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Application of Spin Canceling Cell Design to Several Lattices

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We have previously developed a design approach to minimize the intrinsic spin resonance for lattice. This approach involves ensuring the cancelation of spin kicks due to quadrupoles between spin precessing dipole magnets. We apply this approach to the AGS-Booster, future FCC-hh and FCC-ee Booster rings.

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1. Introduction

A standard FODO cell contains focusing and defocusing quadrupoles with a dipole placed between them. In this situation there is no possibility of cancellation of the spin kick between dipoles. Another type of FODO cell includes the dipoles on either end of the focusing and defocusing string of quadrupoles.

We can estimate the contributions to intrinsic spin resonances by considering the terms of the vertical betatron spin resonance integral [1]

$$w_{\kappa_0} = \lim_{N_T\to\infty} \frac{-1}{2\pi N_T} \int_0^{LN_T} \left[(1+a\gamma)(y'' + \frac{iy'}{\rho}) - i(1+a)(\frac{y}{\rho})' \right] e^{i\kappa_0\theta(s)} \mathrm{d}s. \tag{1}$$

Here L is the lattice length and N_T the number of turns. This equation makes use of the standard Frenet-Serret coordinate system for y and s and θ is the accumulated bending angle with κ_0 the spin resonance tune and a the gyromagnetic anomaly, ρ being the design horizontal bending radius which is large with respect to the orbital coordinates. In this equation y can represent both vertical motion due to betatron motion as well as due to orbit imperfections. It assumes that the spin closed orbit ($\hat{n}_0$) is in the vertical direction since such circular accelerators are dominated by the dipole field which in the absence of strong perturbing fields like spin rotators or snakes set the unperturbed spin precessing axis in the vertical direction.

The dominant term in Eq. 1 is,

$$\int y'' e^{i\kappa_0\theta(s)} ds = \sum_n k l_n y. \tag{2}$$

Here we have expanded the integral into a sum of the contributions from each element in the lattice indexed by n, using the thin lens approximation. Thus the subscript n denotes the values at each n^{th} element. Considering vertical betatron motion y_β we obtain,

$$\sum_n k l_n y_\beta = \sum_n k l_n \sqrt{\beta_n} \cos(\mu_n + \phi) e^{i\kappa_0\theta_n}. \tag{3}$$

Here β_n is the vertical betatron function, μ_n the betatron phase and kl_n the integrated quadrupole normalized gradient and ϕ the initial betatron oscillation phase. A sufficient condition for the cancellation of the intrinsic resonance is if,

$$\begin{aligned} 0 &= \sum_m k l_m \sqrt{\beta_m} \cos(\mu_m) \\ 0 &= \sum_m k l_m \sqrt{\beta_m} \sin(\mu_m). \end{aligned} \tag{4}$$

Here the sum indexed by m is only over a single string of magnets between two dipoles. The spin precessing terms only change in the dipole, so between dipoles they are common and can be factored out. Looking first at the FODO cell example,

$$\text{QF O QD O}. \tag{5}$$

Here QF and QD are focusing and defocusing quads respectively and O is a drift. In this case Eq. 2 would become,

$$\begin{aligned} 0 &= kl_f\sqrt{\beta_f}\cos(\mu_f) + kl_d\sqrt{\beta_d}\cos(\mu_d) \\ 0 &= kl_f\sqrt{\beta_f}\sin(\mu_f) + kl_d\sqrt{\beta_d}\sin(\mu_d) \end{aligned} \tag{6}$$

Since we may always set $\mu_f = 0$ it means that the 2nd equation of Eq. 6 would contain only one term, namely $kl_d\sqrt{\beta_d}\sin(\mu_d)$. For this term to be zero either kl_d or β_d would have to be zero or $\sin(\mu_d)$ which implies that $\mu_d = \pi$. In other words the phase advance across the whole cell would be 2π. So our cell will have no defocusing quad, zero beta function at the quad or have an infinitely large beta function in the cell.

Thus using only two quads per cell wouldn't allow us to construct a viable intrinsic spin resonance canceling cell. Introducing a third quadrupole between the dipoles yields,

$$M = \mathrm{QF}_1 \;\; \mathrm{O} \;\; \mathrm{QD} \;\; \mathrm{O} \;\; \mathrm{QF}_2 \;\; \mathrm{O} \tag{7}$$

In this case Eq. 4 would become,

$$\begin{aligned} 0 &= kl_1\sqrt{\beta_1}\cos(\mu_1) + kl_2\sqrt{\beta_2}\cos(\mu_2) + kl_3\sqrt{\beta_3}\cos(\mu_3) \\ 0 &= kl_1\sqrt{\beta_1}\sin(\mu_1) + kl_2\sqrt{\beta_2}\sin(\mu_2) + kl_3\sqrt{\beta_3}\sin(\mu_3). \end{aligned} \tag{8}$$

2. Accelerator Examples

We explore a few types of accelerators and how they might benefit using these design approaches. We first consider the AGS's Booster, next we look at toy lattices on the scale of the proposed Future Circular Collider (FCC). The key parameters for BNL's AGS Booster are shown in Table 1. The Booster has 36 2.4 m long dipoles with a bending radius of 13.75 m. Each arc FODO cell is 8.4075 m, and quadrupoles are 0.50375 m long. The total circumference is 201.78 m with six super-periods made up of four FODO cells per period, two of which are missing a dipole.

In our test case we inserted three quadrupoles families into each half cell and increased the arc cell length from 8.4075 to 9.5 m to accommodate the new quadrupoles, yielding a circumference of 228m. In this case we first minimized Eq. 8 for each cell type using the MADX Simplex optimizer over a total of 12 families of quadrupoles (three for each of the 4 cells). We did this while controlling the maximum beta functions and dispersion. Next we used the DEPOL algorithm in python to minimize the spin resonance contribution for each cell again controlling the maximum beta functions and dispersion. The results of the quadrupole strengths and twiss parameters for each case are listed in Table 1 and the spin resonances to $a\gamma < 10$ are plotted in Fig. 1c. In all cases we use a standard normalized emittance of 10 mm-mrad for the calculation of the intrinsic spin resonances. Optics for the thin lens optimized lattice is shown in Fig. 1a. The MADX optimization using the thin lens approximation brought Eq. 8 to less than 10^{-8} and significantly reduced the resonance strength, yet we are still left with residual resonance amplitude. Optimization using the full spin resonance integral as calculated in DEPOL improved upon this as shown in Fig. 1b.

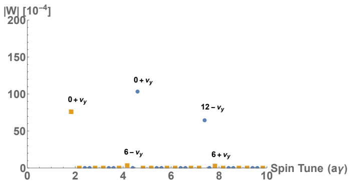
Table 1: Comparison of AGS Booster lattice designs

	Current	Modified thin lens Optimized	Modified DEPOL Optimized
Bending Radius [m]	13.75		
Peak Dipole Field [T]	1.27		
Peak Possible $a\gamma$	10		
Circumference [m]	201.78	228	
Cell length [m]	8.4075		
Quad length [m]	.50375		
No. of Quads	48	144	
Q_x	4.62	4.62	4.9835
Q_y	4.6	1.82	0.739
BetaX MAX [m]	13.3	26.11	60.42
BetaY MAX [m]	13.5	12.16	12.31
Dx MAX [m]	3.95	6	5.6

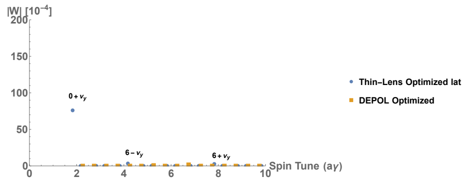
In addition to lengthening the arc cells by about 1 m an additional 4 quadrupoles would need to be added per arc cell or 96 with a maximum peak field of about 15 T/m, while this is technically feasible it is costly. Another approach would be to build a 16 super-periodic lattice around the 36 dipoles and 48 quadrupoles.For this lattice if we kept the vertical tune below the a factor of 1.7928, it would place all the spin resonances outside of the energy range of the Booster. This is because there would be no $0+\nu$ spin resonance since the spin tune could never be lower than a. The next spin resonance would occur no lower than $a\gamma = 16 - a$=14.2072. This wouldn't require satisfying Eq. 8 and one could use traditional FODO cells. To achieve this only hinges on boosting the periodicity to 16, keeping the tune below a and maintaining enough drift space for the RF systems, sextupoles and correctors.

The FCC-hh is proposed which will accelerate protons to energies from 3 TeV using the LHC as a booster to 50 TeV. The rough optics are shown in Fig. 2a for the arc FODO cell and in Table 2 the parameters for a simplified arc only model (no insertion sections) is shown. This simplified model only has dipoles and quadrupoles using the same bending radius, magnet lengths of the dipoles and quadrupoles as defined in "FCC-hh: The Hadron Collider Future Circular Collider Conceptual Design Report Volume 3" [2].

The energy range from 3 to 50 TeV in $a\gamma$ space is 5732 to 95532. In this case a periodicity high enough to avoid spin resonances is not reasonable. However the arc cells could be constructed with three quadrupoles between the dipoles to achieve a major suppression of the spin resonances. In this case using one focusing and 2 defocusing quadrupoles placed in a 1/2 arc cell of 127.23 m long with a 42.58 degree phase advance in the horizontal and 15.48 degree in the vertical plane kept all intrinsics resonances in the energy range less than 1 at 10 mm-mrad normalized emittance. This is low enough that six snakes should be sufficient to control the intrinsic spin resonances. The optics for one arc half cell are plotted in Fig. 2b and the parameters are listed in Table 2. In this case the periodicity is two times higher since our repeating period is from dipole to dipole and the insertion of two more quads increased the path length due to the arc to 99km. This compares to an FCC-hh lattice which following a standard FODO lattice design using an arc cell of 213 m with



(a)



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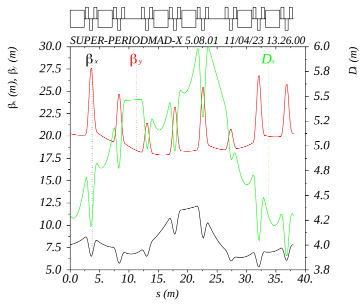


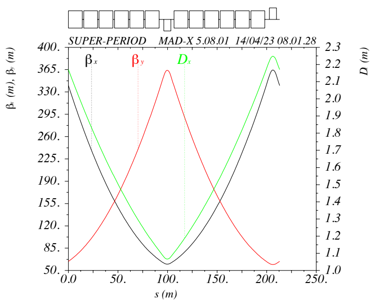
(c)

Figure 1: (a) DEPOL calculated spin resonances using vertical emittance of 10 mm-mrad with the lattice employing the special intrinsic spin resonance suppressing cell. This is compared to a standard FODO cell with dipoles in-between BNL's Booster lattice. Here the strongest intrinsic spin resonances are labeled.(b) Here the thin-lens optimized optics using Eq. 8 and optics optimization using DEPOL algorithm are compared. The strongest intrinsic spin resonances are labeled. (c) Beta functions for horizontal and vertical planes and dispersion in the horizontal plane are shown for one super-period of the new Booster lattice. In this case it is optimized using Eq. 8.

Table 2: FCC-h Arc only lattice comparison

	Standard	3 Quad Optimized
Circumference (arc only) [km]	83.081	99.239
Cell length [m]	213.03	254.46
Periodicity	390	780
Q_x	99.83	92.35
Q_y	100.57	33.71
BetaX MAX [m]	355.5	214.79
BetaY MAX [m]	354.99	485.60
Dx MAX [m]	2.22	2.05
Arc packing factor	0.78	0.66
Peak Quadrupole gradient [T/m]	320[1]	610[2]

an arc path length of 83km, would give resonances greater than 15 at 10 mm-mrad normalized emittance. In Fig. 2c we can see a comparison between the two FCC-hh lattice's intrinsic spin resonance strength. The three quadrupole lattice would also require effectively doubling the peak strength for the quadrupole which might require increasing the length of the quadrupole if the peak fields couldn't be raised. Additionally this construction would reduce the packing factor from 0.78 to 0.66.



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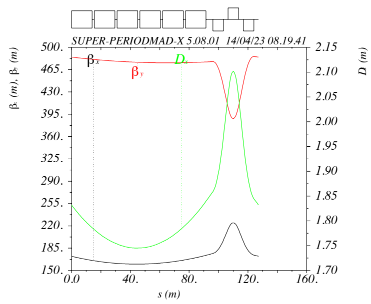

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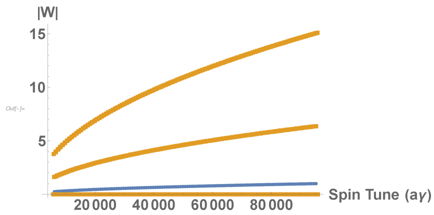

(c)

Figure 2: (a) Beta functions for horizontal and vertical planes and dispersion in the horizontal plane are shown for one FODO cell of the arc FCC-h lattice (per 2019 CDR). (b) Optics shown for one Dipole to Dipole (similar to half standard FODO cell) of my Toy arc only FCC-h lattice. (c) DEPOL calculated spin resonances for a FCC-hh arc only (no insertions) lattice using standard FODO cell construction versus an FCC-hh like lattice with three quad arc cells which suppress the spin resonances.

3. ACKNOWLEDGMENTS

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[2] FCC collaboration, *FCC-hh: The Hadron Collider: Future Circular Collider Conceptual Design Report Volume 3*, *Eur. Phys. J. ST* **228** (2019) 755.