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Experimental overview of CKM metrology from kaon physics

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There is currently a significant tension between the prediction from the unitarity of the first row of the CKM matrix, $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$, and measurements. This is known as the Cabibbo angle anomaly, with the tension corresponding to around three standard deviations. Precision measurements linked to kaon physics can provide insights into V_{ud} and V_{us}. In particular, studies of $K \to \pi\ell\nu$ decays can provide a measurement of V_{us} and the ratio between the rates of $K^{\pm} \to \mu^{\pm}\nu$ and $\pi^{\pm} \to \mu^{\pm}\nu$ decays can provide a measurement of the ratio $|V_{us}|/|V_{ud}|$. Future measurements of these CKM parameters using current and future experiments can play a crucial role in resolving the current tension between observations and the prediction from unitarity. In these proceedings the current state-of-the-art in CKM metrology from kaon physics is presented, and future prospects are summarised.

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1. Introduction

In the Standard Model of particle physics (SM) the coupling of the physical eigenstates of the quarks to the charged-current $W^{\pm}$ bosons is given by [1]

$$-\frac{g}{2}(\overline{u_L}, \overline{c_L}, \overline{t_L})\,\gamma^{\mu}\,W_{\mu}^{+}\,V_{CKM}\begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + \text{h.c.} \tag{1}$$

where $U_L \in [u_L, c_L, t_L]$ and $D_L \in [d_L, s_L, b_L]$ are the physical left-handed up- and down-type quark states, g is a universal gauge coupling strength, and the Cabibbo-Kobayashi-Maskawa (CKM) matrix is

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)\ . \tag{2}$$

In the Wolfenstein parameterisation, given above after the second equality, the importance of parameter $\lambda = sin(\theta_C) = V_{us}$ (where θ_C is the two-generation mixing angle, the Cabibbo angle) is highlighted. In the SM the CKM matrix must be a complex, unitary, 3x3 matrix. The unitarity constraint from the first row elements is given by

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1\ . \tag{3}$$

The PDG reports the current state-of-the art measurements of the matrix element magnitudes as [1]

$$|V_{CKM}^{ij}| = \begin{pmatrix} 0.974 & 0.225 & 0.004 \\ 0.225 & 0.973 & 0.042 \\ 0.009 & 0.041 & 0.9991 \end{pmatrix}\ . \tag{4}$$

Considering $|V_{ub}|^2 \ll [|V_{us}|^2, |V_{ud}|^2]$ equation 3 can be approximated to

$$|V_{ud}|^2 + |V_{us}|^2 = 1\ , \tag{5}$$

to a precision better than 3×10^{-5}. Therefore measurements of $|V_{ud}|^2$ and $|V_{us}|^2$ provide a stringent test of the unitarity of the CKM matrix. Any deviation from unitarity implies the presence of new physics beyond the SM (BSM).

2. Measurements of V_{us}

Studies of the $K \to \pi\ell\nu$ decays (both charged, $K^{\pm} \to \pi^0\ell^{\mp}\nu$ and neutral $K_{L,S} \to \pi^{\pm}\ell^{\mp}\nu$ modes) can be used to extract measurements of V_{us}. This is because the decay rate of these semileptonic kaon decays can be written as [1, 2]

$$\Gamma(K \to \pi\ell\nu(\gamma)) = |V_{us}|^2 \frac{C_K^2 G_F^2 m_K^5}{192\pi^3} S_{EW} |f_+(0)|^2 I_K^{\ell}(\lambda_{K\ell})(1 + 2\Delta_K^{SU(2)} + 2\Delta_{K\ell}^{EM})\ , \tag{6}$$

where $K \in [K^\pm, K_{L,S}]$, $\ell \in [e^\pm, \mu^\pm]$, constant $C_K^2 = \begin{cases} 0.5 & \text{for } K^+ \\ 1 & \text{for } K^0 \end{cases}$ and $S_{EW} = 1.0232$ is a universal short-distance electromagnetic correction. There are several theoretical inputs: $|f_+(0)|^2$ is the hadronic matrix element (form factor) at zero momentum transfer to the $\ell\nu$ system, plus two form-factor corrections $\Delta_K^{SU(2)}$ and $\Delta_{K\ell}^{EM}$ accounting for $SU(2)$-breaking and long-distance electromagnetic effects, respectively. Finally there are experimental inputs: the decay rates $\Gamma(K \to \pi\ell\nu(\gamma))$ (from measurements of the branching ratios, which are fully inclusive with respect to photons in the final state, and kaon lifetimes) and the phase-space integral I_K^ℓ (determined using the measured semileptonic form factors).

Recent experimental measurements of the $K_S \to \pi\ell\nu$ decays by KLOE(2) [3, 4] are examples of important inputs to the determination of V_{us}. The KLOE(2) experiment was located at the DAΦNE ϕ-factory at INFN Laboratori Nazionali di Frascati [2, 5]. An e^+e^- collider was operated at an energy corresponding to the ϕ resonance and neutral kaons were studied from the copious $\phi \to K_L K_S$ decays. The first measurement of the μ channel was reported in [3], providing a branching ratio measurement of $\mathcal{B}(K_S \to \pi\mu\nu) = (4.56 \pm 0.11_{\text{stat}} \pm 0.17_{\text{syst}}) \times 10^{-4}$, while the recent investigation of the e channel reported in [4] provides a result of $\mathcal{B}(K_S \to \pi e\nu) = (7.153 \pm 0.037_{\text{stat}} \pm 0.044_{\text{syst}}) \times 10^{-4}$, with an uncertainty of 0.8%. By normalising each of these results to the branching ratio $\mathcal{B}(K_S \to \pi^+\pi^-)$, also measured by KLOE, the value of $V_{us} f_+(0)$ can be extracted [4].

3. Measurements of V_{us}/V_{ud}

Combining measurements of kaon decays and pion decays allows a precision evaluation of the ratio V_{us}/V_{ud} [6, 7]

$$\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi} = \left(\frac{\Gamma_{K_{\mu 2}} m_{\pi^\pm}}{\Gamma_{\pi_{\mu 2}} m_{K^\pm}}\right)^{\frac{1}{2}} \left(\frac{1 - \frac{m_\mu^2}{m_{\pi^\pm}^2}}{1 - \frac{m_\mu^2}{m_{K^\pm}^2}}\right) \left(1 - \frac{1}{2}\delta_{EM} - \frac{1}{2}\delta_{SU(2)}\right) . \quad (7)$$

Here there are theoretical inputs: f_K/f_π, the ratio of decay constants in the isospin-symmetric limit for the kaon and pion for which lattice-scale uncertainties cancel; δ_{EM}, long-distance electromagnetic correction for $SU(2)$ symmetry breaking; and $\delta_{SU(2)}$ strong isospin symmetry breaking correction ($\delta_{EM} + \delta_{SU(2)}$ is calculated in [8]). The experimental input is the measured decay rates, $\Gamma(K_{\mu 2}) = \Gamma(K^\pm \to \mu^\pm\nu(\gamma))$ and $\Gamma(\pi_{\mu 2}) = \Gamma(\pi^\pm \to \mu^\pm\nu(\gamma))$ [1].

4. Results from measurements of kaon and pion decays

Using the method described in sections 2 and 3 the following results have been established [7]: using $K \to \pi\ell\nu$ ($K_{\ell 3}$) decays (and using $f_+(0) = 0.9698(17)$ [9])

$$|V_{us}|_{(K_{\ell 3})} = 0.22330(35)_{\text{exp}}(39)_{\text{lat}}(8)_{\text{IB}} , \quad (8)$$

with uncertainties from experimental, lattice and internal bremsstrahlung (EM corrections), respectively, and a total relative uncertainty of 0.24%; and using $K^\pm \to \mu^\pm\nu$ ($K_{\mu 2}$) decays (with

$f_K/f_\pi = 1.1978(22)$ [7]),

$$\left|\frac{V_{us}}{V_{ud}}\right|_{(K_{\mu 2})} = 0.23108(23)_{\text{exp}}(42)_{\text{lat}}(16)_{\text{IB}} \ , \tag{9}$$

with a total relative uncertainty of 0.22%.

5. Measurement of V_{ud}

The most precise experimental determination of V_{ud} is derived from super-allowed $0^+ \to 0^+$ nuclear β decays. Using a set of 15 of the most precisely measured of these processes [10] and applying the most up-to-date radiative corrections [7] $|V_{ud}|_\beta = 0.97367(11)_{\text{exp}}(13)_{\text{Rad.Cor.}}(27)_{\text{Nucl.Struc.}}$, with uncertainties of experimental origin, from radiative corrections and nuclear structure modelling, giving a relative uncertainty of 0.33%. Additionally, using the most recent studies of the neutron lifetime τ_n [11], and axial-vector/vector coupling $g_A = G_A/G_V$ [12], provides a result [7] $|V_{ud}|_n = 0.97413(13)_{\text{Rad.Cor.}}(35)_{g_A}(20)_{\tau_n}$, with a similar relative uncertainty of 0.44%. These results are in good agreement and an average can be used [7]:

$$|V_{ud}| = 0.97384(26) \ . \tag{10}$$

6. First row unitarity tests and the Cabibbo angle anomaly

Measurements of V_{ud}, V_{us} and their ratio are displayed in figure 1 (left). Given that equation 5 defines a circle with radius equal to unity in the V_{us} vs V_{ud} plane, the expected unitarity condition is represented by a dashed line. The three sets of measurements are expected to be consistent with each-other and to coincide with the dashed line. However, the combined best fit region, shown in yellow, is inconsistent with the dashed line and numerically the best fit value from experimental measurements is 2.8 standard deviations (2.8σ) from the unitarity expectation. This significant tension is called the Cabibbo angle anomaly [13], with the implied link to the critical $\lambda = V_{us}$ parameter.

7. Unitarity tests and a possible BSM scenario

Equations 8, 9 and 10 report results of measurements of three variables, which are displayed on figure 1. Combining pairs of these results in different ways, and considering the unitarity constraint of equation 5, three parameters can be defined showing the deviation of experimental results from the unitarity prediction, $\Delta^{(i)}_{CKM}$ for $i \in [1, 2, 3]$, with results given by

$$\Delta^{(1)}_{CKM} = |V_{ud}|^2 + |V_{us}|^2_{(K_{\ell 3})} - 1 = 0.00176(56) \quad [-3.1\sigma] \tag{11}$$

$$\Delta^{(2)}_{CKM} = |V_{ud}|^2 \left(1 + |V_{us}/V_{ud}|^2_{(K\mu 2)}\right) - 1 = 0.00098(58) \quad [-1.7\sigma] \tag{12}$$

$$\Delta^{(3)}_{CKM} = |V_{us}|^2_{(K_{\ell 3})} \left(\frac{1}{|V_{us}/V_{ud}|^2_{(K\mu 2)}} + 1\right) - 1 = 0.0164(63) \quad [-2.6\sigma] \ . \tag{13}$$

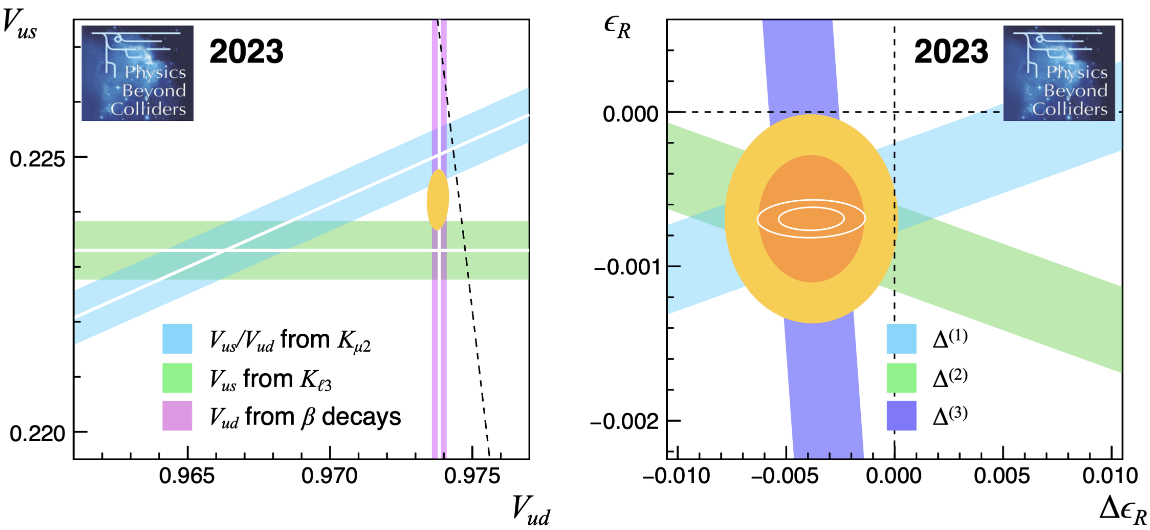

Figure 1: [7, 16] (Left) Summary of measurements related to the unitarity condition of the first row of the CKM matrix. Measurements of V_{us} from $K \to \pi\ell\nu$ decays (section 2), of the ratio V_{us}/V_{ud} from $K^\pm \to \pi^\pm\nu$ and $\pi^\pm \to \mu^\pm\nu$ decays (section 3) and Vud from super-allowed nuclear beta decays (and neutron lifetime measurements, section refsec:MeasurementofVud), are shown as horizontal, diagonal and vertical bands, respectively. The best fit to the data is shown by the orange region (corresponding to one standard deviation) which deviates from the expected unitarity constraint is shown by the dashed line. (Right) Summary of unitarity test constraints $\Delta^{(1)}_{CKM}, \Delta^{(2)}_{CKM}, \Delta^{(3)}_{CKM}$, defined in section 7, as functions of parameters ε_R and $\Delta\varepsilon_R$ of a BSM model representing right-handed currents in the light (u,d) and strange sectors, respectively. The orange regions indicate the best fit to the data.

Each result individually shows a deficit and, as reported in section 6, when using all three results simultaneously the deficit becomes -2.8σ. It is noted in particular that the value of $\Delta^{(3)}_{CKM}$ indicates that there is a 2.6σ deficit with respect to the unitarity prediction, constituting and interesting hint of an anomaly when only using kaon (and pion) decay inputs (section 4).

To attempt to explain this tension BSM scenarios can be considered. In the SM the $W^\pm$ couples only to left-handed chiral fermion states, however in a scenario with couplings to right-handed currents the $\Delta^{(i)}_{CKM}$ parameters can be defined in terms of ε_R and $\Delta\varepsilon_R$ representing the right-handed currents in the light (u, d) sector and difference in right-handed coupling between the light and strange sectors, respectively:

$$\Delta^{(1)}_{CKM} = 2\epsilon_R + 2\Delta\epsilon_R|V_{us}|^2 \tag{14}$$

$$\Delta^{(2)}_{CKM} = 2\epsilon_R - 2\Delta\epsilon_R|V_{us}|^2 \tag{15}$$

$$\Delta^{(3)}_{CKM} = 2\epsilon_R + 2\Delta\epsilon_R(2 - |V_{us}|^2)\ . \tag{16}$$

Using the current experimental results, shown in figure 1, the best fit results are $\epsilon_R = 0.69(27) \times 10^3$ and $\Delta\epsilon_R = 3.9(1.6) \times 10^3$, with significances of 2.5σ and 2.4σ, respectively [7]. Overall this means the $\epsilon_R = \Delta\epsilon_R = 0$ scenario is excluded with a significance of 3.1σ.

8. Future prospects

Kaon physics provides crucial inputs to the first row unitarity tests, and these are currently the least precisely known. With ongoing and future kaon experiments there are significant prospects for impactful updates and new measurements.

The NA62 experiment at the CERN SPS studies K^+ decays-in-flight from a high-intensity 75 GeV/c beam, and is ideally placed to make a high-precision measurement of $R^{K_{\mu3}/K_{\mu2}} = \frac{\mathcal{B}(K^+\to\pi^0\mu^+\nu)}{\mathcal{B}(K^+\to\mu^+\nu)}$. The highly efficient and accurate tracking, triggering and particle identification systems allow large and very pure samples of these decays modes to be collected. Moreover, by using the same data-set, trigger and carefully chosen selection criteria many systematic uncertainties will cancel when evaluating the ratio. The measurement of this ratio in particular is of interest because it can be related to the CKM unitarity tests discussed above because $R^{K_{\mu3}/K_{\mu2}}$ is proportional to r, which is given by [7]

$$r \equiv \left(\frac{1+\Delta^2_{CKM}}{1-\Delta^{(3)}_{CKM}}\right)^{-1/2} = \frac{\left|\frac{V_{us}}{V_{ud}}\right|^2_{(K\mu2)}}{|V_{us}|_{(K_{\ell3})}/|V_{ud}|} = 1 - 2\Delta\epsilon_R \ . \tag{17}$$

The most precise current $K_{\mu2}$ branching ratio measurement [14], with a quoted uncertainty of 0.27%, is highly significant in global fits to kaon data which currently have a relatively poor fit quality [7], which can cause tension with less probable decay modes like $K_{\mu3}$. Therefore, measurement of $R^{K_{\mu3}/K_{\mu2}}$ can provide a qualified statement about the consistency of the kaon data and is sensitive to the presence of right-handed currents ($\Delta\epsilon_R \neq 0$).

Beyond the measurement of $R^{K_{\mu3}/K_{\mu2}}$ NA62 could provide high-precision measurements of all common K^+ decays, in particular focusing on ratios of branching ratios with good control of systematics which will help to over-constrain global fits. Preliminary studies suggest that with a dedicated, stable, low-intensity data-taking for two weeks with minimum-bias triggers will allow such measurements with statistical uncertainties of less than 0.1%.

At CERN the HIKE project [15, 16] has been proposed with a high-intensity upgrade of the beamline of NA62 to perform a new-generation experiment to study K^+ decays in phase 1 and K_L decays in phase 2, as well as operation in beam-dump mode to expand the search for feebly interacting particles in collaboration with the proposed SHADOWS experiment [16, 17]. Additional measurements of both K^+ and K_L decays by HIKE will allow a significant overhaul of the world data entering global fits (especially for the case of K^+). Depending on these measurements the first row unitarity tensions may be resolved or confirmed.

The PIONEER experiment at PSI [18] (which is approved and will begin data-taking in about 5 years) will focus on measurements of the pion beta decay $\pi^+ \to \pi^0 e^+ \nu_e$ in phases 2 and 3, allowing a measurement of V_{ud} independent from nuclear physics. Phase 2 has the goal of measuring $\mathcal{B}(\pi^+ \to \pi^0 e^+ \nu_e)$ with 0.2% precision looking for back-to-back photons from stopped pions, normalising to $\pi^+ \to e^+\nu$, building on the successful strategy of the PIBETA experiment [19]. The challenging long-term goal, for phase 3 of the project, is to reach a precision of 0.02% for the measurement of V_{ud}, competitive with the precision from super-allowed nuclear beta decays.

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9. Conclusions

Measurements of kaon, pion and nuclear beta decays are used to establish the size of the CKM matrix elements V_{us}, V_{ud} and their ratio with precisions of $< 0.5\%$. These measurements can be used to test the unitarity condition for the first row of the CKM matrix. The best fit results in a deficit from the SM unitarity prediction by 2.8σ. Even when only using kaon and pion decay data a deficit of 2.6σ is apparent. This tension, the Cabibbo angle anomaly, may be explained either through invoking new physics such as right-handed currents, even if a statistical fluctuation cannot be excluded. Future improvements to these measurements, for example of kaon decays by the NA62 and/or HIKE experiments or pion decays by the PIONEER experiment, are therefore of significant interest either to confirm the deviation from unitarity or resolve the current tension.

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