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Kaon rare decays: theory overview

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In this proceeding, we review the current theoretical landscape of rare Kaon decays, focusing specifically on flavor-changing neutral current processes. These decays provide valuable indirect avenues for exploring new physics. We will present the Standard Model predictions for the relevant observables, and evaluate their potential for studying new physics effects by comparing them with experimental measurements.

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1. Introduction

The field of kaon physics plays a crucial role in advancing our understanding of beyond the Standard Model (SM) physics. Ongoing experiments like NA62 and KOTO are actively investigating rare kaon decays, while future measurements are anticipated to enhance our capabilities further and open new avenues of exploration. In this proceeding we aim to review the theoretical predictions of rare Kaon decays within the SM, and outline the potential new physics (NP) sensitivity of these decay modes considering the current experimental measurements and upper bounds.

Within the realm of rare kaon decays, the semi-leptonic decays with neutrinos in final states, $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$, hold particular significance as they offer theoretically clean observables, enabling us to detect new physics contributions. Additionally, other rare kaon decays, such as $K^+ \to \pi^+\ell\bar{\ell}$, $K_{L,S} \to \ell\bar{\ell}$, and $K_L \to \pi^0\ell\bar{\ell}$, provide valuable insights into short-distance physics.

For the description of the $b \to s$ transitions we employ the week effective Hamiltonian

$$\mathcal{H}_{\rm eff} = -\frac{4G_F}{\sqrt{2}} V_{td} V_{ts}^* \frac{\alpha_e}{4\pi} \sum_k C_k^\ell O_k^\ell \,, \tag{1}$$

where the relevant local operators are given by

$$O_9^\ell = (\bar{s}\gamma_\mu P_L d)\,(\bar{\ell}\gamma^\mu \ell)\,,\ O_{10}^\ell = (\bar{s}\gamma_\mu P_L d)\,(\bar{\ell}\gamma^\mu\gamma_5 \ell)\,,\ O_L^\ell = (\bar{s}\gamma_\mu P_L d)\,(\bar{\nu}_\ell\,\gamma^\mu(1-\gamma_5)\,\nu_\ell)\,, \tag{2}$$

with NP contributions considered via modifications to the SM Wilson coefficients of the above operators. The SM predictions of the observables given in this review are all according to the `SuperIso` program [1] (see also [2] for more details).

2. $K \to \pi\nu\bar{\nu}$

The first category we examine is the semi-leptonic $K \to \pi\nu\bar{\nu}$ decays. Their remarkable sensitivity to NP contributions, in addition to small theoretical uncertainties, justifies them as the golden channels of rare Kaon decays. The branching ratios of these decays are given by (see e.g. [3])

$$\text{BR}(K_L \to \pi^0\nu\bar{\nu}) = \frac{\kappa_L}{\lambda^{10}} \frac{1}{3} s_W^4 \sum_\ell \text{Im}^2\left[\lambda_t C_L^\ell\right]\,, \tag{3}$$

$$\text{BR}(K^+ \to \pi^+\nu\bar{\nu}) = \frac{\kappa_+(1+\Delta_{\rm EM})}{\lambda^{10}} \frac{1}{3} s_W^4 \sum_\ell \left[\text{Im}^2\left(\lambda_t C_L^\ell\right) + \text{Re}^2\left(-\frac{\lambda_c X_c}{s_W^2} + \lambda_t C_L^\ell\right)\right]\,, \tag{4}$$

where $\lambda_t = V_{td}V_{ts}^*$, $\lambda_c = V_{cd}V_{cs}^*$ and $s_W = \sin\theta_w$, with the sum over the neutrino flavours. The factors κ_+, κ_L and the electromagnetic radiative correction $\Delta_{\rm EM}$ are given in Ref. [4]. The SM prediction of these observables are [2]:

$$\text{BR}(K^+ \to \pi^+\nu\bar{\nu})^{\rm SM} = (7.86 \pm 0.61) \times 10^{-11}\,,\ \text{BR}(K_L \to \pi^0\nu\bar{\nu})^{\rm SM} = (2.68 \pm 0.30) \times 10^{-11}\,, \tag{5}$$

where for both, the main source of uncertainty is parameteric, i.e. from top and charm quark masses and CKM parameters (see e.g. [5, 6]). On the experimental side the most precise measurement for the K^+ decay is given by NA62 [7] while for the K_L decay there is an upper bound by KOTO [8]

$$\text{BR}(K^+ \to \pi^+\nu\bar{\nu})^{\rm exp} = (10.6^{+4.0}_{-3.5} \pm 0.9) \times 10^{-11},\quad \text{BR}(K_L \to \pi^0\nu\bar{\nu})^{\rm exp} < 3.0 \times 10^{-9} @90\%\ \text{CL}. \tag{6}$$

The $K^+ \to \pi^+\nu\bar{\nu}$ measurement with less than 40% uncertainty is consistent with the SM, putting strong constraints on lepton flavour universality conserving (LFUC) new physics effects (see left plot of Fig. 1). However, it is possible to have quite large lepton flavour universality violating (LFUV) new physics contributions. This can be seen in the right plot of Fig. 1, where NP contributions to electrons are considered to be different compared to muons and taus, $\delta C_L^e \neq \delta C_L^\mu (= \delta C_L^\tau)$.

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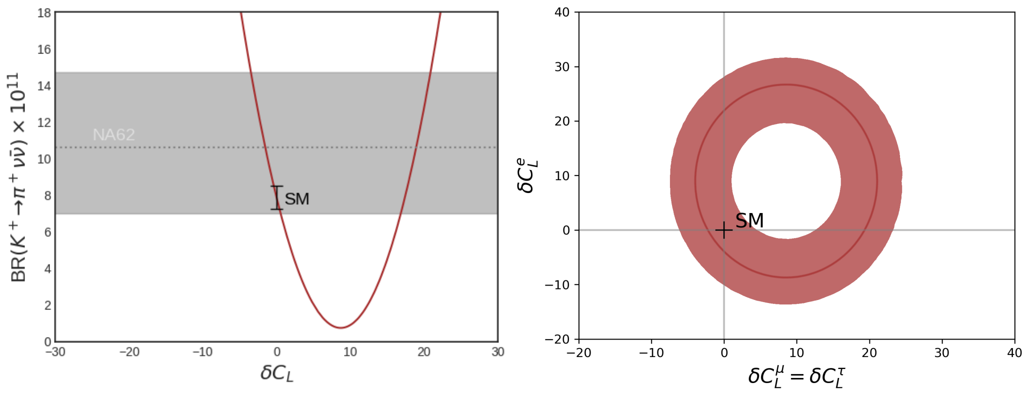

Figure 1: BR($K^+ \to \pi^+ \nu\bar{\nu}$) with LFUC/LFUV new physics contributions on the left/right. On the left, the gray band indicates the NA62 measurement within 1σ. On the right plot, the coloured contour corresponds to 68% CL fitted region.

3. $K^+ \to \pi^+ \ell\bar{\ell}$

An intriguing decay mode, which may serve as an observable to detect LFUV new physics contributions, is $K^+ \to \pi^+ \ell\bar{\ell}$. The branching ratio of this decay is dominated by long-distance contributions via single virtual photon exchange, proportional to the vectorial form factor given by $W(z) = G_F M_K^2 (a_+ + b_+ z) + W^{\pi\pi}(z)$ where $W^{\pi\pi}(z)$ corresponds to the unitarity corrections from the pion loop [9]. The precise theoretical determination of the coefficients a_+ and b_+ remains elusive (see [10] for an outlook on the anticipated advancements in theoretical calculations of these parameters). Nonetheless, any discrepancy in the experimental determinations of the form factor parameters $a_+^{\mu\mu}$ and a_+^{ee}, corresponding to the muon and electron channels, respectively, hints at short-distance LFUV effects [2, 11] as given by $a_+^{\mu\mu} - a_+^{ee} = -\sqrt{2}\,\mathrm{Re}\left[V_{td}V_{ts}^*(C_9^\mu - C_9^e)\right]$. Considering the experimental determination of a_+^{ee} and $a_+^{\mu\mu}$, the bound on LFUV new physics

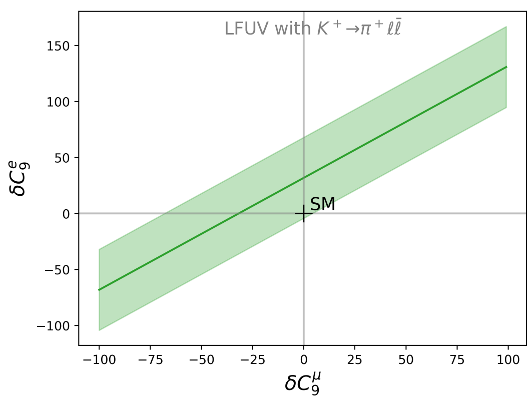

Figure 2: LFUV new physics contributions to $K^+ \to \pi^+ \ell\bar{\ell}$ in the ($C_9^e - C_9^\mu$) plane at 68% CL.

contributions is shown in Fig. 2, where for a_+^{ee} the combination of the E865 [12] and NA48/2 [13] measurements as given in Ref. [14] is considered, while $a_+^{\mu\mu}$ is taken from the recently determined value by NA62 [15].

4. $K \to \ell\bar{\ell}$

The $K_L \to \mu\bar{\mu}$ and $K_S \to \mu\bar{\mu}$ decays offer unique insights; despite significant theoretical uncertainties stemming from dominant long-distance contributions, constraints on new physics parameters can be derived due to potential sizeable effects from short-distance contributions. The branching fractions for these decays, in the absence of scalar and pseudoscalar contributions, are described by [16, 17]

$$\mathrm{BR}(K_S \to \mu\bar{\mu}) = \tau_S \frac{f_K^2 m_K^3 \beta_\mu}{16\pi} \left\{ \beta_\mu^2 \left|N_S^{\mathrm{LD}}\right|^2 + \left(\frac{2m_\mu}{m_K}\frac{G_F\alpha_e}{\sqrt{2}\pi}\right)^2 \mathrm{Im}^2\left[-\lambda_c \frac{Y_c}{s_W^2} + \lambda_t C_{10}^\ell\right]\right\}, \quad (7)$$

$$\mathrm{BR}(K_L \to \mu\bar{\mu}) = \tau_L \frac{f_K^2 m_K^3 \beta_\mu}{16\pi} \left| N_L^{\mathrm{LD}} - \left(\frac{2m_\mu}{m_K}\frac{G_F\alpha_e}{\sqrt{2}\pi}\right) \mathrm{Re}\left[-\lambda_c \frac{Y_c}{s_W^2} + \lambda_t C_{10}^\ell\right]\right|^2, \quad (8)$$

where $\beta_\mu = \sqrt{1 - 4m_\mu^2/M_K^2}$ and Y_c stands for the short-distance charm contribution. The long-distance (LD) contributions as extracted in [17] (see also [16, 18–20]) are: $N_S^{\mathrm{LD}} = (-2.65 + 1.14i) \times 10^{-11}\,(\mathrm{GeV})^{-2}$ and $N_L^{\mathrm{LD}} = \pm\,[0.54(77) - 3.95i] \times 10^{-11}\,(\mathrm{GeV})^{-2}$. The SM predictions for $K_S \to \mu\bar{\mu}$ and $K_L \to \mu\bar{\mu}$ are given by [2] (see also [21])

$$\mathrm{BR}(K_S \to \mu\bar{\mu})^{\mathrm{SM}} = (5.15 \pm 1.50) \times 10^{-12}, \quad (9)$$

$$\mathrm{BR}(K_L \to \mu\bar{\mu})^{\mathrm{SM}} = \begin{cases} \mathrm{LD}(+): & \left(6.82^{+0.77}_{-0.24} \pm 0.04\right) \times 10^{-9}, \\ \mathrm{LD}(-): & \left(8.04^{+1.46}_{-0.97} \pm 0.09\right) \times 10^{-9}, \end{cases} \quad (10)$$

where for $K_L \to \mu\bar{\mu}$, both signs of N_L^{LD} have been considered. It is worth noting that the uncertainty of $K_L \to \mu\bar{\mu}$ (especially for LD+) is highly asymmetric. On the experimental side, $K_L \to \mu\bar{\mu}$ has been measured with less than 2% uncertainty [22]. In contrast, while the theory prediction for $K_S \to \mu\bar{\mu}$ is independent of sign ambiguity, the current upper bound by LHCb [23] is approximately two orders of magnitude larger than the SM prediction, posing challenges in constraining short-distance physics. In Fig. 3, the impact of NP contributions to $K_L \to \mu\bar{\mu}$ and $K_S \to \mu\bar{\mu}$ decay is shown, where for the former both signs of LD contributions are considered.

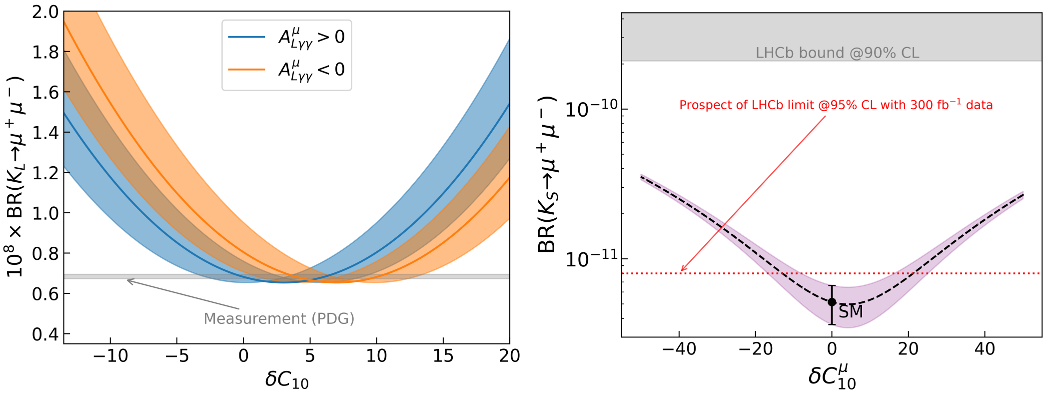

Figure 3: The $K_L \to \mu\bar{\mu}$ and $K_S \to \mu\bar{\mu}$ decays as a function of NP contributions to C_{10}^μ. The coloured bands indicate the 1σ theoretical uncertainty.

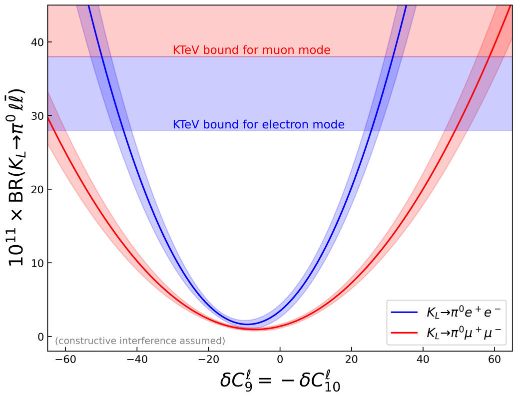

Figure 4: The branching ratio of $K_L \to \pi^0 \ell\bar{\ell}$ as a function of NP contributions to $C_9^\ell = -C_{10}^\ell$ for both the electron and the muon modes. The coloured bands indicate the 1σ theoretical uncertainty.

5. $K_L \to \pi^0 \ell\bar{\ell}$

Finally, the last rare Kaon decay mode we consider in this review is $K_L \to \pi^0 \ell\bar{\ell}$. The branching ratio of this decay in the electron and muon channels are given by

$$\mathrm{BR}(K_L \to \pi^0 \ell\bar{\ell}) = \left(C_{\rm dir}^\ell \pm C_{\rm int}^\ell |a_S| + C_{\rm mix}^\ell |a_S|^2 + C_{\gamma\gamma}^\ell\right) \cdot 10^{-12}\,, \tag{11}$$

with $|a_S| = 1.20 \pm 0.20$. The different components [20] describing the branching ratio are:

	$C_{\rm dir}^\ell$	$C_{\rm int}^\ell$	$C_{\rm mix}^\ell$	$C_{\gamma\gamma}^\ell$
$\ell = e$	$(4.62 \pm 0.24)(w_{7V}^2 + w_{7A}^2)$	$(11.3 \pm 0.3)w_{7V}$	14.5 ± 0.5	≈ 0
$\ell = \mu$	$(1.09 \pm 0.05)(w_{7V}^2 + 2.32 w_{7A}^2)$	$(2.63 \pm 0.06)w_{7V}$	3.36 ± 0.20	5.2 ± 1.6

where $C_{\gamma\gamma}^\ell$ stands for the CP-conserving two-photon contribution, $C_{\rm dir}^\ell$ refers to the direct CP-violating term sensitive to short-distance physics, $C_{\rm mix}^\ell$ indicates the indirect CP-violating contributions from Kaon mixing, and $C_{\rm int}^\ell$ corresponds to the interference between the latter two contributions. The interference can be both destructive and constructive with the latter being theoretically favoured. The $C_{\rm dir}^\ell$ and $C_{\rm int}^\ell$ terms are sensitive to short-distance physics via (see e.g. [24])

$$w_{7V} = \frac{1}{2\pi}\mathrm{Im}\left[\frac{\lambda_t}{1.407 \times 10^{-4}} C_9\right]\,, \quad w_{7A} = \frac{1}{2\pi}\mathrm{Im}\left[\frac{\lambda_t}{1.407 \times 10^{-4}} C_{10}\right]\,, \tag{12}$$

where 1.407×10^{-4} corresponds to the value of λ_t as used by [20]. On the experimental side, the present upper bounds [25, 26] exceed the SM predictions by an order of magnitude. However, even with this upper limit, they provide valuable insights into short-distance physics. Figure 4 illustrates the impact of new physics contributions to $K_L \to \pi^0 \ell\bar{\ell}$, assuming $C_9^\ell = -C_{10}^\ell$.

We conclude this review by noting that a global study [2, 27] of these rare Kaon decays is possible, resulting in stronger bounds and constraints on new physics than what is achievable with each individual observable.

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