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Vector meson spin alignments in high energy reactions

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The global vector meson spin alignment has been reported recently by the STAR Collaboration at the Relativistic Heavy Ion Collider (RHIC) in Brookhaven National Laboratory (BNL). The results not only show that the global polarization effect in non-central heavy ion collisions also exhibits itself in vector meson polarization but also reveal that strong quark spin correlations may exist in the quark matter system, the quark gluon plasma (QGP), produced in the collision processes thus provide new insights in properties of QGP. It shows in particular that the vector meson spin alignment provides a unique opportunity to study the quark spin correlation in the system. Besides the global spin alignment, vector meson spin alignments in the helicity basis have been also observed in many other high energy reactions such as e^+e^-, e^-p and pp collisions many years ago. The results seem to depend strongly on the hadronization mechanism and comprehensive studies become desiring and necessary. The aim of this invited contribution is to present a brief summary of available experimental data and theoretical results in different hadronization mechanisms to stimulate future comprehensive studies.

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1. Introduction

The global polarization effect (GPE) of the quark gluon plasma (QGP) produced in non-central relativistic heavy ion collisions has been predicted [1, 2] in 2004 and confirmed by STAR beam energy scan experiments at phase I (BES I) on Λ-hyperon polarization [3]. The STAR paper was published in 2017 as a cover paper in Nature and attracted much attention in this field [4]. Five years after that publication [3], the STAR Collaboration published their measurements [5] again in Nature on the global vector meson spin alignment with high statistics. The results provide another surprise and bring the studies to another climax recently.

The STAR results [5] on the one hand showed that GPE exhibits itself also on vector meson polarization, just as predicted in [2]. On the other hand, the results seem to be in contradiction to the theoretical results in [1, 2] together with those on Λ-hyperon polarization obtained by STAR in [3]. This is because according to the predictions in [2], the Λ-polarization equals to that of quark while the vector meson spin alignment should be an effect proportional to the square of quark polarization thus much smaller than those observed in STAR experiments [5]. Such results have definitely reveal deep insights into properties of the quark matter system, the QGP, produced in the collision processes hence attracted much attention both experimentally and theoretically [6–14]. Theoretical interpretations have been proposed and new measurements are underway [6–14].

We note that in addition to such studies in relativistic heavy ion collisions, vector meson spin alignments have also been measured in other high energy reactions such as e^+e^-, e^-p and pp collisions many years ago [15–19]. The measurements in those cases were mostly in the helicity frame and the results obtained show quite different features and also attracted much theoretical efforts to describe them [20–27]. These studies seem to be quite different from those on GPE in relativistic heavy ion collisions due to different hadronization mechanisms. The physical meaning behinds them seem even unclear. It is therefore important to bring the studies in all cases together and compare with each other. A summary of these experimental results and theoretical studies is therefore desirable and is in preparation by the speaker and collaborators [28].

The aim of this talk is to present a very brief summary along this line. I would like to briefly summarize the experimental results available from different high energy reactions and discuss the theoretical approaches in two different hadronization mechanisms correspondingly. At the end, I present a short summary and outlook.

2. The global vector meson spin alignment in relativistic heavy ion collisions

The polarization of particles produced in high energy reaction is described by the spin density matrix $\hat{\rho}$. For particles with spin-1/2 such as quarks and anti-quarks, $\hat{\rho}^q$ is a 2×2 Hermitian matrix that can be expanded as

$$\hat{\rho}^q = \frac{1}{2}(1 + \vec{P}_q \cdot \vec{\sigma}), \tag{1}$$

where $\vec{\sigma}$ is the Pauli matrix, $\vec{P}_q = \mathrm{Tr}(\vec{\sigma}\hat{\rho}_q)$ is the polarization vector and is the average of spin in the rest frame of quarks.

For particles with spin-1 such as the vector mesons, the spin density matrix is a 3×3 Hermitian matrix. Choosing a quantization axis, the matrix can be written explicitly in the spin basis $|jm\rangle$,

$$\hat{\rho}^V = \begin{pmatrix} \rho_{11} & \rho_{10} & \rho_{1-1} \\ \rho_{01} & \rho_{00} & \rho_{0-1} \\ \rho_{-11} & \rho_{-10} & \rho_{-1-1} \end{pmatrix}. \tag{2}$$

The diagonal component ρ_{00} is referred as the spin alignment and the spin of vector meson is called aligned if $\rho_{00} \neq 1/3$.

It was pointed out in [1] that in non-central heavy ion collisions, the colliding system possesses a huge angular momentum. Due to QCD spin orbital interaction, such huge angular momentum lead to global polarizations of quarks and anti-quarks produced in the collision process. After hadronization, such global quark polarization leads to global hyperon polarization and global vector meson spin alignment [1, 2].

It is clear that the global hadron polarization depends not only on the global quark polarization but also on the hadronization mechanism. In relativistic heavy ion collisions, it is envisaged that a system consisting of a large number of quarks and anti-quarks is created in the central rapidity and moderate transverse momentum region. Different aspects of experimental data suggest that hadronization of this system proceeds via combination of quarks and/or anti-quarks. This mechanism is phrased as "quark re-combination", or "quark coalescence" or simply as "quark combination". We simply refer it as "the quark combination mechanism" in the following of this paper. Such a hadronization mechanism was used [1, 2] to calculate the global hadron polarization and we summarize the results in the following of this section.

2.1 The theoretical prediction in non-relativistic quark combination models

The simplest case is the non-relativistic quark combination model where we have $q_1 + \bar{q}_2 \to V$. In this case, the physical significance can be demonstrated in the clearest way. Here, it is envisaged that a quark and an anti-quark combine with each other to form a vector meson and spin of the vector meson is just equal to the sum of that of the quark and that of the anti-quark. Hence the spin density matrix and the spin alignment of the vector meson can be calculated from that of quarks and anti-quarks. Such a calculation is straightforward and was carried out in [2]. We summarize the results obtained there [2] in the following.

In [2], the global quark polarization was taken as a constant so that the spin density matrix of quarks and anti-quarks take the diagonal form, i.e.,

$$\hat{\rho}^q = \frac{1}{2}\begin{pmatrix} 1+P_q & 0 \\ 0 & 1-P_q \end{pmatrix}. \tag{3}$$

The spin density matrix of $q_1\bar{q}_2$ systems was taken as the direct product of that of quarks with that of anti-quarks,

$$\hat{\rho}^{q_1\bar{q}_2} = \hat{\rho}^{q_1} \otimes \hat{\rho}^{\bar{q}_2}. \tag{4}$$

The spin density matrix $\hat{\rho}^V$ of vector mesons composed of $q_1\bar{q}_2$ is obtained from $\hat{\rho}^{q_1\bar{q}_2}$ by making the projection to the coupled spin basis $|jm\rangle$, i.e.,

$$\rho^V_{mm'} = \langle jm'|\hat{\rho}^{q_1\bar{q}_2}|jm\rangle, \tag{5}$$

which leads to

$$\rho^V_{m'm} = \sum_{m_i m_i'} \rho^{q_1\bar{q}_2}_{m_i' m_i} \langle j_V m' | m_1' m_2' \rangle \langle m_1 m_2 | j_V m \rangle, \tag{6}$$

where $|j_V m\rangle$ is the spin wave function of V in the constituent quark model. $j_V = 1$ and $m = 0, \pm 1$; and $\langle j_V m | m_1 m_2 \rangle$ is the Clebsch-Gordon coefficient. After the straight forward calculations, we obtain the normalized spin alignment ρ^V_{00} as [2],

$$\rho^V_{00} = \frac{1 - P_{q_1} P_{\bar{q}_2}}{3 + P_{q_1} P_{\bar{q}_2}}. \tag{7}$$

If we take $P_{q_1} = P_{\bar{q}_2} = P_q$ as flavor independent and is the same for quarks and anti-quarks, we simply obtain,

$$\rho^V_{00} = \frac{1 - P_q^2}{3 + P_q^2}. \tag{8}$$

In exact the same way, we obtain the global hyperon polarization $P_H = P_q$ [1].

From Eqs. (7) and (8), we see that, in contrast to the hyperon polarization P_H, the global vector meson spin alignment ρ^V_{00} obtained in quark combination is a quadratic effect of P_q and should be less than $1/3$. We also see that the case considered in [1, 2] is the simplest example where we considered only the spin degree of freedom of quarks, we neglect the fluctuation, the dependence on other degree(s) of freedom, or the spin correlation of quarks and/or anti-quarks.

2.2 The global vector meson spin alignment and quark spin correlations in QGP

The simple case considered in [1, 2] could be over simplified. In this sense, it might not be a surprise to see that the STAR data [3, 5] show that the global vector meson spin alignment of ϕ mesons deviates largely from the square effect of P_Λ. In fact, if we make a small step forward by considering the dependence of P_q on other degree of freedom [10–12, 14], $P_{q_1} P_{\bar{q}_2}$ in Eq. (7) should be replaced by $\langle P_{q_1} P_{\bar{q}_2} \rangle$, i.e.,

$$\rho^V_{00} = \frac{1 - \langle P_{q_1} P_{\bar{q}_2} \rangle}{3 + \langle P_{q_1} P_{\bar{q}_2} \rangle}. \tag{9}$$

The STAR data [3, 5] indicate that

$$\langle P_{q_1} P_{\bar{q}_2} \rangle \neq \langle P_{q_1} \rangle \langle P_{\bar{q}_2} \rangle. \tag{10}$$

which shows that there should be strong correlations between polarization of quarks and that of anti-quarks. Hence, the study of global vector meson spin alignment in heavy ion collisions provides a unique opportunity to study the correlation between polarization of quarks and that of anti-quarks. It opens a window to study the properties of QGP produced in the collisions.

It also emphasized in [14] that the average shown in Eq. (9) is in fact two folded. It includes first an average inside the vector meson then average over the vector mesons in the system. Hence, the spin correlations can be a local correlation or a long range correlation. More measurements are needed to differentiate between such different correlations [14]. In this connection, off-diagonal elements of the spin density matrix of the vector meson and hyperon-hyperon or hyperon-anti-hyperon spin correlations could be good candidates to study.

We note that the situation is different for hyperon polarization where the same calculations lead to the result that the hyperon polarization is a linear combination of those of quarks of different flavors. The coefficients are constants in non-relativistic quark combination models. Hence we receive no contributions from quark spin correlations in the corresponding averages in this case.

Refs. [10–12] present an example where strong spin correlation between quarks and anti-quarks are obtained if strong interactions before hadronization in QGP manifest itself as mediated via vector meson filed. Other possibilities have also been discussed [6–14].

3. Vector meson spin alignments in quark fragmentation

In the QCD framework on high energy reactions, hadron productions in fragmentation mechanism are described by fragmentation functions (FFs). Polarizations of hadrons are expressed in terms of different FFs [25–27] defined via quark-quark correlators. The results of the complete decomposition of quark-quark correlator for spin-1 hadrons were given in [25]. Measurements have been carried out many years ago at LEP for spin alignments of vector mesons [15–18] in the inclusive production process $e^+e^- \to hX$ and significant effects have been observed. These data have attracted many phenomenological studies [20–27] and parameterizations of the corresponding FFs have been obtained [25–27]. We present the main results in the following.

3.1 Vector meson spin alignments from FFs

For the fragmentation of the quark (or anti-quark), the quark-quark correlator is given by,

$$\hat{\Xi}(k;p,S) = \sum_X \int \frac{d^4\xi}{2\pi} e^{-ik\xi} \langle hX|\bar{\psi}(\xi)\mathcal{L}(\xi;\infty)|0\rangle\langle 0|\mathcal{L}^\dagger(0;\infty)\psi(0)|hX\rangle, \tag{11}$$

where k and p denote the 4-momenta of the quark and the hadron respectively, S denotes the spin of the hadron. $\mathcal{L}$ is the well known gauge link that guarantees the gauge invariance of FFs. In the following, for the sake of explicitness of equations, we simply omit the gauge link in the expressions.

The FFs are obtained from $\hat{\Xi}$ by decomposing it in the following steps. First, we expand the 4×4 matrix in Dirac indices $\hat{\Xi}$ in terms of the Γ-matrices, i.e.,

$$\hat{\Xi} = \frac{1}{2}\Big[\Xi_s + i\gamma_5\tilde{\Xi}_{ps} + \gamma^\alpha\Xi_{V\alpha} + \gamma_5\gamma^\alpha\tilde{\Xi}_{A\alpha} + i\sigma^{\alpha\beta}\gamma_5\Xi_{T\alpha\beta}\Big], \tag{12}$$

where the coefficient functions are real and are Lorentz scalar, pseudo-scalar, vector, axial-vector and tensor respectively. Second, we expand these coefficient functions according to their respective Lorentz transformation properties in terms of the basic Lorentz covariants constructed from basic variables at hand. They are expressed as the sum of the basic Lorentz covariants multiplied by scalar functions. These scalar functions are the FFs.

Clearly, the basic Lorentz covariants that we can construct depend strongly on what basic variable(s) that we have at hand. Besides the 4-momenta p and k, we have the variables describing the spin states. Such variables are different for hadrons with different spins and we thus obtain different results for them. For this purpose, we need to decompose the spin density matrix into known matrices multiplied by Lorentz covariants. For spin-1/2 hadrons, this is given by Eq. (1), and the polarization is described by the polarization vector $S = (0, \vec{S})$. For spin-1 hadrons, the

3×3 density matrix $\hat{\rho}$ is usually decomposed as [29], $\hat{\rho}=\frac{1}{3}(\mathbf{1}+\frac{3}{2}S^i\Sigma^i+3T^{ij}\Sigma^{ij})$, where Σ^i is the spin operator of spin-1 particle, and $\Sigma^{ij}=\frac{1}{2}(\Sigma^i\Sigma^j+\Sigma^j\Sigma^i)-\frac{2}{3}\mathbf{1}\delta^{ij}$. The spin polarization tensor $T^{ij}=\mathrm{Tr}(\hat{\rho}\Sigma^{ij})$ and is parameterized as,

$$\mathbf{T}=\frac{1}{2}\begin{pmatrix} -\frac{2}{3}S_{LL}+S_{TT}^{xx} & S_{TT}^{xy} & S_{LT}^{x} \\ S_{TT}^{xy} & -\frac{2}{3}S_{LL}-S_{TT}^{xx} & S_{LT}^{y} \\ S_{LT}^{x} & S_{LT}^{y} & \frac{4}{3}S_{LL} \end{pmatrix}. \tag{13}$$

We see that, for spin-1 hadrons, besides the polarization vector S, we need a tensor polarization part that has five independent components given by a Lorentz scalar S_{LL}, a Lorentz vector $S_{LT}^{\mu}=(0,S_{LT}^x,S_{LT}^y,0)$ and a Lorentz tensor $S_{TT}^{\mu\nu}$ that has two nonzero independent components $S_{TT}^{xx}=-S_{TT}^{yy}$ and $S_{TT}^{xy}=S_{TT}^{yx}$. The spin alignment ρ_{00} is determined by S_{LL} as $\rho_{00}=(1-2S_{LL})/3$.

In the one-dimensional case, we integrate over k^- and $\vec{k}_\perp$ and obtain,

$$\hat{\Xi}(z;p,S)=\sum_X\int\frac{d\xi^-}{2\pi}e^{-ik^+\xi^-}\langle hX|\bar{\psi}(\xi^-)|0\rangle\langle 0|\psi(0)|hX\rangle, \tag{14}$$

where $z\equiv p^+/k^+$. After the Lorentz decomposition, we obtain terms related to S_{LL} as,

$$z\Xi_\alpha(z;p,S)=p^+\bar{n}_\alpha[D_1(z)+S_{LL}D_{1LL}(z)]+\text{power suppressed terms}. \tag{15}$$

We can obtain the expression for $D_1(z)+S_{LL}D_{1LL}(z)$ by reversely solving Eqs. (14) and (15),

$$D_1(z)+S_{LL}D_{1LL}(z)=\sum_X\int\frac{zd\xi^-}{2\pi p^+}e^{-ik^+\xi^-}\langle hX|\bar{\psi}(\xi^-)\gamma^+|0\rangle\langle 0|\psi(0)|hX\rangle, \tag{16}$$

From Eqs. (16), we see clearly that, similar to the well-known unpolarized FF $D_1(z)$, D_{1LL} is independent on the polarization of the initial quark because it is a sum of the spin states of that quark., i.e.,

$$D_1(z)+S_{LL}D_{1LL}(z)=\sum_X\int\frac{zd\xi^-}{2\pi p^+}e^{-ik^+\xi^-}\sum_{\lambda_q=L,R}\langle hX|\bar{\psi}_{\lambda_q}(\xi^-)\gamma^+|0\rangle\langle 0|\psi_{\lambda_q}(0)|hX\rangle, \tag{17}$$

where $\lambda_q=L$ or R denotes the helicity (or chirality) of the quark, and $\psi_{L/R}=(1\pm\gamma_5)\psi/2$ respectively. To compare, we show the result for G_{1L} that describe the longitudinal spin transfer in the fragmentation process

$$z\tilde{\Xi}_\alpha(z;p,S)=\lambda p^+\bar{n}_\alpha G_{1L}(z)+\text{power suppressed terms},$$

$$\lambda G_{1L}(z)=\sum_X\int\frac{zd\xi^-}{2\pi p^+}e^{-ik^+\xi^-}[\langle hX|\bar{\psi}_L(\xi^-)\gamma^+|0\rangle\langle 0|\psi_L(0)|hX\rangle-\langle hX|\bar{\psi}_R(\xi^-)\gamma^+|0\rangle\langle 0|\psi_R(0)|hX\rangle].$$

which depends on the spin of the initial quark explicitly.

The result given by Eq. (17) shows a very important conclusion, i.e., the spin alignment $\rho_{00}=(1-2S_{LL})/3$ for vector meson produced in the fragmentation process $q\to VX$ is determined by D_{1LL} and is independent on the initial polarization of the quark. The conclusion was rather unexpected because the vector meson spin alignment in high energy reactions was first observed in $e^+e^-\to VX$ at LEP [15–18] where the initial quarks and anti-quarks are longitudinally polarized. This is however consistent with space reflection in fragmentation processes where ρ_{00} is space reflection invariant while helicity of the initial quark changes the sign. This conclusion is rather solid since it follows from the general principles of QCD. It can also be tested easily in experiments. In the following we present numerical results for $e^+e^-\to VX$ and $pp\to VX$ as examples.

3.2 Vector meson spin alignments in $e^+e^- \to VX$

Suppose that the quark fragmentation mechanism dominates hadron productions in e^+e^- annihilation at high energies, we can calculate the vector meson alignment for $e^+e^- \to VX$ by extracting the spin-dependent FF $D_{1LL}(z)$ from data available at a given scale and evolve to other scales using the DGLAP QCD evolution equation [32–37]. Such calculations for vector meson spin alignments have been carried out for the first time in [26, 27] by fitting data are available [15–17]. As a comparison, parameterizations of G_{1L} for Λ are also given therein [26, 27] by fitting the data at LEP [30, 31]. Here, we show the fitted results obtained there [26, 27] in Figs. 1 and 2 respectively.

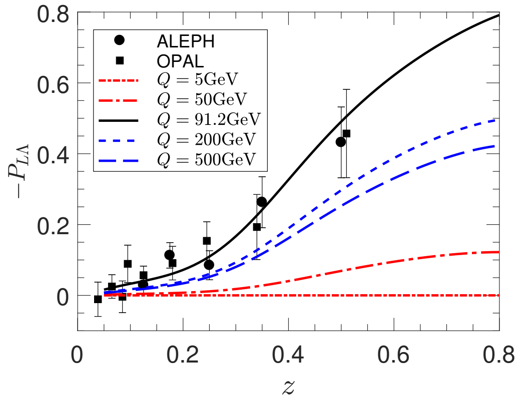

Figure 1: Longitudinal polarization of Λ in $e^+e^- \to \Lambda X$ at high energies. The LEP data are taken from [30, 31]. The solid line is the fit obtained in [26] at LEP energy while those at other energies are calculated results using DGLAP for FFs and energy dependence of P_q. The figure is taken from [26].

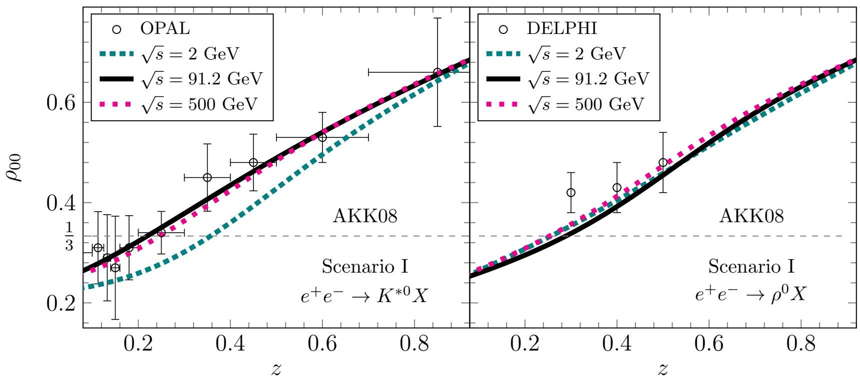

Figure 2: The spin alignments of K^{*0} and ρ^0 in $e^+e^- \to VX$ at the Z-pole fitted in [27] compared with experimental data [15, 16]. The solid line is the fit in [27] at LEP energy while those at other energies are calculated results using DGLAP for FFs. The figure is taken from [26].

From Figs. 1 and 2, we see a distinct feature that there is a strong energy dependence for $P_{L\Lambda}$, whereas that for $\rho_{00}^{K^*}$ is quite weak. The former comes mainly from the energy dependence of P_q while the latter comes mainly from QCD evolution of FFs. In contrast, ρ_{00} changes with Q quite weakly and remains sizable even at lower energies. The relatively rapid change in the energy region around M_Z comes from the influence of relative weight of different quark flavors. This is a clear prediction that can be tested by future experiments.

It is also interesting to see that the LEP data [30, 31] suggest that vector meson spin alignment in fragmentation in the helicity basis is larger than $1/3$ in particular at large z. This indicates that there should a strong spin correlation between the quark and anti-quark that combine with each other to form the vector meson. This is similar to the global spin alignment in heavy ion collisions.

3.3 Vector meson spin alignments in $pp \to VX$

Assuming the universality of FFs, one can calculate vector meson spin alignments in other high energy reactions where the fragmentation mechanism dominates. This applies to hadron production in pp collisions at high transverse momentum p_T and deeply inelastic scattering as well. In [27], such calculations have been carried out for pp collisions. As examples of the results obtained there, we show in in Fig. 3 the results for K^{*0} and ρ^0 in two rapidity regions as functions of p_T.

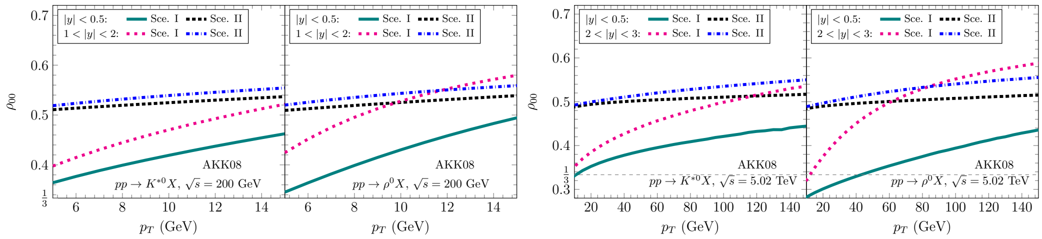

Figure 3: (Color online) Spin alignments of vector mesons in pp collisions at RHIC energy $\sqrt{s} = 200$ GeV and the LHC energy $\sqrt{s} = 5.02$ TeV for K^{*0} and ρ^0 in two rapidity regions as functions of p_T. This figure is taken from [27].

We see that ρ deviates from $1/3$ significantly showing large vector meson spin alignments. Such results can be tested in experiments at RHIC and LHC.

4. Summary and discussions

Vector meson spin alignments have been observed in high energy reactions [5, 15–19]. There are data available are divided into two classes: global vector meson spin alignments in heavy ion collisions [5] and vector meson spin alignments in helicity basis in e^+e^- annihilation into hadrons [15–18]. These data show different features suggesting that vector meson spin alignments are quite different for hadrons produced in different hadronization mechanisms.

In high energy heavy ion collisions in central rapidity and moderate transverse momentum region, hadron productions are dominated by quark combination mechanism. The global vector meson spin alignment in this case not only depends strongly on the polarization of quarks and anti-quarks but also is very sensitive to the spin correlation between them.

In e^+e^- annihilation at high energies, hadron productions are dominated by fragmentation mechanism and are described by fragmentation functions. It has been shown that vector meson spin alignment in the helicity basis in quark fragmentation is independent of the polarization of the initial quark. Parameterizations of corresponding fragmentation functions are obtained and predictions on vector meson spin alignments in different reactions are made [25–27].

It is rather impressive to see the distinct differences and similarities between the global vector meson spin alignment in non-central relativistic heavy ion collisions and those in the helicity basis for mesons produced in the fragmentation mechanism. If we adopt the picture that spin of the vector meson equals to the sum of the spin of the quark and that of the anti-quark, the spin alignment should be determined by the spin correlations between the quark and anti-quark. It is interesting to see whether they have similar mechanisms leading to such spin correlations in different cases. It is also very important to extend the measurements to different high energy reactions at different energies to test the universality of such properties. Such studies provide new insights into properties of QGP and hadronization mechanisms.

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