PROCEEDINGS OF SCIENCE

Relativistic magnetohydrodynamics with spin

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In this work, we present a novel framework of relativistic non-resistive dissipative magnetohydrodynamics for spin-polarized particles. Utilizing a classical relativistic kinetic equation for the distribution function in an extended phase-space of position, momentum, and spin, we derive equations of motion for dissipative currents at first-order in spacetime gradients. Our findings reveal a coupling between fluid vorticity and magnetization via an electromagnetic field, leading to relativistic analogs of the Einstein-de Haas and Barnett effects. Our study provides a tool for a better understanding of the polarization phenomena observed in relativistic heavy-ion collisions.

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1. Introduction

Over the last decades, it has been well established [1] that the strongly interacting matter produced in relativistic nuclear collisions evolves according to principles of relativistic hydrodynamics [2, 3]. It is expected that in non-central collisions this matter may experience large angular momentum and a strong magnetic field [4, 5]. These extreme physical conditions may lead, similarly to the non-relativistic magneto-mechanical effects of Einstein-de Haas [6] and Barnett [7], to spin polarization and magnetization of the matter and, consequently, of the emitted particles [8–10]. The existence of spin polarization phenomenon was recently confirmed experimentally [11–17] triggering vast theoretical developments aiming at finding a unified interpretation of the measured observables [18–36]. In particular, based on fundamental conservation laws, an extension of relativistic hydrodynamics for spin-polarized fluids was proposed [37] giving rise to the rapid development of a new field known as relativistic spin hydrodynamics [38–51, 51–71].

Very recently, a formalism of dissipative non-resistive spin magnetohydrodynamics was constructed, aiming at incorporating into the spin hydrodynamics effects of spin polarization due to the presence of electromagnetic field [72]. In this contribution, we briefly review the framework of [72] and discuss its main implications. Starting from the classical transport equation for the distribution function in an extended phase-space of position, momentum, and spin, in the presence of a magnetic field we derive equations of motion for dissipative currents at first-order in spacetime gradients. It is found that, apart from contributions from various standard hydrodynamic gradients [42, 43], the spin current acquires also effects due to the gradients of electromagnetic field [72]. In particular, we show that the coupling between fluid vorticity and magnetization via an electromagnetic field gives rise to effects similar to that of Einstein-de Haas and Barnett.

We use the following conventions for the metric tensor and Levi-Civita symbol: $g_{\mu\nu} = \mathrm{diag}(+1, -1, -1, -1)$ and $\epsilon^{0123} = -\epsilon_{0123} = 1$. We also use natural units with $c = \hbar = k_B = 1$.

2. Kinetic theory derivation of equations of motion

We consider the classical distribution function of particles with spin in an extended phase-space of space-time position $x \equiv x^\mu$, four-momentum $p \equiv p^\mu$, and intrinsic angular momentum $s \equiv s^{\mu\nu}$, $f \equiv f(x, p, s)$ [39]. The dynamics of f is determined by the following kinetic equation [72]

$$\left(p^\alpha \frac{\partial}{\partial x^\alpha} + m\,\mathcal{F}^\alpha \frac{\partial}{\partial p^\alpha} + m\,\mathcal{S}^{\alpha\beta} \frac{\partial}{\partial s^{\alpha\beta}}\right) f = C[\,f\,]\,, \tag{1}$$

and likewise for anti-particles with the replacement $f \to \bar{f}$. In Eq. (1), the four-momentum $p^\mu = (E_p, \boldsymbol{p})$ is on the mass shell, with $E_p = \sqrt{m^2 + \boldsymbol{p}^2}$ difining the particle energy and m denoting the particle mass, and $C[f]$ is the collision kernel.

In the above equation, $\mathcal{F}^\alpha = dp^\alpha/d\tau$ and $\mathcal{S}^{\alpha\beta} = ds^{\alpha\beta}/d\tau$ (where τ denotes the proper time along the world line) are, respectively, force and torque experienced by a particle moving under influence of electromagnetic field. For composite particles they have the forms

$$\mathcal{F}^\alpha = \frac{\mathfrak{q}}{m} F^{\alpha\beta} p_\beta + \frac{1}{2}\left(\partial^\alpha F^{\beta\gamma}\right) m_{\beta\gamma}, \tag{2}$$

$$\mathcal{S}^{\alpha\beta} = 2\,F^{\gamma[\alpha}\, m^{\beta]}{}_{\gamma} - \frac{2}{m^2}\Big(\chi - \frac{\mathfrak{q}}{m}\Big) F_{\phi\gamma}\, s^{\phi[\alpha}\, p^{\beta]} p^\gamma, \tag{3}$$

where $F^{\mu\nu}$ denotes the electromagnetic field strength tensor and $m^{\alpha\beta} = \chi s^{\alpha\beta}$ is the magnetic dipole moment of particles with χ playing the role of the gyromagnetic ratio [73]. The expressions for the first and second term on the right-hand side of Eq. (2) represent well-known Lorentz and Mathisson force, respectively [73]. On the other hand, the form of the torque in Eq. (3) is less understood. Hence, in this work, we choose to neglect it.

The number current N^λ, the energy-momentum tensor $T_{\rm f}^{\lambda\mu}$, and the spin current $S^{\lambda,\mu\nu}$ of the fluid are expressed, respectively, through the zeroth, first, and "spin" moment of the distribution function [43]

$$N^\lambda = \int_{p,s} p^\lambda \left(f - \bar{f}\right), \tag{4}$$

$$T_{\rm f}^{\lambda\mu} = \int_{p,s} p^\lambda p^\mu \left(f + \bar{f}\right), \tag{5}$$

$$S^{\lambda,\mu\nu} = \int_{p,s} p^\lambda s^{\mu\nu} \left(f + \bar{f}\right), \tag{6}$$

while the polarization-magnetization tensor is given by the formula

$$M^{\mu\nu} = m \int_{p,s} m^{\mu\nu} \left(f - \bar{f}\right). \tag{7}$$

In the above equations we used the shorthand notation $\int_{p,s} \equiv \int dPdS$ with $dP \equiv d^3p/[E_p(2\pi)^3]$ and $dS \equiv m/(\pi\,\textsf{s})\, d^4s\, \delta(s\cdot s + \textsf{s}^2)\, \delta(p\cdot s)$, where the length of the spin vector, $\textsf{s}^2 = \frac{1}{2}\left(1+\frac{1}{2}\right) = \frac{3}{4}$, is defined by the eigenvalue of the Casimir operator.

Presuming that the microscopic interactions preserve fundamental conservation laws the following moments of the collision kernel should vanish:

$$\int_{p,s} C[f] = 0, \qquad \int_{p,s} p^\mu C[f] = 0, \qquad \int_{p,s} s^{\mu\nu} C[f] = 0. \tag{8}$$

Using these properties and Eqs. (4)-(7) one may show that the zeroth, first and "spin" moment of the kinetic equation (1) (assuming no torque) lead, respectively, to the following equations

$$\partial_\mu N^\mu = 0, \qquad \partial_\nu T_{\rm f}^{\mu\nu} = F^\mu{}_\alpha J_{\rm f}^\alpha + \frac{1}{2}\left(\partial^\mu F^{\nu\alpha}\right) M_{\nu\alpha}, \qquad \partial_\lambda S^{\lambda,\mu\nu} = 0, \tag{9}$$

where $J_{\rm f}^\mu = \mathfrak{q} N^\mu$ is a charge current with $\mathfrak{q}$ denoting the electric charge of the particles. Equations (9) constitute the basis for the framework of spin-magnetohydrodynamics.

Assuming Landau's definition of four-velocity u of the fluid, $T_{\rm f}^{\mu\nu} u_\nu = \epsilon u^\mu$, where ϵ is the energy density, the particle current, and the stress-energy tensor are given by

$$N^\mu = n u^\mu + n^\mu, \quad T_{\rm f}^{\mu\nu} = \epsilon u^\mu u^\nu - (P + \Pi)\,\Delta^{\mu\nu} + \pi^{\mu\nu} \tag{10}$$

where n is the net particle number density, n^μ particle number diffusion, P is the pressure, Π and $\pi^{\mu\nu}$ are the bulk and shear viscous pressures, and $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$. Since we are interested in the formulation of magnetohydrodynamics with spin in the non-resistive limit, we have

$$F^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta} u_\alpha B_\beta, \tag{11}$$

where B^μ is the magnetic field four-vector satisfying the well-known Maxwell equations, see Ref. [72]. The field strength tensor and polarization-magnetization tensors are related to each other by $H^{\mu\nu} - M^{\mu\nu} = F^{\mu\nu}$, where $H^{\mu\nu}$ is the induction tensor.

3. Dynamics of dissipative currents

To derive constitutive relations for dissipative quantities in Eqs. (10), we consider the kinetic equation (1), with the collision term treated in relaxation-time approximation (RTA) [74]

$$\left(p^\alpha \frac{\partial}{\partial x^\alpha} + m\,\mathcal{F}^\alpha \frac{\partial}{\partial p^\alpha}\right) f = -\,(u\cdot p)\,\frac{f - f_{\rm eq}}{\tau_{\rm eq}} \equiv -\,(u\cdot p)\,\frac{\delta f}{\tau_{\rm eq}}, \tag{12}$$

where $f_{\rm eq}$ is the equilibrium distribution function and relaxation time $\tau_{\rm eq}$ is assumed to be independent of particle momentum and energy. Note that, within the RTA, the zeroth and first moments (see, respectively, the first and second equation in (8)) of the right-hand side of Eq. (12) vanish when Landau frame and matching conditions are used. Moreover, imposing the matching condition [43]

$$u_\lambda \delta S^{\lambda,\mu\nu} \equiv u_\lambda\left(S^{\lambda,\mu\nu} - S^{\lambda,\mu\nu}_{\rm eq}\right) = 0, \tag{13}$$

where $\delta S^{\lambda,\mu\nu}$ is the dissipative part of the spin current, also the spin moment (see the third equation in (8)) vanishes.

Herein, we assume the equilibrium distribution to have the Fermi-Dirac form,

$$f_{\rm eq} = \left\{1 + \exp\left[\beta(u\cdot p) - \xi - \frac{1}{2}\,\omega_{\mu\nu}s^{\mu\nu}\right]\right\}^{-1}, \tag{14}$$

and similarly for anti-particles with $\xi \to -\xi$, where $\xi \equiv \mu\beta$ and $\beta \equiv 1/T$. Here, $\omega_{\mu\nu}$ plays the role of Lagrange multiplier corresponding to spin conservation [37] and is related to spin polarization observable via Pauli-Lubanski four-vector [38, 39]. Considering the limit of small polarization, we can keep only terms up to linear in $\omega^{\mu\nu}$ and write

$$f_{\rm eq} = f_0 + \frac{1}{2}\omega_{\mu\nu}s^{\mu\nu} f_0(1 - f_0), \tag{15}$$

where $f_0 \equiv \{1 + \exp\left[\beta(u\cdot p) - \xi\right]\}^{-1}$.

The dissipative quantities defined in Eqs. (10) and (13) are given in terms of the non-equilibrium corrections to the distribution function,

$$n^\mu = \int_{p,s} p^{\langle\mu\rangle}\left(\delta f - \delta\bar f\right), \qquad \Pi = \int_{p,s}\left(-\frac{1}{3}\right) p^{\langle\mu\rangle}p_{\langle\mu\rangle}\left(\delta f + \delta\bar f\right), \tag{16}$$

$$\pi^{\mu\nu} = \int_{p,s} p^{\langle\mu}p^{\nu\rangle}\left(\delta f + \delta\bar f\right), \qquad \delta S^{\lambda,\mu\nu} = \int_{p,s} p^\lambda s^{\mu\nu}\left(\delta f + \delta\bar f\right), \tag{17}$$

where used the notation $X^{\langle\mu\rangle} \equiv \Delta^\mu_\alpha X^\alpha$ and $X^{\langle\mu\nu\rangle} \equiv \Delta^{\mu\nu}_{\alpha\beta}X^{\alpha\beta}$.

To obtain the relativistic Navier-Stokes expressions for the dissipative quantities, using Eq. (12) we evaluate the non-equilibrium corrections to the phase-space distribution functions up to first-order in hydrodynamic gradients. In this way, for particles we get

$$\begin{aligned}\delta f_1 = & -\frac{\tau_{\rm eq}}{(u\cdot p)}\left[p^\alpha\partial_\alpha + \frac{m\,\chi}{2}\left(\partial^\alpha F^{\beta\gamma}\right)s_{\beta\gamma}\partial^{(p)}_\alpha\right] f_{\rm eq} \\ & + \frac{\tau_{\rm eq}}{(u\cdot p)}\mathfrak{q}F^{\alpha\beta}p_\beta\partial^{(p)}_\alpha\left[\frac{\tau_{\rm eq}}{(u\cdot p)}\left\{p^\rho\partial_\rho + \frac{m\,\chi}{2}\left(\partial^\rho F^{\phi\kappa}\right)s_{\phi\kappa}\partial^{(p)}_\rho\right\} f_{\rm eq}\right],\end{aligned} \tag{18}$$

where, $\partial^{(p)}_\alpha \equiv \frac{\partial}{\partial p^\alpha}$ is the partial derivative with respect to particle momenta. Anti-particle analogue of δf_1 may be obtained from Eq. (18) by the replacement $f \to \bar{f}$, $\xi \to -\xi$, $\mathfrak{q} \to -\mathfrak{q}$ and, $\chi \to -\chi$.

Substituting the non-equilibrium corrections to distribution functions in Eqs. (16)-(17), we get the following general form of constitutive relations for the currents $X^{\mu_1\cdots\mu_s} \in \{n^\mu, \Pi, \pi^{\mu\nu}, \delta S^{\lambda,\mu\nu}\}$ at first order in gradients

$$\begin{aligned} X^{\mu_1\dots\mu_s} =&\tau_{\rm eq}\Big[\beta^{\mu_1\dots\mu_s}_{X\Pi}\,\theta + \beta^{\mu_1\dots\mu_s\,\alpha}_{Xa}\dot{u}_\alpha + \beta^{\mu_1\dots\mu_s\,\alpha}_{Xn}\left(\nabla_\alpha\xi\right) + \beta^{\mu_1\dots\mu_s\,\alpha\beta}_{XF}\left(\nabla_\alpha B_\beta\right) \\ &+ \beta^{\mu_1\dots\mu_s\,\alpha\beta}_{X\pi}\sigma_{\alpha\beta} + \beta^{\mu_1\dots\mu_s\,\alpha\beta}_{X\Omega}\Omega_{\alpha\beta} + \beta^{\mu_1\dots\mu_s\,\alpha\beta\gamma}_{X\Sigma}\left(\nabla_\alpha\omega_{\beta\gamma}\right)\Big], \end{aligned} \tag{19}$$

where we used the notation: $\theta \equiv \partial_\alpha u^\alpha$, $\dot{X} \equiv u^\alpha\partial_\alpha X$, $\nabla^\mu \equiv \partial^{\langle\mu\rangle}$, $\sigma^{\mu\nu} \equiv \partial^{\langle\mu}u^{\nu\rangle}$ and $\Omega_{\mu\nu} \equiv (\partial_\mu u_\nu - \partial_\nu u_\mu)/2$. The explicit expressions for the tensorial transport coefficients β may be found in Ref. [72]. Here it is sufficient to note that the dissipative currents are affected by various hydrodynamic gradients, including those of magnetic field.

4. Discussion

Based on the above formalism we make some important observations and conclusions:

1. ***Relativistic Barnett and Einstein-de Haas effects.*** Plugging equilibrium distribution functions into Eq. (7) one may show that the equilibrium magnetization tensor reads [72]

$$M^{\mu\nu}_{\rm eq} = a_1\,\omega^{\mu\nu} + a_2\,u^{[\mu}u_\gamma\omega^{\nu]\gamma}. \tag{20}$$

Since in global equilibrium, the spin polarization tensor ω corresponds to the thermal vorticity tensor ϖ [18, 19, 24, 37–39, 52, 54], from Eq. (20) we conclude that the vorticity of the fluid is related to its magnetization. Hence, Eq. (20) leads to relativistic analogs of the well-known Barnett [7] and Einstein-de Haas [6] effects.

2. ***Spin polarization due to the coupling between thermal vorticity and electromagnetic field.*** Using Eq. (13), one may derive the following evolution equation for $\omega^{\mu\nu}$

$$\begin{aligned} \dot{\omega}^{\mu\nu} =& \mathcal{D}^{[\mu\nu]}_\Pi\,\theta + \mathcal{D}^{[\mu\nu]\gamma}_a\dot{u}_\gamma + \mathcal{D}^{[\mu\nu]\gamma}_{\rm n}\left(\nabla_\gamma\xi\right) + \mathcal{D}^{[\mu\nu]\rho\kappa}_B\left(\nabla_\rho B_\kappa\right) \\ &+ \mathcal{D}^{[\mu\nu]\rho\kappa}_\pi\sigma_{\rho\kappa} + \mathcal{D}^{[\mu\nu]\rho\kappa}_\Omega\Omega_{\rho\kappa} + \mathcal{D}^{[\mu\nu]\phi\rho\kappa}_\Sigma\left(\nabla_\phi\omega_{\rho\kappa}\right), \end{aligned} \tag{21}$$

where the tensorial coefficients, $\mathcal{D}$, contain equilibrium quantities, see Ref. [72]. From Eq. (21) we observe that among different gradient terms, there is a coupling of spin polarization tensor to the fluid vorticity represented by Ω. The coefficient $\mathcal{D}_\Omega$ multiplying this term vanishes when the electromagnetic field is absent which implies that the conversion between spin polarization and vorticity proceeds via coupling with electromagnetic field.

3. ***Dissipative gradient terms.*** Demanding the positivity of the divergence of the entropy current (given by the Boltzmann H-theorem) one can show that only the following gradient terms in Eqs. (19) are dissipative

$$\Pi = -\zeta\theta, \qquad n^\mu = \kappa^{\mu\alpha}\left(\nabla_\alpha\xi\right), \qquad \pi^{\mu\nu} = \eta^{\mu\nu\alpha\beta}\sigma_{\alpha\beta}, \tag{22}$$

$$\delta S^{\mu,\alpha\beta} = \Sigma^{\mu\alpha\beta\lambda\gamma\rho}\left(\nabla_\lambda\omega_{\gamma\rho}\right), \tag{23}$$

where, comparing Eq. (19) and Eqs. (22)-(23), the dissipative transport coefficients read: $\zeta = -\tau_{\rm eq}\beta_{\Pi\Pi}$, $\kappa^{\mu\alpha} = \tau_{\rm eq}\beta^{\langle\mu\rangle\alpha}_{nn}$, $\eta^{\mu\nu\alpha\beta} = \tau_{\rm eq}\beta^{\langle\mu\nu\rangle\alpha\beta}_{\pi\pi}$ and $\Sigma^{\lambda\mu\nu\alpha\beta\gamma} = \tau_{\rm eq}B^{\lambda,[\mu\nu]\alpha\beta\gamma}_\Sigma$.

5. Summary and outlook

In this work, we reviewed a recently developed framework of relativistic dissipative non-resistive magnetohydrodynamics for spin-polarized particles. Using the relativistic kinetic equation for the distribution function in an extended phase space of space-time position, momentum, and spin with the kinetic kernel treated in the relaxation time approximation, we calculated equations of motion for dissipative currents at first-order in gradients. The resulting equations of motion contain various transport coefficients, both dissipative and non-dissipative, which were distinguished using the positivity of the entropy production law. We have shown the emergence of the coupling between the magnetization and the vorticity of the fluid, which constitutes a mechanism leading to relativistic analogs of the Einstein-de Hass and Barnett effects. Furthermore, our analysis reveals that the relationship between magnetic fields and spin polarization occurs at the gradient level. In the context of relativistic heavy-ion collisions, our model offers a new perspective on explaining the splitting of the polarization signal for Λ and anti-Λ particles commonly attributed to the interaction between the magnetic field and the intrinsic magnetic moments of the emitted particles.

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PoS(SPIN2023)241

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