]](#page-7-10), [\[19\]](#page-7-11)) because it takes the measurement errors into account. The GLS periodogram is shown in Figure [1.](#page-3-0)

2.2 *Fermi***-LAT Data**

Archived γ -ray data from the *Fermi*-LAT spacecraft [\[20\]](#page-7-12) were retrieved from the *Fermi* Science Support Center (FSSC^{[3](#page-2-2)}). The new Pass 8 LAT [\[21\]](#page-7-13) γ -ray data was collected from the date range 01 January 2009, 00:00:00 to 02 December 2023, 00:00:00 between the energy range 0.5-10 GeV. The data was analysed with the *Fermi* Sciencetools (version v11r5p3) software provided by FSSC. A standard unbinned likelihood analysis was performed on the γ -ray events from a ROI of 0.6[°] centered around J0317 (RA=49.31602858[°], DEC=-85.54043285[°], J2000). Only SOURCE class events (evclass=128 and evtype=3) were selected and γ -ray photons from Earth's limb were excluded by choosing a zenith angle less than 90° . High quality data were obtained by using the expression "(DATA_QUAL>0) $&&$ (LAT_CONFIG==1)" utilizing the gtm KTIME command. The background emission was modelled using the "gll_iem_v07.fits" file for the galactic diffuse emission and the "iso_P8R3_SOURCE_V3_v1.txt"^{[4](#page-2-3)} file for extra-galactic diffuse emissions. Events were then further filtered by using the tool gtsrcprob to only include photons originating from J0317 having a probability greater than 80%. These filtered event files were then used to search for pulsed γ -ray emission using the gtpsearch command. The Rayleigh periodograms are shown in Figures

¹<https://heasarc.gsfc.nasa.gov/cgi-bin/W3Browse/w3browse.pl>

²<https://heasarc.gsfc.nasa.gov/docs/software/heasoft/>

³<https://fermi.gsfc.nasa.gov/ssc/>

⁴<https://fermi.gsfc.nasa.gov/ssc/data/access/lat/BackgroundModels.html>

[2](#page-4-0) and [4.](#page-5-0) The γ -ray events were then folded using ephemeris [\[4\]](#page-6-4) and TEMPO2 [\[22\]](#page-7-14) timing analysis software with the *Fermi* plug-in [\[23\]](#page-7-15). The folded γ -ray light curves are shown in Figures [3](#page-4-0) and [5.](#page-5-0)

3. Results and Discussions

3.1 X-ray Results

Figure 1: GLS Periodogram of X-ray data showing the spin period and half the spin period of J0317. Note this periodogram has been normalized according to [\[17\]](#page-7-9). The FAP=0.60 (40%, blue dotted horizontal line) corresponds to P∼725.72s, FAP=0.01 (99%, magenta dash-dotted line) corresponds to the second highest peak in the periodogram P∼568.58s and FAP=3.28×10−⁵ (99.997%, yellow dotted line) corresponds to P∼362.82s.

The GLS periodogram is shown in Figure [1](#page-3-0) and it shows possible X-ray pulsations at a period of $P=362.82\pm0.04$ s. All uncertainty on periods were estimated using the bootstrap resampling method [\[24\]](#page-8-0). This period is in agreement with half the spin period of WD of P∼362.9s found by [\[4\]](#page-6-4) using the extreme ultraviolet data. The false-alarm probability (FAP) of this peak is FAP=3.28 \times 10⁻⁵. The FAP is the probability that a signal with no periodic component would lead to a peak in the periodogram of that height [\[25\]](#page-8-1). The FAP can be calculated by using

$$
FAP(P_n) = 1 - [1 - prob(P > P_n)]^M
$$
 (6)

where M is the number of independent frequencies (can be estimated as the number of peaks in the periodogram) and P_n is the power threshold (see [\[17\]](#page-7-9) for details). The spin period of P∼725.72s in the X-rays of the WD was found (shown by green dotted line in Figure [1\)](#page-3-0) but it was not significant (FAP∼0.60). This could possibly suggest that there might be pulsed soft X-ray emission emanating from J0317 but additional soft X-ray observations are required to confirm this result. [\[4\]](#page-6-4) suggested that the additional periodicity at half the spin period of the WD in their EUV data could be due

to surface abundance inhomogeneities at both the magnetic poles. The possible detection of soft X-ray pulsations at a similar period could suggest that the EUV and X-ray photons are produced at the same region on the WD. The X-rays could also be emitted by pair-produced particles flowing back towards the polar caps (PCs) or regions close to the PCs of the WD [\[14\]](#page-7-6).

3.2 *Fermi***-LAT -ray Results**

and lumber of Pulse phase H -test TS Event time (MJD)

Figure 2: Rayleigh Periodogram showing pulsations at spin period of WD with $-log(P_r)=5.60$ (FAP=2.51×10⁻⁶=99.9997%) at ~4.59 σ .

Figure 3: Folded γ -ray (0.5-10 GeV) light curve with spin period P=724.648s using ephemeris by [\[4\]](#page-6-4). The H-test Test Statistic is shown in the bottom panel.

The Rayleigh periodogram of the γ -ray data is shown in Figure [2](#page-4-0) with a prominent peak at a period P=724.648±0.001s with a -log(P_r)=5.60 at a significance of ~4.59 σ . This means that there is a $\approx 10^{-5.60}$ probability that this peak could be associated with noise. The significance was roughly estimated by selecting regions on either side of the Rayleigh periodogram excluding the highest peak and then calculating the average -log(P_r) of these regions. This average -log(P_r) was then subtracted from the $-log(P_r)$ of the highest peak and the result was squared to give the variance. The standard deviation was calculated by taking the square root of the variance and this standard deviation represents the significance of the possible spin period^{[5](#page-4-1)}. The periodogram was produced using the ephemeris by [\[4\]](#page-6-4) and using a sampling frequency $v=2.12\times10^{-11}Hz$ with a number of trial frequencies set to 10^6 . This sampling frequency corresponds to 0.01 times the Fourier resolution $(\nu=2.12\times10^{-9}Hz)$. The Fourier resolution is the frequency spacing of a fast Fourier transform (FFT, [\[26\]](#page-8-2)) and is given by $\Delta f = 1/T$, where T is the length of the observation. The γ -ray light curve is shown in Figure [3](#page-4-0) folded on the spin period of the WD using the same ephemeris, shows that there is possible evidence of pulsed γ -ray emission at the spin period of this WD. This emission could possibly be from electrons moving along the strong magnetic field lines above the PCs of the WD emitting curvature photons. Upon first glance, the folded light curve seems to be single-peaked at a phase of about 0.45 but further inspection reveals that there is a second, albeit faint, peak emerging at a phase of about 0.85. The H-test [\[27\]](#page-8-3) test statistic (TS) is shown in the bottom panel of Figure [3](#page-4-0) and shows cumulative power at this spin period with time.

Given the weak second peak in the folded γ -ray light curve (see Figure [3\)](#page-4-0), a search for γ -ray pulsations at half the spin period of the WD was done using the same ephemeris, sampling frequency

⁵The exact procedure was followed to calculate the significance for the possible first harmonic.

Figure 4: Rayleigh Periodogram showing pulsations at half the spin period of WD with $-log(P_r)=5.19$ (FAP=6.46×10⁻⁶=99.9994%) at $~\sim 4.18\sigma.$

Figure 5: Folded γ -ray (0.5-10 GeV) light curve with half the spin period P=361.685s using ephemeris by [\[4\]](#page-6-4). The H-test Test Statistic is shown in the bottom panel.

and number of trials. Pulsations were found at a period of $P=361.685\pm0.001$ s with $-\log(P_r)=5.19$ at a significance of ~[4.](#page-5-0)18 σ shown in Figure 4. The single-peaked folded γ -ray light curve is shown in Figure [5.](#page-5-0) The corresponding H-test TS at this period is also shown at the bottom panel of Figure [5.](#page-5-0) For reasons that are still under investigation, both the spin period and its first harmonic in the γ -rays are lower than the ultraviolet [\[4\]](#page-6-4) and recent optical periods (see Table [1\)](#page-6-0). It is interesting that we found possible γ -ray pulsations close to but significantly different from the spin period and its first harmonic because this could possibly suggest that we are receiving γ -ray emission from both poles of the WD.

4. Conclusion

In this paper, we analyzed archival soft X-ray (0.1-2.4keV) data of EUVE J0317-85.5 from the ROSAT space telescope and found possible evidence of pulsed X-ray emission at a period of P=362.82±0.04s (FAP=3.28×10⁻⁵=99.997%) which corresponds to half the spin period of the white dwarf (P≈725s). Possible pulsations were also found at a spin period of P∼725.72s but it was not significant (FAP∼0.60=40%) above the background noise. This could suggest that X-ray emission emanates from the white dwarf poles or that there might be surface abundance inhomogeneities on the white dwarf as suggested by [\[4\]](#page-6-4). The pulsed X-ray emission could also be due to the heating of the polar caps by the backward flow of pair-produced charged particles but additional high resolution soft X-ray observations are required to strengthen this claim. We also analyzed *Fermi*-LAT data in the energy range 0.5-10 GeV from 2009 to 2023 and found possible γ -ray pulsations at a period P=724.648±0.001s (-log(P_r)=5.60, FAP=2.51×10⁻⁶=99.9997%, ~4.59 σ) and a period P=361.685±0.001s (-log(P_r)=5.19, FAP=6.46×10⁻⁶=99.9994%, ~4.18 σ) which was close to but significantly different from the spin period and its first harmonic of the white dwarf. See Table [1](#page-6-0) for a summary of pulsed periods. The possible detection of pulsed γ -ray emission at both these periods could suggest that the white dwarf is orientated nearly orthogonal to our line of sight. We suggest that the γ -ray pulsations from EUVE J0317-85.5 could be due to curvature radiation given

Wavebands	$P_{spin}(s)$	$P_{spin/2}(s)$	$\text{FAP}(P_{spin})$	$\text{FAP}(P_{spin/2})$
$X-rays (0.1-2.4 keV)$	725.72 ± 0.20	362.82 ± 0.04	0.60	3.28×10^{-5}
γ -rays (0.5-10 GeV)	724.648 ± 0.001	361.685 ± 0.001	2.51×10^{-6}	6.46×10^{-6}
Optical	725.76+0.14	$362.46 + 0.10$	0.0	1.41×10^{-8}

Table 1: Summary of spin periods and first harmonics for different wavebands. Recent ∼9 hour optical (no filters were used) observations with Bootes-6 robotic telescope are also included for comparison. Errors on pulsed periods were estimated using the bootstrap resampling method [\[24\]](#page-8-0).

the high magnetic field and relatively fast rotation period of this isolated white dwarf. These results suggest that EUVE J0317-85.5 does appear to behave as a pulsar but future radio observation are required to classify this source as a white dwarf pulsar.

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